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Brief paper

Distributed control for uniform circumnavigation of ring-coupled unicycles[☆]

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ABSTRACT

The paper studies the general circumnavigation problem for a team of unicycle-type agents, with the goal of achieving specific circular formations and circling on different orbits centered at a target of interest. A novel distributed solution is proposed, in which the control laws are heterogeneous for the agents such that some agents are repellant from the target while attractive to its unique neighbor and some agents are attractive to the target while repellant from its neighbor. A systematic procedure is developed to design the control parameters according to the specific radii of the orbits and the formations that the agents are desired to converge to. Moreover, this control scheme uses a minimum number of information flow links between the agents and local measurements of relative position only. Based on the block diagonalization of circulant matrices by a Fourier transform, asymptotic convergence properties are analyzed rigorously. The validity of the proposed control algorithm is also demonstrated through numerical simulations.

1. Introduction

The circumnavigation problem, in which the task is to circumnavigate a target of interest by one agent or a network of autonomous agents, has many applications in security and surveillance (Shames, Dasgupta, Fidan, & Anderson, 2012), satellite formation flying (Milam, Petit, & Murray, 2001), source seeking (Moore & Canudas-de Wit, 2010), etc. In a space mission, a formation made up of numerous micro satellites is desired to orbit around a large satellite for the tasks of monitoring, maintenance and repairing, in which the fundamental issue is the autonomous formation by the micro satellites and also the relative orbit formation of these micro satellites with respect to the large satellite. In a security and surveillance mission, it is often required to have a single or a group of agents to circle around a target or an area of interest for monitoring or gathering information. Depending on different applications, there may be different control specifications, but in general there is a fundamental requirement of achieving circular formations on specific orbits around a target of interest.

Many circumnavigation algorithms have been studied recently in the literature for a single agent. For holonomic agents, a continuous-time nonlinear periodically time-varying algorithm is developed in Deghat, Shames, Anderson, and Yu (2010) and Shames et al. (2012), which adaptively estimates the position of the target and moves the agent to a trajectory encircling it based on the distance or bearing measurement to the target. The bearing-based circumnavigation algorithm is also extended to the unicycle-like agent in Deghat et al. (2012). Moreover, for the unicycle-like agent, a range-only strategy is presented in Matveev, Teimoori, and Savkin (2011) using a sliding mode approach. The study of circumnavigation by a team of autonomous agents has also attracted a lot of attention recently. Compared with the single-agent case, besides circumnavigating around the target on a specific orbit, the agents should also achieve an optimal configuration surrounding the target through group coordination, e.g., to uniform the spacing next to each other. A big challenge here is how to achieve coordination in a distributed way. Some of the early works in this line include (Yamaguchi, 1999) where a group of holonomic mobile agents are used.

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A cyclic pursuit-based strategy is proposed in Kim and Sugie (2007) which achieves uniform circumnavigation by decoupling the target tracking and inter-agent coordination tasks. Moving-target enclosing strategy is proposed in Guo, Yan, and Lin (2010) by Guo et al. For nonholonomic agents, Ceccarelli et al. present a switching control scheme to drive a group of unicycle-type vehicles to achieve circular motions around a static target in Ceccarelli, Di Marco, Garulli, and Giannitrapani (2008). However, this control law does not result in an even spacing formation, which is achieved in Lan, Yan, and Lin (2010) by using a hybrid control strategy. Moreover, there are also a lot of works on circular formations (Chen & Zhang, 2011; Marshall, Broucke, & Francis, 2004, 2006; Sinha & Ghose, 2007). Although in these works the orbit center depends on the agents' initial states rather than the target, the ideas developed are still very helpful in dealing with the circumnavigation problem if the agents also interact with the target.

However, most aforementioned works assume that the goal is to achieve a circular formation on the same orbit for all the agents. But in many applications such as satellite formation flying, the micro satellites may need to stay on different orbits to form a large aperture; In the target enclosing problem, the agents may need to stay on different orbits to perform different missions, for example, the inner agents surveil the target while the outer agents protect the inner ones against possible attacks. In this paper, we study the general circumnavigation problem with provable stability properties for desired circular formations in which autonomous agents can circle on different orbits around the target. A new distributed control strategy is proposed, which combines attraction/repulsion from its pursuing agent as well as the target of interest. The strategy was originally developed in Zheng, Lin, Fu, and Sun (2013) to achieve uniform distribution on the same orbit when circling about a specific target. But in order to solve the coordinated circumnavigation problem on different orbits, this paper generalizes the idea by considering non-identical control laws for the agents. That is, under the proposed control strategy, some agents may be repellant from the target while attractive to the pursuing agent, and some agents may be attractive to the target while repellant from the pursuing agent. We then show how the control parameters are designed in a systematic way according to the specific radii of the orbits and the formations that the agents are desired to converge to. Moreover, we show that among all equilibrium formations achieving uniform circumnavigation, only two of them are asymptotically stable, corresponding to the clockwise or counterclockwise motions around the target. The stability analysis technique is based on the block diagonalization of circulant matrices by a Fourier transform. Simulations are also given to demonstrate the effectiveness of the proposed distributed control strategy to achieve coordinated circumnavigation formation.

The work is a generalization of cyclic pursuit strategies studied in Marshall et al. (2004), Zheng, Lin, and Yan (2009) and cyclic repelling strategies studied in Zheng et al. (2013) so that the combination of attraction and repulsion can be used to realize general uniform circumnavigation around a target of interest on different orbits. Moreover, different from the distributed circumnavigation control laws for multiple unicycles developed in Ceccarelli et al. (2008) and Lan et al. (2010) that are switching over time, the control law in this paper is time-invariant and continuous. In addition, it uses only local information of relative positions and is simple and easily implementable, which is important from the engineering standpoint.

2. Problem formulation

Consider a team of N autonomous unicycle-like agents moving in the plane $\mathbb{R}^2$ and suppose in the plane there is a point-like stationary target or beacon $\mathcal{T}$. Our objective is to make all agents asymptotically converge to a formation while navigating on concentric orbits centered at the target. The task should be carried out based on locally available information which can be obtained by individual agents through onboard sensors, e.g., an omnidirectional camera.

Denote $\mathbb{N} = \{1, 2, \ldots, N\}$. The kinematic model of the unicycle-like agent is described as follows subject to nonholonomic constraints. For each agent $i \in \mathbb{N}$,

$$\dot{x}_i = v_i \cos\theta_i, \qquad \dot{y}_i = v_i \sin\theta_i, \qquad \dot{\theta}_i = \omega_i, \tag{1}$$

where (x_i, y_i) is the Cartesian coordinate of the ith agent in the world frame and θ_i gives the orientation of agent i, also in the world frame. The longitudinal velocity v_i and angular velocity ω_i are the control inputs which we need to design.

To accomplish the mission, we make the following assumptions:

(1) The agents interact each other forming a directed ring with minimum information exchange. That is, agent i detects agent $(i \mod N) + 1$ in its local frame. Throughout the paper, modulo N operation is used to identify agents, i.e., agent $N + i$ is the same as agent i.
(2) The global posture information (x_i, y_i, θ_i) is unavailable. The difference of orientations $\theta_j - \theta_i$ is also unavailable. Each agent i can only measure the relative position $\mu_b^{(i)}$ of the target and the relative position $\mu_+^{(i)}$ of agent $i + 1$ in its own local frame.
(3) Each agent i knows the predefined radius r_i of its circular orbit and the one for its neighbor r_{i+1}, obtained for example, through local communication.

Remark 1. For simplicity, in the following analysis, it is assumed that the target can be detected by all the agents all the time. This assumption, however, can be relaxed in practice. For example, when an agent cannot detect the target, it can just pursue its neighboring agent. If some of the agents detect the target, then eventually all the agents can be steered to the vicinity of the target. In that case, it is reasonable to assume that all the agents can detect the target simultaneously.

Denote by (x_b, y_b) the coordinate of the target in the world frame. The relationship between the relative measurements in local frames and the global coordinate in the world frame is given as

$$\mu_b^{(i)} = R(\theta_i)\begin{bmatrix} x_b - x_i \\ y_b - y_i \end{bmatrix}, \qquad \mu_+^{(i)} = R(\theta_i)\begin{bmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \end{bmatrix},$$

where $R(\theta_i) = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix}$ is the rotation matrix. We define $\psi_i := \operatorname{atan2}(y_{i+1} - y_b, x_{i+1} - x_b) - \operatorname{atan2}(y_i - y_b, x_i - x_b)$ and also let $\psi_i \in [0, 2\pi)$.[3]

Now we are ready to give the formal problem statement for the uniform circumnavigation problem studied in the paper.

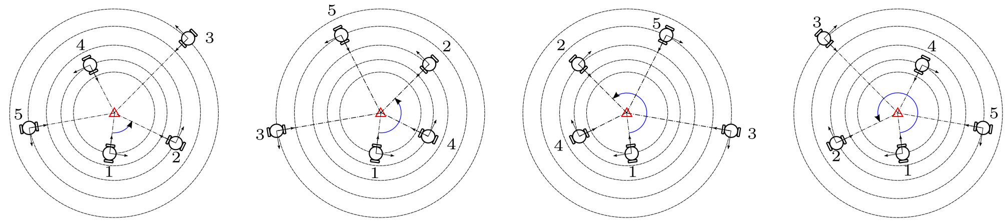
The uniform circumnavigation problem *For given radii $r_1, r_2, \ldots, r_N$, design a local control law of v_i and ω_i for each agent i with model (1) such that*

(1) $\lim_{t\to\infty} \|(x_i, y_i) - (x_b, y_b)\| = r_i,\ i = 1, \ldots, N$,
(2) $\lim_{t\to\infty} \omega_1 = \cdots = \lim_{t\to\infty} \omega_N = \bar{\omega}$,
(3) $\lim_{t\to\infty} v_1/r_1 = \cdots = \lim_{t\to\infty} v_N/r_N = \bar{v}$,
(4) $\lim_{t\to\infty} \psi_1 = \cdots = \lim_{t\to\infty} \psi_N = \bar{\psi}$,

where $\bar{v}$ and $\bar{\omega}$ are some constants and $\bar{\psi} = \frac{2d\pi}{N}$ for some $d \in \{1, 2, \ldots, N - 1\}$.

Remark 2. In the uniform circumnavigation problem, formations are not unique to satisfy the specifications. In general, there are $N - 1$ possible formations because $\bar{\psi}$ can be any value $\frac{2d\pi}{N}$ for $d \in \{1, \ldots, N - 1\}$. See Fig. 1 for an example of all possible four formations of five agents.

[3] The function $\operatorname{atan2}(y, x)$ represents a two-argument arctangent function returning the angle of point (x, y) as a numeric value in $[0, 2\pi)$.



(a) $\bar{\psi} = \frac{2\pi}{5}$ $(d = 1)$. (b) $\bar{\psi} = \frac{4\pi}{5}$ $(d = 2)$. (c) $\bar{\psi} = \frac{6\pi}{5}$ $(d = 3)$. (d) $\bar{\psi} = \frac{8\pi}{5}$ $(d = 4)$.

Fig. 1. Examples of uniform circumnavigation formations.

3. Controller synthesis and analysis

3.1. Controller synthesis

In this paper, we propose and examine the following simple continuous-time control law:

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} k_v & 0 \\ 0 & k_\omega \delta_i \end{bmatrix} \left[(1 - a_i)\mu_+^{(i)} + a_i \mu_b^{(i)} \right]. \tag{2}$$

In the equation above, k_v and k_ω are positive control gains and assumed to be identical for all agents, and δ_i is a positive design parameter used to control the radius of the achieved circumnavigating orbit. Moreover, by setting a_i to be different values, different individual actions are taken:

- When $a_i < 0$, agent i pursues agent $i + 1$ while repelling itself from the target.
- When $a_i = 0$, agent i pursues agent $i+1$ irrespective of the target. Especially, when $a_i = 0$ for all i, the control law (2) becomes the purely cyclic pursuit control law which has been studied in Zheng et al. (2009).
- When $a_i \in (0, 1)$, agent i pursues both the target and agent $i+1$.
- When $a_i = 1$, agent i pursues the target independently and it can be shown that $(x_i, y_i) \to (x_b, y_b)$ as $t \to \infty$.
- When $a_i > 1$, agent i pursues the target while repelling itself from agent $i + 1$.

To solve the uniform circumnavigation problem on different orbits, each agent i needs to use (2) with different a_i and δ_i, which makes the analysis difficult. To overcome the difficulty, in the following we show that by designing a_i and δ_i properly, the control laws can be transformed to be identical. This is done by mapping all the agents to *virtual agents* on the same orbit. Given positive $r_1, r_2, \ldots, r_N$, each agent i's corresponding virtual agent is defined in the following way:

$$\tilde{x}_i = (x_i - x_b)/r_i, \qquad \tilde{y}_i = (y_i - y_b)/r_i, \qquad \tilde{\theta}_i = \theta_i. \tag{3}$$

So through the coordinate transformation (3), agent i is mapped to a virtual agent with respect to the target $\mathcal{T}$.

The following theorem states that the all virtual agents use identical control laws when a_i and δ_i are designed properly.

Theorem 1. *For positive $r_1, r_2, \ldots, r_N$, if we design δ_i and a_i in (2) as*

$$\delta_i = 1/r_i \quad \text{and} \quad a_i = 1 - c\frac{r_i}{r_{i+1}} \tag{4}$$

for a constant c, then each virtual agents i is governed by

$$\frac{d}{dt}\tilde{x}_i = \tilde{v}_i \cos\tilde{\theta}_i, \qquad \frac{d}{dt}\tilde{y}_i = \tilde{v}_i \sin\tilde{\theta}_i, \qquad \frac{d}{dt}\tilde{\theta}_i = \tilde{\omega}_i \tag{5}$$

with identical control law

$$\begin{bmatrix} \tilde{v}_i \\ \tilde{\omega}_i \end{bmatrix} = \begin{bmatrix} k_v & 0 \\ 0 & k_\omega \end{bmatrix} R(\tilde{\theta}_i) \left\{ c \begin{bmatrix} \tilde{x}_{i+1} \\ \tilde{y}_{i+1} \end{bmatrix} - \begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix} \right\}. \tag{6}$$

Proof. Calculating the derivatives of $(\tilde{x}_i, \tilde{y}_i, \tilde{\theta}_i)$ in (3) with respect to time, we obtain

$$\frac{d}{dt}\tilde{x}_i = \frac{v_i}{r_i}\cos\tilde{\theta}_i, \qquad \frac{d}{dt}\tilde{y}_i = \frac{v_i}{r_i}\sin\tilde{\theta}_i, \qquad \frac{d}{dt}\tilde{\theta}_i = \omega_i.$$

Replacing (x_i, y_i, θ_i) with $(\tilde{x}_i, \tilde{y}_i, \tilde{\theta}_i)$ in (2), we obtain

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} k_v & 0 \\ 0 & k_\omega \delta_i \end{bmatrix} R(\tilde{\theta}_i) \left\{ (1 - a_i) r_{i+1} \begin{bmatrix} \tilde{x}_{i+1} \\ \tilde{y}_{i+1} \end{bmatrix} - r_i \begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix} \right\}.$$

When $\delta_i = 1/r_i$ and $a_i = 1 - c\frac{r_i}{r_{i+1}}$, combining the above two equations leads to (5) and (6). ∎

3.2. Boundedness analysis

In this subsection we show how to choose the system parameter c in (4) to guarantee that the trajectories of all (x_i, y_i)'s are bounded.

Let us use $\sec(\cdot)$ to denote the secant function and use $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ to denote the floor and ceiling functions, respectively. The following theorem establishes the boundedness of the agents' trajectories when c lies in certain range.

Theorem 2. *For a group of N agents under control law (2) with two parameters designed as in Theorem 1, their trajectories are ultimately bounded if*

$$c \in \mathcal{I} := \left(\sec\left(\frac{2\lfloor N/2 \rfloor \pi}{N}\right), 1 \right).$$

We prove Theorem 2 by showing that the trajectories of all virtual agents $(\tilde{x}_i, \tilde{y}_i)$'s are bounded. To do that, we define

$$\begin{bmatrix} \xi_i \\ \eta_i \end{bmatrix} := \begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix} + \frac{k_v}{k_\omega} \begin{bmatrix} \cos\tilde{\theta}_i \\ \sin\tilde{\theta}_i \end{bmatrix},$$

which is the relative position of a point with a distance k_v/k_ω ahead of the virtual agent i. Differentiating (ξ_i, η_i) and applying (6) result in

$$\begin{aligned} [\dot{\xi}_i \; \dot{\eta}_i]^T &= k_v \left\{ c \begin{bmatrix} \xi_{i+1} \\ \eta_{i+1} \end{bmatrix} - \begin{bmatrix} \xi_i \\ \eta_i \end{bmatrix} \right\} \\ &\quad + \frac{k_v^2}{k_\omega} \left\{ -c \begin{bmatrix} \cos\tilde{\theta}_{i+1} \\ \sin\tilde{\theta}_{i+1} \end{bmatrix} + \begin{bmatrix} \cos\tilde{\theta}_i \\ \sin\tilde{\theta}_i \end{bmatrix} \right\}. \end{aligned}$$

To describe the system more concisely, let us define $\xi = [\xi_1 \; \cdots \; \xi_N]^T$, $\eta = [\eta_1 \; \cdots \; \eta_N]^T$, $\cos\tilde{\theta} = [\cos\tilde{\theta}_1 \; \cdots \; \cos\tilde{\theta}_N]^T$, and $\sin\tilde{\theta} = [\sin\tilde{\theta}_1 \; \cdots \; \sin\tilde{\theta}_N]^T$. We also use $\text{circ}[A_1 \;\; A_2 \;\; \cdots \;\; A_N]$ to represent a (block) circulant matrix with the first (block) row $[A_1 \;\; A_2 \;\; \cdots \;\; A_N]$(Block circulant matrix will also be used in Section 3.4). It is obtained that

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = -k_v \begin{bmatrix} M & \mathbf{0} \\ \mathbf{0} & M \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \frac{k_v^2}{k_\omega} \begin{bmatrix} M & \mathbf{0} \\ \mathbf{0} & M \end{bmatrix} \begin{bmatrix} \cos\tilde{\theta} \\ \sin\tilde{\theta} \end{bmatrix}, \tag{7}$$

where $M = \text{circ}[1 \quad -c \quad 0 \quad \cdots \quad 0]$.

Proof of Theorem 2. Let $V = \frac{1}{2}(\xi^T\xi + \eta^T\eta)$. Taking the time derivative of V along the solution of (7), we obtain

$$\dot{V} = -k_v \begin{bmatrix}\xi\\ \eta\end{bmatrix}^T \begin{bmatrix}\bar{M} & \mathbf{0}\\ \mathbf{0} & \bar{M}\end{bmatrix}\begin{bmatrix}\xi\\ \eta\end{bmatrix} + \frac{k_v^2}{k_\omega}\begin{bmatrix}\xi\\ \eta\end{bmatrix}^T \begin{bmatrix}M & \mathbf{0}\\ \mathbf{0} & M\end{bmatrix}\begin{bmatrix}\cos\tilde{\theta}\\ \sin\tilde{\theta}\end{bmatrix}$$

where $\bar{M} = \frac{M+M^T}{2} = \text{circ}\begin{bmatrix}1 & -\frac{c}{2} & 0 & \cdots & 0 & -\frac{c}{2}\end{bmatrix}$. According to the properties of circular matrix (Davis, 1994), the eigenvalues of $\bar{M}$ are $\lambda_k = 1 - \frac{c}{2}\left(\varphi_k + \varphi_k^{N-1}\right) = 1 - c\cos\left(\frac{2k\pi}{N}\right)$ with $k = 0, 1, \ldots, N-1$, where $\varphi_k = e^{2j\pi k/N}$. To make $\min_k(\lambda_k) > 0$, it requires that $c \in \mathcal{I}$. Hence we have

$$\dot{V} \le -k_v \min_k(\lambda_k)\begin{bmatrix}\xi\\ \eta\end{bmatrix}^T\begin{bmatrix}\xi\\ \eta\end{bmatrix} + \frac{k_v^2}{k_\omega}\begin{bmatrix}\xi\\ \eta\end{bmatrix}^T\begin{bmatrix}M & \mathbf{0}\\ \mathbf{0} & M\end{bmatrix}\begin{bmatrix}\cos\tilde{\theta}\\ \sin\tilde{\theta}\end{bmatrix}.$$

Then $\dot{V} < 0$ for $\left\|\begin{bmatrix}\xi\\ \eta\end{bmatrix}\right\| > \frac{k_v}{k_\omega}\|M\|\sqrt{N}/\min_k(\lambda_k)$. By the ultimate boundedness result in Khalil (2002, Section 4.8), it follows that the trajectories are bounded, so is the original system. ■

3.3. Equilibrium formations

In this subsection we analyze how to design k_v and k_ω for the local control law (2) to achieve desired circumnavigation formations.

We investigate the collective behaviors of agents under local control laws (2) by looking at the dynamics of v_i, ω_i, and $\beta_i := \theta_{i+1} - \theta_i$. For collective behaviors satisfying the specifications given in the uniform circumnavigation problem statement, it must hold that $\dot{v}_i \equiv 0$ and $\dot{\omega}_i \equiv 0$ for all i, which correspond to uniform circular motions. Also, if all the agents perform concentric circular motions around the target, then the value of β_i becomes equal to the value of ψ_i. Taking derivatives of $[v_i, \omega_i, \beta_i]^T$ and considering the choice of δ_i and a_i as in Theorem 1, we have

$$\begin{aligned}\dot{v}_i &= \frac{r_i}{r_{i+1}}ck_v v_{i+1}\cos\beta_i - k_v v_i + \frac{r_i k_v \omega_i^2}{k_w},\\ \dot{\omega}_i &= \frac{1}{r_{i+1}}ck_\omega v_{i+1}\sin\beta_i - \frac{k_\omega v_i\omega_i}{r_i k_v},\\ \dot{\beta}_i &= \omega_{i+1} - \omega_i.\end{aligned} \tag{8}$$

Let $\chi_i = [v_i, \omega_i, \beta_i]^T$ and we can write each subsystem (8) as

$$\dot{\chi}_i = f_i(\chi_i, \chi_{i+1}). \tag{9}$$

Let $\chi = [\chi_1^T, \chi_2^T, \ldots, \chi_N^T]^T$. Then the overall system can be written as

$$\dot{\chi} = \hat{f}(\chi). \tag{10}$$

For the overall system (10), it is not very hard to verify that

$$\{\chi | v_i = r_i\bar{v}, \omega_i = \bar{\omega}, \beta_i = \bar{\beta}, \forall i\} \tag{11}$$

with $\bar{v} = \frac{\bar{\omega}^2}{k_\omega(1 - c\cos\bar{\beta})}$, $\bar{\omega} = k_v c\sin\bar{\beta}$, and $\bar{\beta} = \frac{2d\pi}{N}$, $d \in \{1, \ldots, N-1\}$, is an equilibrium of system (10) by substituting it to (8).

We say a group of N agents with local control law (2) *achieves uniform circumnavigation in the steady state* on the orbits of radii $r_1, r_2, \ldots, r_N$ if at an equilibrium of (10) (namely, $\dot{v}_i \equiv 0$, $\dot{\omega}_i \equiv 0$, and $\dot{\beta}_i \equiv 0$), the specifications (1)–(4) given in the uniform circumnavigation problem statement are all satisfied.

The following theorem reveals that uniform circumnavigation can be achieved in the steady state with suitable choices of control parameters k_v and k_ω for local control law (2).

Theorem 3. *For positive $r_1, r_2, \ldots, r_N$, suppose a_i and δ_i are chosen as in Theorem 1 with $c \in \mathcal{I}$. A group of N agents with local control law (2) can achieve uniform circumnavigation in the steady state on the orbits of radii $r_1, r_2, \ldots, r_N$ if*

$$\frac{k_v}{k_\omega} = \frac{1 - c\cos\bar{\psi}}{|c\sin\bar{\psi}|}$$

where $\bar{\psi} = \frac{2d\pi}{N}$ with $d \in \{1, \ldots, N-1\}$.

The proof of Theorem 3 requires the following lemma.

Lemma 4 (*Zheng et al., 2013, Theorem 2*). *Suppose a_i and δ_i are chosen as in Theorem 1 with $c \in \mathcal{I}$. If $\dot{v}_i \equiv 0$, $\dot{\omega}_i \equiv 0$ for all i, then*

- *all the agents move on concentric circles with equal angular speed around the target (x_b, y_b) or*
- *all the agents remain stationary at the target (x_b, y_b).*

Proof of Theorem 3. It has been proven in Lemma 4 that all agents remain stationary at the target or move on concentric circles with equal angular speed around the target when $\dot{v}_i \equiv 0$ and $\dot{\omega}_i \equiv 0$. Because the stationary equilibrium does not solve the uniform distributed circumnavigation problem, it is not considered in this paper. Moreover, as we point out in (11),

$$\{\chi | v_i = r_i\bar{v}, \omega_i = \bar{\omega}, \beta_i = \bar{\beta}, \forall i\}$$

is an equilibrium of system (10). So it can be concluded that $\psi_i = \beta_i$ for all i and so $\psi_1 = \cdots = \psi_N = \bar{\psi} = \bar{\beta} = \frac{2d\pi}{N}$ with $d \in \{1, \ldots, N-1\}$. Furthermore, for concentric motions around the target we can know that for any i,

$$\|(x_i, y_i) - (x_b, y_b)\| = \left|\frac{r_i\bar{v}}{\bar{\omega}}\right| = r_i\left|\frac{k_v c\sin\bar{\beta}}{k_\omega(1 - c\cos\bar{\beta})}\right|. \tag{12}$$

Therefore, if $\frac{k_v}{k_\omega} = \frac{1 - c\cos\bar{\psi}}{|c\sin\bar{\psi}|}$, then $\|(x_i, y_i) - (x_b, y_b)\| = r_i$, with which we can then conclude that the N agents achieve uniform circumnavigation on the orbits of radii $r_1, \ldots, r_N$. ■

3.4. Stability analysis

In this subsection we investigate stability properties of each equilibrium formation (11), that is, to show whether a group of N agents can asymptotically achieve uniform circumnavigation and which one is asymptotically stable. The main result below shows that under the proposed control law (2) only two equilibria (11) are asymptotically stable for specific values of $c \in \mathcal{I}$.

Theorem 5. *Consider the equilibria (11) with $d \in \{1, \ldots, N-1\}$. The following holds:*

- *Case I: There exists an $\varepsilon > 0$ sufficiently small such that when $c = 1 - \varepsilon$ only the equilibria with $d = 1$ and $d = N-1$ are asymptotically stable.*
- *Case II: For odd N, when $c = -1$ only the equilibria with $d = \lfloor N/2 \rfloor$ and $d = \lceil N/2 \rceil$ are asymptotically stable.*
- *Case III: For even N, there exists an $\varepsilon > 0$ sufficiently small such that when $c = -1 + \varepsilon$ only the equilibria with $d = N/2 \pm 1$ are asymptotically stable.*

To prove Theorem 5, we let $\bar{\chi} = [r_1\bar{v}, \bar{\omega}, \bar{\beta}, \ldots, r_N\bar{v}, \bar{\omega}, \bar{\beta}]^T$ be an equilibrium (11). The linearized model about the equilibrium for each subsystem (9) is then obtained, which has the following form $\dot{\tilde{\chi}}_i = A_i\tilde{\chi}_i + B_i\tilde{\chi}_{i+1}$, where $\tilde{\chi}_i = \chi_i - [r_i\bar{v}, \bar{\omega}, \bar{\beta}]^T$ and the Jacobian matrices A_i and B_i are calculated as

$$A_i = \begin{bmatrix} -k_v & \dfrac{2k_v r_i\bar{\omega}}{k_\omega} & -cr_ik_v\bar{v}\sin\bar{\beta}\\ \dfrac{-k_\omega\bar{\omega}}{k_v r_i} & \dfrac{-k_\omega\bar{v}}{k_v} & ck_\omega\bar{v}\cos\bar{\beta}\\ 0 & -1 & 0\end{bmatrix},$$

$$B_i = \begin{bmatrix} \dfrac{cr_i k_v \cos\bar{\beta}}{r_{i+1}} & 0 & 0 \\ \dfrac{ck_\omega \sin\bar{\beta}}{r_{i+1}} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Moreover, the linearized model of the overall system (10) has the form $\dot{\tilde{\chi}} = S^{-1}\hat{A}S\tilde{\chi}$, where $\tilde{\chi} = \chi - \bar{\chi}$, $S = \mathrm{diag}(1/r_1, 1, 1, \ldots, 1/r_N, 1, 1)$, and $\hat{A}$ is the block circulant matrix $\hat{A} = \mathrm{circ}[A, B, \mathbf{0}_{3\times 3}, \ldots, \mathbf{0}_{3\times 3}]$ with

$$A = \begin{bmatrix} -k_v & \dfrac{2k_v\bar{\omega}}{k_\omega} & -ck_v\bar{v}\sin\bar{\beta} \\ \dfrac{-k_\omega\bar{\omega}}{k_v} & \dfrac{-k_\omega\bar{v}}{k_v} & ck_\omega\bar{v}\cos\bar{\beta} \\ 0 & -1 & 0 \end{bmatrix}, \tag{13}$$

$$B = \begin{bmatrix} ck_v\cos\bar{\beta} & 0 & 0 \\ ck_\omega\sin\bar{\beta} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Define $D_i := A + \varphi^{i-1}B$, $i \in \mathbb{N}$, where $\varphi = e^{2j\pi/N}$. Consider the characteristic polynomial of D_i:

$$p_i(\lambda) := \lambda^3 + \varrho_1\lambda^2 + \varrho_2\lambda + \varrho_3, \tag{14}$$

where $\varrho_1, \varrho_2, \varrho_3$ are complex numbers. We define a Hermitian matrix associated with $p_i(\lambda)$ as

$$H_i := \begin{bmatrix} \varrho_1 + \bar{\varrho}_1 & \varrho_2 - \bar{\varrho}_2 & \varrho_3 + \bar{\varrho}_3 \\ -\varrho_2 + \bar{\varrho}_2 & \varrho_1\bar{\varrho}_2 + \varrho_2\bar{\varrho}_1 - \varrho_3 - \bar{\varrho}_3 & \varrho_3\bar{\varrho}_1 - \varrho_1\bar{\varrho}_3 \\ \varrho_3 + \bar{\varrho}_3 & \varrho_1\bar{\varrho}_3 - \varrho_3\bar{\varrho}_1 & \varrho_2\bar{\varrho}_3 + \varrho_3\bar{\varrho}_2 \end{bmatrix}, \tag{15}$$

where $\bar{\varrho}_1$, $\bar{\varrho}_2$, and $\bar{\varrho}_3$ denote the complex conjugates of $\varrho_1, \varrho_2, \varrho_3$ respectively.

The following lemmas are useful to prove Theorem 5.

Lemma 6. *For $c \in \mathcal{I}/\{0\}$, an equilibrium (11) is asymptotically stable if the leading principal minors h_{i1}, h_{i2}, h_{i3} of H_i are positive with all $i \in \mathbb{N}/\{1\}$.*

We first provide two lemmas about h_{i1} and h_{i2}.

Lemma 7. *For $i \in \mathbb{N}/\{1\}$, the inequality $h_{i1} > 0$ holds when*

- *N is odd and $c \in [-1, 1)$ or*
- *N is even and $c \in (-1, 1)$.*

Lemma 8. *For $i \in \mathbb{N}/\{1\}$, the inequality $h_{i2} > 0$ holds when*

- *$c = 1$ and $d \neq N/2$ or*
- *$c = -1$.*

See Appendix for the proofs of Lemmas 6–8.

Proof of Theorem 5. Case I: According to Lemmas 7 and 8 and considering the continuity of functions, we obtain that h_{i1} and h_{i2} are positive when $c = 1 - \varepsilon$, $d \neq N/2$, and $i \in \mathbb{N}/\{1\}$,

Now we calculate h_{i3} with $c = 1$ and obtain

$$h_{i3}|_{c=1} = 32k_v^9(c_r+1)^2(c_i-1)(c_i-c_r)^3(2c_rc_i^2 + 2c_i^2 + 4c_r^2c_i - 5c_rc_i - 8c_i - 5c_r^2 + 2c_r + 8),$$

where $c_r = \cos\bar{\beta}$ and $c_i = \cos\frac{2(i-1)\pi}{N}$.

It can be proven that the last term of $h_{i3}|_{c=1}$: $2c_rc_i^2 + 2c_i^2 + \cdots + 2c_r + 8 > 0$. The sign of $h_{i3}|_{c=1}$ is therefore the same as $\tilde{h}_{i3}|_{c=1} = (c_r+1)^2(c_i-1)(c_i-c_r)^3$. When $i \in \mathbb{N}/\{1\}$ and $d \neq N/2$, $\tilde{h}_{i3}|_{c=1} > 0$ is equivalent to $\cos\frac{2d\pi}{N} > \cos\frac{2(i-1)\pi}{N}$. Hence $d = 1$ or $d = N-1$. Notice that the equality holds only when $i = 2$ or $i = N$. The Taylor series of the third leading principal minor $h_{i3}|_{c_i=c_r}$ at $c = 1$ is

$$-64k_v^9(c_r-1)^3(c_r+1)^5(3c_r-4)(c-1) + o(c-1). \tag{16}$$

Table 1
Specification and designed control parameters.

i	1	2	3	4	5	6
r_i	2	2	2	3	3	3
$\bar{\psi}$			$2\pi/3$ or $4\pi/3$			
c			-0.9			
δ_i	1/2	1/2	1/2	1/3	1/3	1/3
a_i	1.900	1.900	1.600	1.900	1.900	2.350
k_v			2.750			
k_ω			3.897			

It can be checked that the coefficient of $c-1$ is less than 0. So when $c_i = c_r$, (16) becomes positive if $c = 1 - \varepsilon$ where $\varepsilon > 0$ is small enough. So when $c = 1 - \varepsilon$, only the equilibria with $d = 1$ and $d = N-1$ satisfy $h_{i1}, h_{i2}, h_{i3} > 0$ when $i \in \mathbb{N}/\{1\}$. According to Lemma 6, these equilibria are asymptotically stable.

Case II: Similar to Case I, it is obtained that both h_{i1} and h_{i2} are positive when $c = -1$ and $i \in \mathbb{N}/\{1\}$.

We calculate h_{i3} with $c = -1$ and obtain that

$$h_{i3}|_{c=-1} = -32k_v^9(c_r-1)^2(c_i-1)(c_i+c_r)^3(2c_rc_i^2 - 2c_i^2 - 4c_r^2c_i - 5c_rc_i + 8c_i + 5c_r^2 + 2c_r - 8).$$

Similarly, it can be proven that the last term of $h_{i3}|_{c=-1}$: $2c_rc_i^2 - 2c_i^2 - \cdots + 2c_r - 8 < 0$. The sign of $h_{i3}|_{c=-1}$ is therefore the same as $\tilde{h}_{i3}|_{c=-1} = (c_i-1)(c_i+c_r)^3$. When $i \in \mathbb{N}/\{1\}$, $\tilde{h}_{i3}|_{c=-1} > 0$ is equivalent to $\cos\frac{2d\pi}{N} < -\cos\frac{2(i-1)\pi}{N}$. from which we obtain $d = \lfloor N/2 \rfloor$ or $d = \lceil N/2 \rceil$.

Case III: From the proof of Case II, when $c = -1$ and N is even, $\tilde{h}_3|_{c=-1} > 0$ for all $i \in \mathbb{N}/\{1\}$ requires that $d = N/2$, which is the case $\bar{v} = 0$ and $\bar{\omega} = 0$ by Theorem 3. Therefore we consider $d = N/2 \pm 1$ instead as this is a more interesting case corresponding to the uniform circumnavigation formation. In this case, when $i \in \mathbb{N}/\{1\}$, $\tilde{h}_{i3}|_{c=-1} \geq 0$ and $\tilde{h}_{i3}|_{c=-1} = 0$, only when $i = 2$ or $i = N$. The Taylor series of the third leading principal minor $h_{i3}|_{c_i=-c_r}$ at $c = -1$ is

$$-64k_v^9(c_r-1)^5(c_r+1)^3(3c_r+4)(c+1) + o(c+1). \tag{17}$$

It can be proven that the coefficient of $c+1$ is positive when $c_i = -c_r$, so (17) is positive if $c = -1 + \varepsilon$ and $\varepsilon > 0$ is small enough. Hence equilibria with $d = N/2 \pm 1$ are asymptotically stable. ■

Algorithm 1 (*Summary of Control Design*)**.** Given predefined radii $r_1, r_2, \ldots, r_N$, each agent determines its control parameters as follows:

(1) According to Theorem 5, for desired stable formations in uniform circumnavigation choose c to be either -1, $-1+\varepsilon$, or $1-\varepsilon$ with $\varepsilon > 0$ sufficiently small to make $\tilde{h}_{i2}$ and $\tilde{h}_{i3}$ positive for all $i \in N/\{1\}$.
(2) Choose a_i and δ_i according to Theorem 1.
(3) Choose k_v and k_ω according to the desired stable formations and Theorem 3.

4. Simulations

In this section, we give an example of six agents to demonstrate how to design the control parameters based on given specifications and validate our obtained results. Their predefined radii r_i and desired $\bar{\psi}$ are given in Table 1. Then the control parameters c, δ_i, a_i, k_v and k_ω are designed following Algorithm 1. For those control parameters, the trajectories of the agents are shown in Fig. 2. The initial postures of the agents are represented by blank wedges and final postures by filled wedges. From Fig. 2, we can see that the desired formation is achieved on the desired orbits.

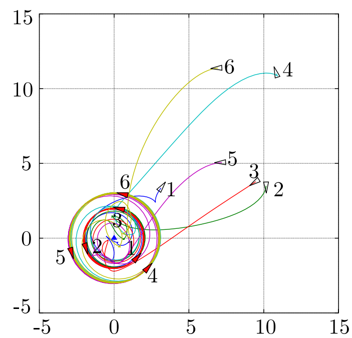


Fig. 2. Trajectories of the robots under the control parameters given in Table 1.

5. Conclusions

In this paper, a distributed control law for a network of nonholonomic agents, which are deployed to achieve collective circular motions around a target of interest, has been proposed. We show that the collective behavior of the multi-agent system can be shaped by choosing different control parameters. A sufficient condition for the boundedness of the trajectories is provided. We then show that by setting a particular control parameter to different values, the agents can achieve various circumnavigation patterns around the target.

Several interesting developments of this work can be regarded as future topics.

- From a theoretical point of view, it would be of great significance to provide global stability analysis for the collective behaviors.
- Another topic is to consider an undirected ring interaction amongst the agents. In a previous work (Zheng, Lin, & Cao, 2011), we showed how a pursuit strategy drives a group of unicycles with general connected and undirected graph to rendezvous. If undirected ring-coupled unicycles pursue the target while repel their neighbors, it is expected a static circular formation around the target can be formed.
- From the analysis in Section 3.3, it is known that every configuration in the set (11) is an equilibrium of the system. In this paper, we give rigorous stability analysis for specific values of c by using the idea of perturbation analysis. To study the stability of all $c \in \mathcal{I}$ is much more difficult and we are still working on it.
- Under the proposed controller, desired circumnavigation formations can be achieved only when all the member agents work normally. It is of practical signification to consider the case when some agents fail. In this case, to perform the task successfully, for example, a new interaction network involving the normal agents may be rebuilt through a distributed negotiation mechanism.

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Appendix

The proof of Lemma 6 requires the following three lemmas.

Lemma 9 (*Davis, 1994*)**.** *The eigenvalues of*

$$\hat{A} = \text{circ}[A, B, \mathbf{0}_{3\times3}, \ldots, \mathbf{0}_{3\times3}]$$

are the collection of all eigenvalues of $D_i = A + \varphi^{i-1}B$ with all $i \in \mathbb{N}$ where $\varphi = e^{2j\pi/N}$.

Lemma 10. *For A and B defined in (13), the eigenvalues of $D_1 = A + B$ are*

$$\lambda_1 = 0, \qquad \lambda_2 = k_v(-1 + c\cos\bar{\beta}), \qquad \lambda_3 = \frac{-k_v c^2 \sin^2\bar{\beta}}{1 - c\cos\bar{\beta}}.$$

Lemma 11 (*Barnett, 1983, Theorem 3.16*)**.** *The polynomial $p_i(\lambda)$ defined in (14) has all roots of negative real parts if and only if H_i defined in (15) is positive definite.*

Proof of Lemma 6. When $c \in \mathcal{I}/\{0\}$ and $i = 1$, Lemma 10 tells us that D_1 has one zero eigenvalues λ_1 and two negative eigenvalues λ_2 and λ_3. However, the zero eigenvalue can be proven to not affect the stability of the system. Similar to Zheng et al. (2009, Lemma 2), this zero eigenvalue occurs because of the definition of β_i which intrinsically satisfies $\sum_i \beta_i = 2k\pi$, $k \in \mathbb{N}$.

So to determine the local stability of each equilibrium, we need to locate the eigenvalues of D_i's with all $i \in \mathbb{N}/\{1\}$. This is not a trivial step considering that D_i is a complex matrix. However, since to locate the eigenvalues of D_i is equivalent to locate the roots of its characteristic polynomial $p_i(\lambda)$, Lemma 11 is then used. We denote the leading principal minors of H_i as h_{i1}, h_{i2}, and h_{i3} and recall the fact that a Hermitian matrix H_i is positive definite if and only if its leading principal minors are positive, which accomplishes the proof. ■

Proof of Lemma 7. We calculate $h_{i1} = 2k_v(c^2\frac{1-c_r^2}{1-c\cdot c_r} + 1 - c\cdot c_r c_i)$, where $c_r = \cos\bar{\beta}$ and $c_i = \cos\frac{2(i-1)\pi}{N}$.

When $c \in (-1, 1)$, we have $(1-c_r^2)/(1-c\cdot c_r) > 0$ and $1 - c\cdot c_r c_i > 0$, so $h_{i1} > 0$. When N is odd and $c = -1$, we have $h_{i1} = 2k_v(c_r c_i - c_r + 2)$. Moreover, notice that $c_r < 1$, so $h_{i1} > 0$. ■

Proof of Lemma 8. We calculate h_{i2} with $c = \pm 1$ and obtain that

$$\begin{aligned} h_{i2}|_{c=1} &= 8k_v^4(c_r+1)(c_r c_i^3 + 2c_r^2 c_i^2 + 2c_r c_i^2 - 3c_r^2 c_i \\ &\quad - 8c_r c_i - 2c_i + c_r^2 + 3c_r + 4), \\ h_{i2}|_{c=-1} &= 8k_v^4(c_r-1)(c_r c_i^3 - 2c_r^2 c_i^2 + 2c_r c_i^2 + 3c_r^2 c_i \\ &\quad - 8c_r c_i + 2c_i - c_r^2 + 3c_r - 4). \end{aligned}$$

We consider $h_{i2}|_{c=1}$ first. By checking the value of $f = \frac{h_{i2}|_{c=1}}{8k_v^4}$, it is found that $f \geq 0$ and $f = 0$ when $c_r = -1$ or $(c_r = 1) \wedge (c_i = 1)$. When $d \in \mathbb{N}/\{N\}$, $c_r \neq 1$. So if $c_r \neq -1$, i.e., $d \neq N/2$, then $h_{i2}|_{c=1} > 0$. Similarly, we can prove that $h_{i2}|_{c=-1} > 0$. ■

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