Abstract—The paper improves the cross-correlation method about the condition monitoring for train suspension systems. The faults of both the spring and the damper are discussed and the improved technique quantifies the result of the detection in a great accuracy. A side-view model of the vehicle is employed and the simulation verifies the effectiveness of this method.

I. INTRODUCTION

The train suspension systems are the mechanisms which connect the body, bogie and wheels, which are composed of several springs and dampers. When the vehicle moves along the track, the suspensions can reduce the vibrations of the body and make the passenger feel more comfortable. It is better to detect the faults as early as possible to avoid further destruction, which makes the vehicle more secure. What’s more, the scheduled maintenance can be adjusted according to the results of the detection and the expense related can be reduced greatly. Meanwhile, the number of the late arrivals of the train will be less, which improves customer satisfaction.

The development of the computer techniques makes it possible to sample data accurately and to analyze the faults in a short period of time. And the model-based approaches especially describe the inner relationship among each component. The detecting methods are carried out according to the sensors available and frequency domain signal analysis is employed to find the phenomenon related to the faults. The parameter estimation method based on physical model works well when the model is known in advance. However, the suspension system is dynamic and time-varying and classic detecting methods have to deal with high-order models, which is always extremely difficult.

The paper improves the cross-correlation method and considers the situation that both the spring and damper may go wrong. The symmetrical configurations of the model are taken advantage of and a set of sensors helps measure both the acceleration and angular acceleration of the bogie. When faults occur, the imbalance of the front and rear suspensions can be reflected by the cross-correlation, which describes the condition of the suspensions.

The cross-correlation method developed before assumes that the springs always work well. The improved method allows the spring to go wrong and the condition of both the springs and dampers can be detected accurately. It is also crucial to ensure that uncertainties from input signals or disturbances can be suppressed effectively.

The basic idea of the traditional technique is explained in Section II according to a conventional bogie. The deduction of the cross-correlation is given in Section III. In Section IV, the cross-correlation method is improved on the basis of Section III.

II. MODELING OF SUSPENSION SYSTEM

The train suspension consists of body, bogie and wheel sets. The dynamic behavior of a railway vehicle is also governed by the wheel-rail contact mechanism. The side-view model of the bogie is employed and it is shown in Figure 1. Physical equations of the motions of the bogie may be given in Equations 1 and 2, which is hard to find the interactions between the faults and the movements of the bogie. The different inputs from two wheels make the bogie nutates and the same inputs from two wheels make the bogie moves vertically. The secondary suspension is also involved here. The front and rear suspensions are no more symmetrical when faults occur and the application of Newton’s laws of motion to the model lead to the equations as follows:

\[
\begin{align*}
L_{tx}(C_p1 - C_p2)\dot{z}_b + 2K_p1z_t1 + 2K_p2z_t2 + F_d & = C_p1z_t1 + C_p2z_t2 + K_p1z_t1 + K_p2z_t2 + F_d \\
L_{bx}(C_p1 - C_p2)\dot{z}_b + 2L_{bx}(K_p1 + K_p2)\dot{\phi}_b & = C_p1z_t1 + C_p2z_t2 + K_p1z_t1 + K_p2z_t2 + F_d
\end{align*}
\]

Where \( C_{p1}, C_{p2} \) the damping coefficient of primary suspensions(front and rear); \( F_d \), force from the secondary suspension; \( L_{xb} \), the inertia of the bogie; \( L_{bx} \), semi wheel space; \( m_b \), the bogie mass; \( K_{p1}, K_{p2} \), stiffness constant of primary suspensions(front and rear); \( z_b \), bogie bounce displacement; \( \phi_b \), bogie angular displacement; \( z_1, z_2 \) track vertical...
The cross-correlation method has been described before, but it focuses merely on the damping failure, which ignores the fault of the spring. The model of a traditional bogie vehicle is explored to study the novel fault detection method. Compared to the conventional method, both the dampers and the springs in the primary suspension are studied. The track irregularities are generated in the simulation as the white noise with a limited power spectrum. The vehicle speed is 50 m/s in the simulations.

When the vehicle moves along the track, the sensors sample the data in a fixed period of $T$, the conditions of time $j$ are listed as below. $m_b z_{j} + I_b \phi_{j} / L_{bx}$ and $m_b z_{j} - I_b \phi_{j} / L_{bx}$ are novel conditions added according to the equations

$$m_b z_{j+i} + I_b \phi_{j+i} / L_{bx} + 2 C_{p1} U_{v_{j+i}} + 2 K_{p1} z_{v_{j+i}} + F_{d}^{i+j}$$

$$= 2 C_{p1} z_{v_{j+i}} + 2 K_{p1} z_{s_{j+i}} - 2 K_{p1} U_{s_{j+i}} + F_{d}^{i+j}$$

$$m_b z_{j-i} - I_b \phi_{j-i} / L_{bx} + 2 C_{p2} v_{j-i} + 2 K_{p2} z_{v_{j-i}} + 2 K_{p2} v_{s_{j-i}} + F_{d}^{i-j}$$

$$= 2 C_{p2} z_{v_{j-i}} + 2 K_{p2} z_{s_{j-i}} - 2 K_{p2} v_{s_{j-i}} + F_{d}^{i-j}$$

Where $F_{d}^{i+j}$, $F_{d}^{i-j}$ the force from the secondary suspension (They hardly affect the final result, so they are omitted). The track inputs to the two wheels are the same, but at the time difference of $T_s$ (for the semi-wheel space of $L_{bx}$ and the vehicle speed of $v$). Therefore, the cross-correlation of the rack inputs of the wheels is the highest at the time shift of $T_s$ and it is 0 in any
different points, taking into account that the track inputs at the two suspensions are independent of each other at any time.

\[ \sum_{i=0}^{n-1} z_{s1}^{i+1} z_{s2}^{i} = 0 \quad (k \neq 0) \]  

Furthermore, there are correlations between the inputs and the conditions of the bogie. The relationship of each condition is presented below and their cross-correlation can be written as:

\[ \sum_{i=0}^{n-1} z_{y}^{i} = 0 \quad (k \neq 0) \]  

\[ \sum_{i=0}^{n-1} u_{a}^{i} u_{b}^{i} = (2/T^2) \sum_{i=0}^{n-1} (u_{a}^{i} u_{b}^{i}) \]  

\[ m_{b} z_{b}^{i} + I_{b} \phi_{b}^{i} / L_{b} \]  

\[ = (m_{b} + I_{b} / L_{b}^{2}) u_{a}^{i} / 2 + (m_{b} - I_{b} / L_{b}^{2}) v_{a}^{i} / 2 \]  

\[ m_{b} z_{b}^{i} - I_{b} \phi_{b}^{i} / L_{b} \]  

\[ = (m_{b} + I_{b} / L_{b}^{2}) v_{a}^{i} / 2 + (m_{b} - I_{b} / L_{b}^{2}) u_{a}^{i} / 2 \]  

From the equation (6)-(12), the deduction is as follows.

\[ \sum_{i=0}^{n-1} z_{p}^{i} z_{q}^{i} = \left( (C_{pl1} + C_{p21}) (m_{b} + I_{b} / L_{b}^{2}) + 4C_{p1} C_{p21} \right) (2/T^2) \sum_{i=0}^{n-1} (u_{a}^{i} u_{b}^{i}) + 4C_{p1} C_{p21} \]  

\[ = (C_{pl1} + C_{p21}) (m_{b} + I_{b} / L_{b}^{2}) / (2/T^2) \]  

\[ \sum_{i=0}^{n-1} z_{p}^{i} z_{q}^{i} = \frac{4(C_{pl1} + C_{p21}) (m_{b} + I_{b} / L_{b}^{2})}{(2/T^2)} \sum_{i=0}^{n-1} (u_{a}^{i} u_{b}^{i}) \]  

where \( \sum_{i=0}^{n-1} z_{p}^{i} z_{q}^{i} \) and \( \sum_{i=0}^{n-1} u_{a}^{i} u_{b}^{i} \) are known. If \( \sum_{i=0}^{n-1} z_{s1}^{i} z_{s2}^{i} \) can be calculated, the coefficients \( (C_{pl1}, C_{p21}, K_{pl0}, K_{p20}) \) shall be easy to get. Consider the fact that the inputs are hard to measure up, the conventional method cannot detect the condition easily.

IV. IMPROVEMENT OF THE TRADITIONAL METHOD

Under normal circumstances, the coefficients of the suspensions are set as \( C_{pl0}, K_{pl0}, C_{p20}, K_{p20} \). Then \( C_{pl0} = C_{p20}, K_{pl0} = K_{p20} \). The method is presented in three steps.

Firstly, with the help of a set of sensors, the cross-correlation of the accelerations of the front and rear suspension shall be calculated when no fault occurs. In the same way, the cross-correlation of the conditions \( m_{b} z_{b}^{i} + I_{b} \phi_{b}^{i} / L_{b} \) and \( m_{b} z_{b}^{i} - I_{b} \phi_{b}^{i} / L_{b} \) are easy to acquire. Secondly, the faulty component shall be located with the help of the measurements of the sensors. The conventional cross-correlation method presents the way how to detect the fault of the dampers. When the springs go wrong, the cross-correlation becomes larger than before. Thirdly, the detection method shall be presented separately in the cases.

Case 1: two faults occur (only one kind of components goes wrong). Assume \( K_{p12} = 0.5K_{p10}, K_{p22} = 0.9K_{p20}, C_{p12} = C_{p10} = C_{p22} = C_{p20} \). Two sets of sensors are used to measure separately up the data in two sampling period (\( T_{1}, T_{2} \)).

The cross-correlation of the acceleration of the front and rear suspensions \( A_{21} = 5363.5044, A_{22} = 684.6135 \) as well as the cross-correlation of the new conditions \( B_{21} = 5.1706 \times 10^{9}, B_{22} = 6.6446 \times 10^{8} \) are calculated.

In time of detection, the vehicle shall travel in the same track with the speed of \( v \), the acceleration and angular acceleration of the bogie are acquired in sampling period of \( T \). Two sets of sensors are used to measure separately up the data in two sampling period (\( T_{1} = 0.001, T_{2} = 0.002 \)). The cross-correlation of the acceleration of the front and rear suspensions \( C_{21} = 5444.0380, C_{22} = 704.2403 \) as well as the cross-correlation of the new conditions \( D_{21} = 5.2447 \times 10^{9}, D_{22} = 6.8345 \times 10^{8} \) are calculated. To simplify the following deduction, the following equation shall be applied.

\[ W_{21} = 4C_{p10} C_{p20} / T_{1}^2 + 4(K_{p10} - C_{p10} / T_{1})(K_{p20} - C_{p20} / T_{1}) \]  

\[ R_{21} = 4C_{p10} C_{p20} / T_{1}^2 + 4(K_{p12} - C_{p12} / T_{1})(K_{p22} - C_{p22} / T_{1}) \]  

\[ Q_{21} = T_{2}^2 (C_{p20} + C_{p10})(m_{b} + I_{b} / L_{b}^2) / T_{1} + 4C_{p10} C_{p20} / 2 \]  

The following equation shall be applied to simplify the equation.

\[ (B_{21} - Q_{21} A_{21}) / W_{21} = (D_{21} - P_{21} C_{21}) / R_{21} \]  

\[ (B_{22} - Q_{22} A_{22}) / W_{22} = (D_{22} - P_{22} C_{22}) / R_{22} \]

\[ M_{1} = W_{21}(D_{21} - P_{21} C_{21}) / (B_{21} - Q_{21} A_{21}) - 2C_{p10} C_{p20} / T_{1}^2 \]

\[ M_{2} = (D_{22} - P_{22} C_{22}) / (B_{22} - Q_{22} A_{22}) W_{22} - 2C_{p10} C_{p20} / T_{1}^2 \]

Then the equation (19) and (20) are changed to another form.

\[ C_{p10}(K_{p22} + K_{p12}) / T_{1} + K_{p12} K_{p22} = M_{1} \]

\[ -C_{p10}(K_{p22} + K_{p12}) / T_{2} + K_{p12} K_{p22} = M_{2} \]

To make the result simple, the following equations shall be employed.

\[ a_{1} = T_{2}(M_{1} - M_{2}) / (T_{1} - T_{2}) / C_{p10} \]

\[ b_{1} = (M_{1} T_{2} - M_{2} T_{1}) / (T_{1} - T_{2}) \]

Then the coefficients of the springs are calculated below.

\[ K_{p12} = (a_{1} \pm \sqrt{a_{1}^2 - 4b_{1}}) / 2 \]

There are two possible results:

1. \( K_{p12} = 877444.8006 \), \( K_{p22} = 452167.6636 \)
2. \( K_{p12} = 452167.6636 \), \( K_{p22} = 877444.8006 \)

However, it is impossible to distinguish which spring is damaged more severely. The RMS (root mean square) helps solve the problem.
The monitoring of the accelerations of the front and rear suspension by sensors are depicted in figure 3 and 4. Meanwhile, the running RMS shows that the front suspension is destructed more severely than the rear one.

V. CONCLUSION

This paper described an efficacious method for the condition monitoring of the train suspensions using cross-correlations between the measurements of the accelerometers. The method has been shown to be very sensitive and reliable in distinguishing different fault conditions. It is advantageous in dealing with complex cases (more than two faults). The measurements required are simple to obtain and the process is simple, which means easy to implement practically. Although the vertical suspensions are the subject of the study in this paper, the technique may be equally applied to monitor conditions of other suspensions where the interactions may be introduced by the component failures including primary, secondary, vertical, and lateral suspensions.

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REFERENCE
