Two-Degree-of-Freedom Control of a Dual-Stage Actuator Positioning System for Short-Span Tracking

Jinchuan Zheng, Weizhou Su, and Minyue Fu

Abstract—This paper presents a new two-degree-of-freedom (2DOF) control design method for a dual-stage actuator (DSA) positioning system consisting of a linear motor (LM) and a piezo actuator (PA). The 2DOF controller is proposed to achieve disturbance rejection and short-span tracking in the PA range and is designed using the doubly coprime factorization approach, with which the closed-loop transfer function is expressed explicitly in terms of design parameters. This greatly simplifies the optimization of design parameters in meeting desired specifications. We further study how to use the design parameters to deal with specific problems in the DSA, i.e., control allocation and trajectory planning. Experimental results demonstrate the practical implementation of the DSA control system and verify its effectiveness for step tracking and disturbance rejection and its robust performance under load changes.

I. INTRODUCTION

Dual-stage actuator (DSA) servo systems are characterized by a structural design with two actuators connected in series along a common axis. The primary actuator (coarse actuator) is of long travel range but with poor accuracy and slow response time. The secondary actuator (fine actuator) is typically of higher precision and faster response but with a limited travel range. By combining the DSA system with a properly designed servo controller, the two actuators are complementary to each other providing long travel range, high positioning accuracy and fast response. The DSA servomechanism has been commonly used in the industry, e.g., the dual-stage hard disk drive (HDD) actuator [1], the dual-stage machine tools [2], and the dual-stage XY positioning tables [3].

The control design for the DSA is a challenging task because of the specific characteristics in the DSA systems: 1) the DSA system is a dual-input single-output (DISO) system, which means that for a given desired trajectory, inputs to the two actuators are not unique. Thus, a proper control strategy is required for control allocation. 2) The secondary actuator typically has a very limited travel range, which results in a severe actuator saturation problem. A number of approaches have been reported to deal with dual-stage control problems. Control design for reference following can be found in [4], [5]. In [6] and [7], the secondary actuator saturation problem is explicitly taken into account during the control design. In [8], a decoupled track-seeking controller is developed to enable high-speed short-span seeking for a dual-stage HDD servo system. Further, short and long-span seeking controls are incorporated in a single control scheme with fast settling time [9], [10]. The literature has also demonstrated successful applications of some new control theories and design methods to DSA servo systems. For example, robust control is used to overcome plant uncertainty and to maintain performance [3], [5]. Repetitive control is used to suppress periodic disturbances and vibrations [2]. Nonlinear control is applied to handle the actuator saturation [9] or to enhance the seeking performance [10].

In this paper, we present a new control design method for a DSA positioning system consisting of a linear motor (LM) and a piezo actuator (PA). We focus on the development of a two-degree-of-freedom (2DOF) controller for disturbance rejection and step tracking in the PA range. A doubly coprime factorization (DCF) [11] is used for the 2DOF controller design because it provides the advantages that: 1) it parameterizes all linear internally stabilized 2DOF controller by two free design parameters; 2) it offers a unifying design method to solve the tracking and disturbance rejection problems; 3) the derived frequency transfer functions of disturbance rejection response and seeking response are simply expressed and they are uniquely in terms of the design parameters, which makes the relationship between the design parameters and the desired specifications explicit. Finally, we verify the effectiveness of the DSA controller through experiment results. For tracking control beyond the PA range, it is not the purpose of this paper and thereby is not given in detail. Interested readers can refer to [10] for the control strategy in this range, where the LM is with a proximate time-optimal servomechanism (PTOS) controller [12] and the PA is activated only when the tracking error enters the PA range.

Throughout this paper, we use the following notation. For any signal \( u(t) \), we denote its Laplace transform by \( \hat{u}(s) \). \(|\cdot|\) denotes the Euclidean vector norm and \( \|\cdot\|_2 \) the norm in space \( L_2 \).

II. 2DOF CONTROL DESIGN BASED ON DCF

The 2DOF control systems are the most general feedback configuration in linear control schemes. Fig. 1 shows a generic structure for this class of systems. In this setup, \( G \) denotes the given linear time-invariant (LTI) plant model, \( \omega \) denotes the known LTI stable and proper weight, and \( K \) denotes the 2DOF controller to be designed. The signals \( r \), \( y \), \( u \), and \( d \) represent, respectively, the step reference signal,
the system output, the control input, and the disturbance with energy bounded by $\delta^2$, i.e., $\|d\|_2^2 \leq \delta^2$.

In this paper, we consider the asymptotic tracking and disturbance rejection problem for the system in Fig. 1. We need to design the controller $K$ such that the closed-loop system is internally stable and the system output $y$ asymptotically tracks a step signal $r(t) = v$, $t \geq 0$ for all disturbance $d \in L_2$ with $\|d\|_2 \leq \delta$. The measure of the tracking performance is defined as

$$J = \int_0^\infty \|e(t)\|^2 dt,$$

where $e(t) = r(t) - y(t)$ denotes the tracking error. Obviously, $J$ depends on the disturbance $d$. We thus consider the worst value of $J$ over all possible $d$ as the performance index for the tracking and disturbance rejection problem, i.e.,

$$\sup_{\|d\|_2 \leq \delta} J.$$  
(2)

Therefore, it is our interest to seek a controller $K$ among all possible stabilizing 2DOF controllers to achieve the minimum value of (2) defined by

$$J_{opt} = \inf_K \sup_{\|d\|_2 \leq \delta} J.$$  
(3)

The DCF is a well-suited approach to solve (3). Let $\mathcal{RH}_\infty$ denotes the set of all stable, proper, rational transfer function matrices. Let also the right and left coprime factorizations of $G$ be given by

$$G = ND^{-1} = \tilde{D}^{-1}\tilde{N},$$  
(4)

where $N$, $D$, $\tilde{N}$, $\tilde{D} \in \mathcal{RH}_\infty$ and satisfy the doubly Bezout identity

$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & Y \\ N & X \end{bmatrix} = I$$  
(5)

for some $X$, $Y$, $\tilde{X}$, $\tilde{Y} \in \mathcal{RH}_\infty$.

In [13], Nett has proposed explicit formulas for the doubly coprime fractional representation of an LTI system in terms of its state-space realization. This method is numerically easy to use. To do this, we first represent the plant model $G(s)$ in state-space as follows:

$$G(s) = C(sI - A)^{-1}B,$$  
(6)

and $(A - LC)$ are both Hurwitz. Thus, a DCF of $G$ is given by

$$\begin{align*}
N(s) &= C(sI - A + BF)^{-1}B \\
D(s) &= I - F(sI - A + BF)^{-1}B \\
\tilde{N}(s) &= C(sI - A + LC)^{-1}B \\
\tilde{D}(s) &= I - C(sI - A + LC)^{-1}L \\
X(s) &= I + C(sI - A + BF)^{-1}L \\
\tilde{X}(s) &= I + F(sI - A + LC)^{-1}B \\
Y(s) &= -F(sI - A + BF)^{-1}L \\
\tilde{Y}(s) &= -F(sI - A + LC)^{-1}L.
\end{align*}$$  
(7)

According to [11], all linear internally stabilizing 2DOF controllers $K = [K_1 \ K_2]$ can be parameterized by

$$\begin{align*}
\hat{u} &= K_1\hat{r} + K_2\hat{y}, \\
K_1 &= (\tilde{X} - R\tilde{N})^{-1}Q, \\
K_2 &= (\tilde{X} - R\tilde{N})^{-1}(\tilde{Y} - R\tilde{D}),
\end{align*}$$  
(8) (9) (10)

where $Q, R \in \mathcal{RH}_\infty$ are the free parameters to be designed. By substituting the controllers $K_1$, $K_2$ and the factorized plant model (4) into Fig. 1, we can easily obtain the following input-output relationship in frequency domain

$$\hat{y} = T_{yr}\hat{r} + T_{yTd}\hat{d},$$  
(11)

with

$$T_{yr} = NQ, \quad T_{yTd} = (X - NR)\tilde{N}W,$$

where $T_{yr}$ and $T_{yTd}$ denote the closed-loop responses from the reference and disturbance to the system output, respectively. It is advantageous that the closed-loop response functions are expressed by the design parameters $Q$ and $R$ explicitly. Hence, from Parseval’s theorem, we have

$$J = \int_0^\infty \|e(t)\|^2 dt = \|\hat{r} - \hat{y}\|^2 T_{yr}^*T_{yr} + \|\hat{y}\|^2 T_{yTd}^*T_{yTd}$$  
(12)

Then, the following result is clear.

**Theorem 1** [14]: Let $G$ have non-minimum phase (NMP) zeros $z_1, z_2, \ldots, z_m$ with corresponding Blaschke vectors $\eta_1, \eta_2, \ldots, \eta_m$. Then the minimax tracking performance of asymptotical tracking and disturbance rejection of the system is given by

$$J_{opt} = \inf_Q \| (I - T_{yr})\hat{r} \|^2_2 + \delta^2 \inf_R \| T_{yTd}\|^2_2$$  
(13)

$$= 2 \sum_{i=1}^m \text{Re}(z_i) \cos^2\angle(\eta_i, v) + \delta^2 \inf_R \| T_{yTd}\|^2_2$$  
(14)

**Remark I**: The theorem reveals that the optimal tracking performance with the 2DOF controller is a sum of two terms as shown in (13). The first term is the optimal tracking performance of the system without the disturbance input $d$, while the second one is the best achievable performance of disturbance attenuation of the system without the reference signal $r$. These two optimal problems have been studied in [15], [16], respectively, the results therein are then applied to yield (14).
optimization problems in terms of the free parameters invertible, stable and minimum phase, the resulting to the special case where the plant must be proper, right more of the constraints. In addition, the optimal at the same time. Instead, the designer has to deal with one or servomechanisms, these strict conditions are rarely satisfied in practical design approach to a DSA servo system, which consists of a DSA positioning system. Therefore, the relation to the reference signal of the system. Therefore, the one that yields \( J_{opt} = 0 \). However, this option only applies to the special case where the plant must be proper, right invertible, stable and minimum phase, the resulting \( R \) and \( Q \) are proper and the control input has no saturation. In practical servomechanisms, these strict conditions are rarely satisfied at the same time. Instead, the designer has to deal with one or more of the constraints. In addition, the optimal \( Q \) would be related to the reference signal of the system. Therefore, the design of \( R \) and \( Q \) requires some extra techniques to obtain a practical servo system without degrading the tracking performance significantly. In general, we can attempt to design \( R \) and \( Q \) such that \( T_{yd} = (X - NR)NW \rightarrow 0 \) and \( T_{yr} = NQ \rightarrow I \) in the frequency of interest according to the design specifications [17]. Under this circumstance, a suboptimal controller is achieved to approximate the optimal one that yields (14), while to handle the constraints at hand. The performance of the resulting servo system will then compromise among the optimal tracking, robustness, and easy implementation (e.g., least controller order). The design examples along this line include [18] that aims for optimal step responses of an unstable and NMP flexible beam, and [19] that handles control design with actuator torque constraints.

In the next section, we will apply such a 2DOF controller design approach to a DSA servo system, which consists of a DISO plant with saturation for both actuators. In particular, we will address the design of \( R \) that determines the control allocation of the two actuators for disturbance rejection, and the robust stability of the feedback loop. On the other hand, \( Q \) is designed to generate the desired trajectories in response to a step reference for the two actuators, as such, the overall system output can obtain a fast and smooth response.

III. APPLICATION TO A DSA CONTROL SYSTEM

A. Plant Modeling

The DSA positioning system is depicted in Fig. 2, which consists of a primary stage driven by an LM and a secondary stage driven by a PA. The LM has a 0.5 m travel range, while the PA has a limited travel range of \( \pm 15 \mu m \). The nonlinear friction force of the LM is overcome by a precompensator, see [10] for details. The resonance of the PA stage flexure is actively damped by its integrated control electronics. In this setup, we can simply ignore the coupling forces between the two actuators. After these manipulations, the DSA plant can be depicted by Fig. 3. The LM model is approximated by:

\[
G_1 = \frac{\hat{y}_1}{\hat{u}_1} = \frac{k_1}{s^2},
\]

(15)

where \( y_1 \) is the LM position output, \( u_1 \) is the control input with \(|u_1| \leq \bar{u}_1 = 1 \) V, and \( k_1 = 1.5 \times 10^7 \). The PA model is approximated by:

\[
G_2 = \frac{\hat{y}_2}{\hat{u}_2} = \frac{k_2}{s^2 + as + b}.
\]

(16)

where \( y_2 \) is the PA position output relative to the LM, \( u_2 \) is the control input with \(|u_2| \leq \bar{u}_2 = 5 \) V, and \( k_2 = 3.0 \times 10^6 \), \( a = 1810 \), \( b = 1.0 \times 10^6 \). Fig. 4 shows the frequency responses of the LM and PA system, which verify the accuracy of the identified models in the frequency of interest.

In Fig. 3, the control output \( y \), i.e., the absolute position of the PA, is the only available measured output for feedback
control. Hence, the overall DSA model $G$ can be represented as a DISO linear system

$$\dot{y} = G\dot{u} = [G_1 \ G_2 \ \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix}].$$  \hspace{1cm} (17)

To get the DCF of $G$ by (7), we transform $G$ into a state-space form, whose system matrices are given by

$$A = \begin{bmatrix}
A_1 & 0 \\
0 & A_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -b & -a
\end{bmatrix},$$

$$B = \begin{bmatrix}
B_1 & 0 \\
0 & B_2
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & k_1 \\
0 & k_2
\end{bmatrix},$$

$$C = \begin{bmatrix}
C_1 \\
C_2
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 1 & 0
\end{bmatrix},$$

where $A_{1,2}, B_{1,2}$ and $C_{1,2}$ are the state-space representation of the LM and PA, respectively.

For the disturbance source, we are concerned with a shock disturbance acting on the LM. The half sine wave with a duration of 10 ms is typically used as the standard industry shock test [20]. Thus, we can model the disturbance as

$$W = \begin{bmatrix}
0.05 \\
0.0008s^{-1} + 1 \\
0
\end{bmatrix}, \quad d = \begin{cases} 
\sin(314t), & t \in [0 \ 0.01] \\
0, & \text{otherwise}
\end{cases}.$$  \hspace{1cm} (21)

Obviously, we have $\|d\|_2 \leq \delta = 0.071$.

### B. 2DOF Controller Design

The 2DOF controller for the DSA step responses within the PA Range should satisfy the following specifications:

1) The overshoot should be kept under 1 $\mu$m.
2) The control inputs to the LM and PA are not saturated, i.e., should not exceed $\pm 1$ and $\pm 5$ V, respectively.
3) In response to a step reference, the displacement of PA should settle down to zero at steady state such that it can further response to a sequential reference.
4) The DSA servo system should have gain margin larger than 6 dB and phase margin more than 50 deg.

We will present a step-by-step design procedure.

#### Step 1: DCF of $G$

According to (7), we should first select $F$ and $L$ such that $(A - BF)$ and $(A - LC)$ are both Hurwitz. Clearly, $F$ is a state feedback gain matrix, and $L$ is a state estimator gain matrix. Since there is no coupling between the LM and the PA, the gains $F$ and $L$ can be partitioned as

$$F = \begin{bmatrix}
F_1 & 0 \\
0 & F_2
\end{bmatrix}, \quad L = \begin{bmatrix}
L_1 \\
L_2
\end{bmatrix}.$$  \hspace{1cm} (19)

Hence, we can individually design the gains for the LM and PA loops by using the pole placement method such that the PA loop should have a faster dynamics than the LM loop, and the estimator is faster than the state feedback loop. To do this, we select $F_1 = [0.0024 \ 2.2 \times 10^{-5}]$ and $L_1 = [3037 \ 3.8 \times 10^{6}]^T$ to make the LM loop and its estimator have a bandwidth of 30 and 200 Hz, respectively, and select $F_2 = [-0.286 \ 3.7 \times 10^{-4}]$ and $L_2 = [243 \ -3.6 \times 10^5]^T$ for the counterparts of the PA with 60 and 250 Hz bandwidths, respectively. Then, the DCF of $G$ can be computed by (7).

#### Step 2: Design of $R$

For disturbance rejection, we should make the disturbance rejection function $T_{yd} = (X - N^TR)NW \to 0$ in the low frequencies. Let $R = [R_1 \ R_2]^T$ and $N = [N_1 \ N_2]$, we then take

$$R_1 = N_1^{-1}Xr_1(s),$$  \hspace{1cm} (19)

$$R_2 = N_2^{-1}Xr_2(s),$$  \hspace{1cm} (20)

with

$$r_1(s) = \frac{1 - \beta}{(\eta s + 1)^2},$$  \hspace{1cm} (21)

$$r_2(s) = \frac{\beta}{(\eta s + 1)^2},$$  \hspace{1cm} (22)

where $\eta > 0$ and $\beta \in [0 \ 1]$ are tuning scalars. Note that the order of $r_{1,2}$ is chosen to make $R_{1,2}$ proper at least. Then, we have

$$T_{yd} = (1 - \frac{1}{(\eta s + 1)^2})X\tilde{N}W.$$  \hspace{1cm} (23)

We can see that the term $(1 - 1/(\eta s + 1)^2)$ introduces low gains in low frequencies for disturbance rejection. Moreover, the available frequency region for the disturbance rejection problem can be increased with a smaller $\eta$. Since stability margin is also required for robustness, we can study the open-loop characteristics of the DSA, which is defined by

$$OL(s) = GK_2 = \frac{Y_1G_1 + Y_2G_2 - X*(\eta s + 1)^2)}{X(1 - 1/(\eta s + 1)^2)},$$  \hspace{1cm} (24)

where $Y_1$ and $Y_2$ are, respectively, the elements of $Y$ with $Y = [Y_1 \ Y_2]^T$. It is clear that the open-loop transfer function is related to $\eta$ only. However, the relationship between the stability margin and $\eta$ is implicit. Hence, we have to tune $\eta$ by trial and error such that a suitable stability margin and disturbance rejection function in (23) are both achieved.

Next, we discuss how to select $\beta$. In fact, for a given $T_{yd}$ (or equivalently a given position output), $\beta$ is related to the allocation of the control efforts of the two actuators. Typically, the LM works mainly for the low-frequency movement, while the PA responses more for high-frequency disturbance. With such allocation in frequency domain, it is possible to take full advantage of the PA to bypass the LM uncertainty in the high-frequency band and improve the servo bandwidth. A key point to analyze the control allocation of the two actuators is the intersection of the two paths in frequency domain. Specifically, we can analyze the ratio of the open-loop systems of the two actuators. This idea is identical to the so-called PQ method [4]. Let the controller $K_2 = [K_{21} \ K_{22}]^T$, and then define the open-loop systems of the LM and PA as $OL_1 = G_1K_{21}$ and $OL_2 = G_2K_{22}$, respectively. Then we can obtain the ratio of $OL_1$ and $OL_2$ as

$$\Gamma = \frac{OL_1}{OL_2} = \frac{X(1 - \beta) - (\eta s + 1)^2Y_1G_1}{X\beta - (\eta s + 1)^2Y_2G_2}. $$  \hspace{1cm} (25)
We can see that $\Gamma$ is a function of $\beta$ provided that $\eta$ is determined. In order to make the two actuators have maximum cooperation, $\Gamma$ is chosen to give a roll-off characteristics and a phase margin of at least 60 deg at the 0-dB crossover frequency [4].

In our case, we choose $\eta = 1/(2\pi 250)$, and the corresponding Bode plot of the $T_{yd}$ is shown in Fig. 5. Based on (25) we then choose $\beta = 0.8$, which achieves a phase margin of 102 degree for $\Gamma$ function as shown in Fig. 6. We also observe that the 0-dB crossover frequency (the hand-off frequency) in Fig. 6 is decreased with larger $\beta$. This indicates a large disturbance rejection contribution from the PA, but it also tends to saturate the PA. To check the stability margin, Fig. 7 shows the open-loop system of the DSA, which indicates that the DSA open-loop system ($OL(s)$) is dominated by the LM loop ($OL_1(s)$) in the low frequency while the PA loop ($OL_2(s)$) in the high frequency. Furthermore, the DSA system achieves a phase margin of 54 deg at 146 Hz, and a gain margin of $\infty$. Compared with the LM loop whose phase margin is only 13 deg at 84 Hz, we can see that the PA loop improves the stability margin and pushes the open-loop frequency bandwidth.

**Step 3: Design of $Q$**

Let $Q = [Q_1 \quad Q_2]^T$. Due to the fact that $G_1$ and $G_2$ are minimum phase, we thus aim at the design of $Q_1$ and $Q_2$ such that $T_{yr} = N_1 Q_1 + N_2 Q_2 \to 1$ has a high frequency bandwidth and the control inputs for a step response are within both actuators’ control limits. Furthermore, it is required that the displacement of PA settles down to zero at steady state. This means that $y_1(\infty) = r$ and $y_2(\infty) = 0$ should be satisfied for a step response with amplitude $r$ and the disturbance with $d(\infty) = 0$. Hence, we first analyze the individual position outputs of the two actuators. Partition $D$ as

$$D = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix},$$

and suppose $d = 0$, it is thus easy to get

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = \begin{bmatrix} N_1 D_1^{-1} & 0 \\ 0 & N_2 D_2^{-1} \end{bmatrix} DQ \hat{r}$$

$$= \begin{bmatrix} N_1 Q_1 \\ N_2 Q_2 \end{bmatrix} \hat{r}.$$  (27)

We can see that the step responses of the two actuators are completely decoupled in terms of $Q_1$ and $Q_2$. As the transfer functions $N_1$ and $N_2$ have been properly designed in *Step 1* to individually reflect the LM and PA closed-loop dynamics, we can then interpret $Q_1$ and $Q_2$ as the trajectory planning functions for the two actuators.

From Theorem 1, we can infer that the minimal $\| (I - NQ)\hat{r} \|_2^2$ achievable is zero as the DSA model has no NMP zeros. This can be completed by selecting $Q_1 = N_1^{-1}$ and $Q_2 = 0$, which, however, is not a practical solution due to the improper $Q_1$ and the saturation of $u_1$. In order to compromise between the tracking speed and the limitation...
of the control input, we choose $Q_1$ and $Q_2$ as
\begin{align*}
Q_1 & = N_1(0)^{-1}, \quad (28) \\
Q_2 & = \gamma N_2(0)^{-1}(1 - N_1 N_1(0)^{-1}), \quad (29)
\end{align*}
where $\gamma \in [0, 1]$ is a tuning parameter. It is obvious that $N_1(0) Q_1(0) = 1$ and $N_2(0) Q_2(0) = 0$, which imply that
\[ y(\infty) = y_1(\infty) + y_2(\infty) = r + 0 = r. \quad (30) \]
Moreover, define the LM and PA closed-loop dynamics by
\begin{align*}
T_1 & = N_1 N_1(0)^{-1}, \\
T_2 & = N_2 N_2(0)^{-1}.
\end{align*}
We then have the step response transfer function of the DSA
\[ T_{yr} = T_1 + \gamma T_2(1 - T_1). \quad (33) \]
It is clear that when $\gamma$ varies from 0 to 1, the cut-off frequency of $T_{yr}$ switches from that of $T_1$ to that of $T_2$. On the other hand, we can see from (29) that the PA will follow the scaled tracking error of the LM loop ($\gamma(1 - N_1 N_1(0)^{-1}) r$), where $\gamma$ actually determines the contribution of the PA to the overall position output. Since the PA has a faster response than the LM loop, it is preferable to have a maximal position output of the PA. Thus, we should maximize $\gamma \in [0, 1]$ subject to
\begin{align*}
\|T_{yr}\|_{\infty} & \leq 1.067, \quad (34) \\
\|u_2\|_{\infty} & \leq 5 \text{ V}, \quad (35)
\end{align*}
where the constraint (34) is introduced for an overshoot under 1 $\mu$m, while (35) is for no saturation of the PA. For the LM, its control input $u_1$ is generally not saturated for step responses within the PA range. Otherwise, we have to go back to Step 1 and reduce $F_1$ for slower LM dynamics. Although this iteration can be avoided by adding extra tunable dynamics to (28) to generate a slower trajectory for the LM, we believe it is not cost-effective as the selection of $Q_1$ as a constant gain can reduce the overall controller order.

In our case, we obtain $\gamma = 0.5$ to meet the requirement. Fig. 8 shows the Bode plot of the closed-loop systems for the DSA($T_{yr}$), the LM($T_1$), and the PA($T_2$), respectively. We can see that the DSA frequency bandwidth is located between that of the LM loop and that of the PA loop, which indicates that the DSA servo should be faster than the LM loop but slower than the PA loop.

C. Controller Implementation

Fig. 9 shows a transformed 2DOF controller structure for practical implementation. The reason is that the lumped 2DOF controller computed by (9) and (10) results in four sub-controllers, each with an order of 24. Instead, the equivalent 2DOF controller structure in Fig. 9 separates the lumped controller into several elements, each is numerically easy to compute and only appears once in the controller. Hence, the computation time is greatly reduced and the computation accuracy is improved.

IV. EXPERIMENTAL RESULTS

Experiments are conducted on the DSA positioning system to verify the effectiveness of the proposed DSA controller. For comparison, we also carry out the experiments for the single-stage (SS) servo system, where the LM is controlled by a PTOS controller [10] and the PA is switched off. The controller is implemented by a real-time DSP system (dSPACE-DS1103) with the sampling frequency of 5 kHz. Fig. 10 shows the step reference command and the shock disturbance acting on the LM for performance test. Fig. 11 shows the tracking result. From Fig. 11(a), we can see that the PA is effective to speed up the step response and to cancel out the LM position deviation due to the shock disturbance occurring at $t = 0.1$ s. As such, the dual-stage servo significantly outperforms the single-stage servo as shown in Fig. 11(b) in terms of the settling time and disturbance rejection. Note that the high frequency oscillations in the responses are due to the sensor quantization noise that limits the position accuracy within $\pm 1 \mu$m. Further, we calculate the corresponding performance cost $J(e)$ defined by (1) and show it in Fig. 11(c), which indicates a smaller $J(e)$ achievable by the DSA compared to the SS servo. Although the $J_{opt}$ derived in (14) for the DSA under study can be close to 0, it is impractical due to the actuator saturation limitation. Therefore, it is used for benchmark only. Finally, we evaluate the robust performance against various payloads mounting on the motor platform. Table I summarizes the results for various step references and with or without 1 kg payload carried by the DSA. The results clearly show a smaller difference of the specifications between with payload and without payload.
This verifies the robustness of the proposed controller on our DSA application.

V. CONCLUSION

We have revealed that the tracking and disturbance rejection problems can be decoupled into two independent optimization problems under the 2DOF control framework. Then, each problem can be separately solved by the design of the free parameters in the 2DOF controller, which is parameterized based on the DCF approach. The 2DOF controller is applied to an actual DSA system for disturbance rejection and step tracking in the PA range. Experimental results demonstrated that the proposed DSA control system can significantly speed up the step response and enhance the shock disturbance rejection compared with the single-stage servo system. Further, the performance is robust within an acceptable level when the DSA is subject to payload changes.

REFERENCES


TABLE I

<table>
<thead>
<tr>
<th>Step reference (μm)</th>
<th>Settling Time (ms)</th>
<th>Performance cost: J(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w/o load</td>
<td>w/ load</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>20</td>
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<td>15</td>
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Fig. 11. Experimental tracking control and disturbance rejection.