A Class of Weak Khartitonov Regions for Robust Stability of Linear Uncertain Systems

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Abstract—In this note, the Khartitonov’s theorems are generalized to the problem of so-called weak Khartitonov regions for robust stability of linear uncertain systems. Given a polytope of (characteristic) polynomials $P$ and a stability region $D$ in the complex plane, $P$ is called $D$-stable if the zeros of every polynomial in $P$ is contained in $D$. It is of interest to know whether the $D$-stability of the vertices of $P$ implies the $D$-stability of $P$. A simple approach is developed which unifies and generalizes many known results on this problem.

I. INTRODUCTION

Consider a family of characteristic polynomials $P$ associated with a linear dynamic system containing parameter perturbations

$$
\mathcal{P} = \left\{ p(s, q) = \sum_{i=0}^{m} a_i(q) s^i : q \in \mathcal{Q} \right\}, \quad a_i(q) \neq 0, \forall q \in \mathcal{Q}
$$

where

$$
q = [q_1, q_2, \ldots, q_m]^T
$$

is the vector of perturbation parameters with each $q_i$ varying in the bounding rectangle

$$
\mathcal{Q} = \{ t_i + jw_i : t_i \leq t \leq t_i, w_i \leq w \leq w_i \} \subseteq \mathbb{C},
$$

$$
\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \cdots \times \mathcal{Q}_m
$$

is the bounding set of $q$, and $a_i(q)$ is the $i$th coefficient of $p(s, q)$. It is assumed that $a_i(q)$ are affine functions of $q$ and that each $\mathcal{Q}_i$ contains zero. Under these assumptions, we can rewrite $p(s, q)$ in (1) as

$$
p(s, q) = p^0(s) + \sum_{i=1}^{m} q_i p_i(s)
$$

where $p^0(s)$ is the nominal polynomial which is obtained from $p(s, q)$ by setting $q = 0$, and $p_i(s)$ are the perturbation polynomials, obtained from $p(s, q) = p^0(s)$ by setting $q_i = 1$ and $q_k = 0, \forall k \neq i$. Accordingly, the family of polynomials $P$ in (1) becomes the so-called polytope of polynomials and can be rewritten as

$$
P = \left\{ p^0(s) + \sum_{i=1}^{m} q_i p_i(s) : q_i \in \mathcal{Q}_i, \quad i = 1, 2, \cdots, m \right\}.
$$

Many systems with parameter variations can be captured by the aforementioned description. A simple example is the so-called interval polynomial for which

$$
p(s, q) = p^0(s) + \sum_{i=0}^{n} q_i s^i,
$$

i.e., each coefficient of the polynomial lies in a given interval which reflects the inaccuracy of the coefficient due to modeling or estimation error. Another trivial example is

$$
p(s, q) = p(s, [k, r]) = s^2 + (2 + r + k)s + r + k
$$

which is the characteristic polynomial of the unity feedback system with open-loop transfer function equal to $k(s+1)/(s+2)(s+r)$, where $k$ and $r$ are uncertain gain and time constant, respectively. For further engineering motivation of this type of polynomials, the reader is referred to, among numerous papers and books, [1]-[3] and the references therein.

For convenience, we denote

$$
p = \{ p^0(s), p_1(s), \ldots, p_m(s) \}.
$$

The set of vertex polynomials of $P$ is given by

$$
V_P = \{ p(s, q) : q_i \in [q_{i1}, q_{i2}, q_{i3}, q_{ia}], \quad i = 1, 2, \cdots, m \}
$$

where $q_{i1}, q_{i2}, q_{i3},$ and $q_{ia}$ are the vertices of $\mathcal{Q}_i$. Note that if the perturbation parameter $q_i$ is purely real, the $\mathcal{Q}_i$ becomes an interval and the number of its vertices is dropped to two.

Given the family of (characteristic) polynomials as in (1) and a stability region $D$ in $\mathbb{C}$ (the complex plane), it is of interest to determine whether the zeros of every polynomial in $P$ are contained in $D$. The stability regions are usually subsets of $C_\infty$ (the open left-half plane) for continuous-time systems, or subsets of the open unit disk for discrete-time systems.

We now give the definitions of $D$-stability, anti-$D$-stability, and weak Khartitonov regions. In the following, $D^c$ and $\partial D$ denote the complement and the boundary of $D$, respectively.

**Definition 1.1** [1, 4]: Given an open set $D \subseteq \mathbb{C}$, a polynomial $p(s)$ is called $D$-stable (respectively, anti-$D$-stable) if every zero of $p(s)$ is contained in $D$ (respectively, $D^c$, including $\partial D$). A family of polynomials $P$ is called $D$-stable (respectively anti-$D$-stable) if every polynomial in $P$ is $D$-stable (respectively anti-$D$-stable).

**Definition 1.2**: Let $p$ be given in (8). A set $D \subseteq \mathbb{C}$ is called a weak Khartitonov region with respect to $p$ if the following condition holds: For an arbitrary bounding set $\mathcal{Q}$ of the form (4) and (3), $P$ in (6) is $D$-stable if and only if $V_P$ in (9) is $D$-stable.

The notion of weak Khartitonov region comes from the seminal work by Khartitonov [5, 6] where he considered the special case for which $D = C_\infty$ and $P$ is an interval polynomial as in (7). He showed that $P$ is $C_\infty$-stable if and only if $V_P$ is $C_\infty$-stable, and furthermore, if and only if eight special vertex polynomials in $V_P$...
are $C_\text{-stable}$ or four special ones when the coefficients of the interval polynomials are purely real. Since the later result requires checking much less number of polynomials than the former one and this number is independent of the polynomial degree, it is often referred to as the strong version while the former result the weak version.

The objective of this paper is as follows: given a family of polynomials $P$ as in (6) and a stability region $D \subset C$, determine whether $D$ is a weak Khariitonov region. The most pertinent results to this note are those by Petersen [7], [4], Soh and Berger [8], Soh [9], Hollot and Bartlett [10], Kraus et al. [11], and Biadas and Garloff [12]. In [7], the regions in $C_-$ which can be mapped onto $C_+$ by the so-called strongly admissible rational functions [13] are considered and a number of interesting regions are found to be weak Khariitonov regions. In [4], it is shown that $C_-$ is a weak Khariitonov region if $p^r(s)$ are all anti-$D$-stable. In [8], [9], some sectors in the left-hand plane are proven to be weak Khariitonov regions provided that the polynomial coefficients are real. In [10], [11], some special conditions on $p^r(s)$ are found for the open unit disc to be a weak Khariitonov region. In [12], Polynomials with even or odd perturbations, i.e., each $p^r(s)$ is either an even or odd polynomial, are shown to be Hurwitz if the vertex polynomials are Hurwitz.

In this note, a new approach to the problem of weak Khariitonov regions is developed using the concept of decreasing phase property for stability region $D$ defined as follows.

**Definition 1.3:** Given a stability region $D \subset C$ and the polynomial vector $p$ in (8), $D$ is said to hold the decreasing phase property if, for an arbitrary $n$th order $D$-stable polynomial $f(s)$ and $1 \leq i \leq m$, $\arg p^r(s)/f(s)$ is monotonically decreasing except at $p^r(s) = 0$ when $s$ traverses on $\partial D$ in the counterclockwise direction (or, for short, monotonically decreasing on $\partial D$).

We now end our introduction with a key theorem which links the problem of weak Khariitonov regions to the decreasing phase property discussed previously.

**Theorem 1.1 (see the Appendix for Proof):** Let an open set $D \subset C$ and $p = (p^0(s), p^1(s), \ldots, p^m(s))$ be given. Then $D$ is a weak Khariitonov region with respect to $p$ if $D$ holds the decreasing phase property.

**II. WEAK KHARIITONOV REGIONS**

In this section, Theorem 1.1 is used to derive a number of useful weak Khariitonov regions for both continuous-time and discrete-time systems. These results unify and generalize many known results in [4]–[12].

**Theorem 2.1:** Any (rotated) open-half plane

$$D = \{x + jy: a + bx + cy < 0\}, \quad a, b, c \in \mathbb{R} \quad (10)$$

is a weak Khariitonov region with respect to $p$ in (8) for any $p^r(s)$ if $p^r(s), i = 1, 2, \ldots, m$ are anti-$D$-stable.

**Proof:** Suppose $p^r(s), i = 1, 2, \ldots, m$ are anti-$D$-stable. Let $f(s)$ be an arbitrary $n$th order $D$-stable polynomial. It is straightforward to see that $\arg p^r(s)/f(s)$ is monotonically decreasing on $\partial D$ because $\arg p^r(s)$ (respectively, $\arg f(s)$) are monotonically noncreasing (respectively, increasing). Therefore, it follows from Theorem 1.1 that $D$ is a weak Khariitonov region with respect to $p$.

**Remark:** The aforementioned theorem is an extension to the main result in [4] where $D = C_-$ is considered.

**Corollary 2.1 [7]:** Any open-half plane

$$D = \{x + jy: x < -a + by\}, \quad a, b \in \mathbb{R} \quad (11)$$

is a weak Khariitonov region with respect to $p = (p^0(s), 1, s, \ldots, s^n)$ for any $p^r(s)$ of $n$th-order. In particular, $C_-$ is a weak Khariitonov region with respect to the aforementioned $p$ in [5].

**Theorem 2.2 [7], [8]:** Any open region

$$D = \{x + jy: a_i - b_j | x | < y < -a_i + b_j | x |, x < -a_j\}, \quad a_i \geq 0, b_j > 0 \quad (12)$$

is a weak Khariitonov region with respect to $p = (p^r(s), 1, s, \ldots, s^n)$ for any $p^r(s)$ of $n$th-order.

**Proof:** The proof is essentially identical to that of Theorem 2.1.

**Theorem 2.3:** Any open circular region

$$D = \{s: c + r \exp(i\theta) : 0 \leq r < c, 0 \leq \theta \leq 2\pi\}, \quad c \in \mathbb{C}, r > 0 \quad (13)$$

is a weak Khariitonov region with respect to $p$ in (8) for any $p^r(s)$ if $p^r(s), i = 1, 2, \ldots, m$ are anti-$D$-stable.

**Proof:** Let $f(s)$ be any $n$th-order $D$-stable polynomial. From Theorem 1.1, it is sufficient to show that $\arg p^r(s)/f(s), 1 \leq i \leq m,$ is monotonically decreasing when $s$ traverses on $\partial D$. Let $z_1$ and $z_2$ be any zeros of $f(s)$ and $p^r(s)$, respectively, see Fig. 1. We claim that $\arg (s - z_1)/(s - z_2)$ is monotonically decreasing. To see this, we divide $\partial D$ into $L_1$ and $L_2$ according to the tangent points $A$ and $B$ in Fig. 1. When $s$ traverses on $L_1$, $\arg (s - z_1)/(s - z_2)$ is obviously decreasing because $\arg (s - z_1)$ is increasing and $\arg (s - z_2)$ is decreasing. Now suppose $s$ traverses on $L_2$ and $\theta$ is increased by $\partial D$. Note that both $\arg (s - z_1)$ and $\arg (s - z_2)$ are increased. Therefore, we need to prove that the increment $\partial \theta_1$ of $\arg (s - z_1)$ is greater than the increment $\partial \theta_2$ of $\arg (s - z_2)$. This is not difficult to see from Fig. 1 because $\partial \theta_1 > \partial \theta_2$, $\partial \theta_2 \leq \partial \theta_1$, and $\partial \theta_1 = \partial \theta_1 = \partial \theta_2 / 2$. Consequently, $\arg (s - z_1)/(s - z_2)$ is monotonically decreasing on $L_2$. Hence, our claim holds. We then conclude that $\arg p^r(s)/f(s)$ is monotonically decreasing on $\partial D$ because the number of zeros of $p^r(s)$ is no more than that of $f(s)$.

**Corollary 2.2 [7]:** Any open circular regions

$$D = \{x + jy : (x - a)^2 + y^2 < r^2\}, \quad 0 \leq r \leq a \quad (14)$$

and

$$D = \{x + jy : (x - a)^2 + y^2 < r^2\}, \quad 0 \leq r \leq a \leq 1/2 \quad (15)$$

are weak Khariitonov regions with respect to $p = (p^r(s), 1, s, \ldots, s^n)$ for any $p^r(s)$ of $n$th-order.

**Theorem 2.4:** Any open parabolic region

$$D = \{x + jy : (ax)^2 - (by)^2 > 1, x < 0\}, \quad a, b > 0 \quad (16)$$

is a weak Khariitonov region with respect to $p^r(s) = (p^r(s), 1, s, \ldots, s^n)$ for any $p^r(s)$ of $n$th-order.
Proof: The proof is essentially identical to that of Theorem 2.1.

Theorem 2.5 [9]: Any region
\[ D = \{ x + jy : x < 0, a \leq |y| < b \leq |x| \}, \quad b \geq a > 0 \]  
(17)
is a weak Kharitonov region with respect to \( p(s) = (p_d(s), 1, s, \ldots, s^n) \) for any \( p_d(s) \) of \( n \)-th order provided that the parameters \( q_i, i = 1, 2, \ldots, n \) and the coefficients of \( p_d(s) \) are real.

Proof: Let \( f(s) \) be any \( n \)-th order \( D \)-stable polynomial with zeros given by \( z_1, z_2, \ldots, z_n \), where \( z_n \) denotes the complex conjugate of \( z_k \), \( \text{Im}(z_n) > 0 \). By Theorem 1.1, it is sufficient to show that arg \( s'f(s) \) is monotonously decreasing as \( s \) traverses \( \partial D \) for any \( 0 \leq i \leq n \). Note that \( \partial D \) is given by \( |y| = b \) or \( |y| = a \). We first observe that arg \( s' \) is fixed on \( \partial D \). Therefore, we only need to show that arg \( (s - z_k)(s - z_n) \) is monotonously increasing on each \( \partial D \). This holds trivially on \( |y| = b \). On \( y = a \), this holds because arg \( (s - z_k) \) increases faster than arg \( (s - z_n) \). A similar argument applies to \( y = -a \). Therefore, arg \( s'f(s) \) is monotonously decreasing on \( \partial D \).

Theorem 2.6: Every open convex region \( D \subset C \) is a weak Kharitonov region with respect to \( p_d(s) \) for any \( p_d(s) \).

Proof: Let \( f(s) \) be any \( D \)-stable polynomial. It is obvious that arg \( 1/f(s) \) is monotonously decreasing on \( \partial D \). Therefore, it follows from Theorem 1.1 that \( D \) is a weak Kharitonov region with respect to \( p_d(s) \).

Theorem 2.7 [12]: Let \( p \) be given in (8) satisfying the following condition: for each \( i = 1, 2, \ldots, m \), either \( \text{Re} p_i(j\omega) = 0 \) or \( \text{Im} p_i(j\omega) = 0 \). Then \( C_p \) is a weak Kharitonov region with respect to \( p \).

Proof: Let \( f(s) \) be an arbitrary \( n \)-th order \( C_p \)-stable polynomial. From Theorem 1.1, it is sufficient to show that arg \( p_i(j\omega)/f(j\omega) \) is monotonously decreasing when \( \omega \) increases except at \( p_i(j\omega) = 0 \), \( i = 1, 2, \ldots, m \). This is obvious because arg \( f(j\omega)/p_i(j\omega) \) is either purely real or purely imaginary without phase change.

Theorem 2.8 [11]: The open unit disk is a weak Kharitonov region with respect to
\[ p = \left( p_d(s), 1 + s^n, 1 - s^n, s + s^{-1}, s - s^{-1}, \ldots, s^{n/2} + s^{-n/2}, s^{n/2} - s^{-n/2} \right) \]
for any \( p_d(s) \) of \( n \)-th order, where \([ \cdot ]\) denote the integer part.

Proof: Let \( D \) be the open unit disk and \( f(s) \) be any \( n \)-th order \( D \)-stable polynomial. Note that \( \partial D = \{ \exp(j\theta) : 0 \leq \theta \leq 2\pi \} \). By applying Theorem 1.1, it is sufficient to show that arg \( s' \pm z^{n-i}/f(s) \) is monotonously decreasing for any \( i \in [n/2] \) when \( s \) traverses \( \partial D \). Note that
\[ \exp(j\theta) = \exp(j(n - i)\theta) \]
\[ = \exp(j\theta) \left( \exp(j(i - n/2)\theta) \pm \exp(j(n/2 - i)\theta) \right) \]
\[ = \begin{cases} 2 \cos \left( j(i - n/2)\theta \right) & \exp(jn/2 \theta) \\ 2j \sin \left( j(i - n/2)\theta \right) & \exp(jn/2 \theta) \end{cases} \]

and its phase is either \( \pi \theta / 2 \) or \( (\pi + \pi n/2) \). Let \( z_k, k = 1, 2, \ldots, n \) be the zeros of \( f(s) \), \( z = \exp(j\theta) \) and suppose \( \theta \) is increased by \( \pi \theta / 2 \), as shown in Fig. 2. Then, arg \( s' \pm z^{n-i} \) is increased by \( \pi \theta / 2 \). On the other hand, arg \( f(s) \) is increased by more than \( \pi \theta / 2 \) because \( \pi \theta / 2 > \pi \theta \). Consequently, arg \( s'f(s) \) is monotonously decreasing on \( \partial D \).

Theorem 2.9 [10]: The unit disk is a weak Kharitonov region with respect to \( p = (p_d(s), 1, s, \ldots, s^{n/2}) \) for any \( p(s) \) of \( n \)-th order.

Proof: The proof of this Theorem is exactly the same as that of Theorem 2.8 except that \( s' \pm s^{n-i} \) is replaced by \( s' \) and that arg \( s' \) is increased by only \( 2\pi \) rather than \( \pi \theta / 2 \) when \( \theta \) is increased by \( \pi \theta / 2 \).

To summarize, the weak Kharitonov regions are tabulated in Tables I and II for continuous-time and discrete-time systems, respectively. It should be noted, however, more weak Kharitonov regions can be constructed by 1) applying Theorem 1.1 on other special uncertain polynomial (e.g., low-order polynomial, 2) using the fact that the intersection of weak Kharitonov regions is a weak Kharitonov region [7]; 3) relaxing the requirement of the decreasing phase property in Theorem 1.1.

APPENDIX

Proof of Theorem 1.1

The following lemma is essential in the proof of Theorem 1.1.

Lemma 1: Given an open stability region \( D \subset C \) and \( n \)-th order \( D \)-stable polynomials \( f_k(s) \) and \( f_k(s) + f_k(s) \) with positive leading coefficients. Suppose arg \( f_k(s)/f_k(s) \) is monotonously decreasing on \( \partial D \). Then, the polynomial
\[ f(s, \alpha) = f_k(s) + \alpha f_k(s) \]
is \( D \)-stable for all \( 0 < \alpha < 1 \).

Proof: Let \( \Gamma \subset C \) denote the trajectory of \( f(s)/f(s) \) as \( s \) traverses \( \partial D \), i.e.,
\[ \Gamma = \{ f(s)/f(s) : s \in \partial D \} \]

Since \( f_k(s) \) is \( D \)-stable and \( \text{deg} f_k(s) \leq \text{deg} f_k(s) \), \( \Gamma \) is a bounded and closed curve. Therefore, arg \( f(s)/f(s) \) is monotonously decreasing implying that \( \Gamma \) encloses the origin. On the other hand, the point \( -1 + j\theta \) is not encircled by \( \Gamma \) because \( f_k(s) + f_k(s) \) is \( D \)-stable (principle of argument). Consequently, using the facts that arg \( f(s)/f_k(s) \) is monotonously decreasing again and that \( \Gamma \) encloses the origin, the interval \( (-\infty, -1) \) is not encircled by \( \Gamma \). In particular, the point \( 1/\alpha + j\theta \) is not encircled by \( \Gamma \). Therefore, \( f(s) + \alpha f_k(s) \) is \( D \)-stable (principle of argument).

Proof of Theorem 1.1: Suppose \( V_p \) is \( D \)-stable. Define, for \( i = 1, 2, \ldots, m \)
\[ g_{2i-1}(s) = \left( \hat{t}_i - t_i \right) p_i(s) \]
\[ g_{2i}(s) = \hat{t}_i - t_i, p_i(s) \]
TABLE I
WEAK KHARITONOV REGIONS FOR CONTINUOUS-TIME SYSTEMS

<table>
<thead>
<tr>
<th>$p$</th>
<th>$D$</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p_0(s), p_1(s), \ldots, p_m(s))$</td>
<td>(10), (13)</td>
<td>$p_0(s)$ is anti-$D$-stable</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$\left{ \begin{array}{l} \omega_1 - \omega_i \ \omega_i - \omega_i \ 0 \end{array} \right.$</td>
<td>$\Re p_0(j \omega) = 0$ or $\Im p_0(j \omega) = 0, 1 \leq i \leq m$</td>
</tr>
<tr>
<td>$(p_0(s), 1, s, \ldots, s^n)$</td>
<td>(11), (12), (14), (16)</td>
<td>none</td>
</tr>
<tr>
<td>$(p_0(s), 1)$</td>
<td>any open convex set</td>
<td>real parameters and coefficients</td>
</tr>
</tbody>
</table>

TABLE II
WEAK KHARITONOV REGIONS FOR DISCRETE-TIME SYSTEMS

<table>
<thead>
<tr>
<th>$p$</th>
<th>$D$</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p_0(s), p_1(s), \ldots, p_m(s))$</td>
<td>the open unit disk or any open circular region inside of it</td>
<td>$p_0(s)$ is anti-$D$-stable</td>
</tr>
<tr>
<td>$(p_0(s), 1, s, \ldots, s^n)$</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>$(p_0(s), 1, s, \ldots, \frac{1}{s^n-1})$</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>$(p_0(s), 1, s^n, \frac{1}{s^n-1})$</td>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

\[
\alpha_{2l-1} = \begin{cases} 1 - \frac{\omega_i}{\omega_i} - l_i, & \text{if } l_i \neq 1, \\ 1 - \frac{\omega_i}{\omega_i}, & \text{if } l_i = 1, \\ 0, & \text{if } l_i = \omega_i, \end{cases}
\]

\[
\alpha_{2l} = \begin{cases} \omega_i - \omega_i, & \text{if } \omega_i \neq \omega_i, \\ \omega_i - \omega_i, & \text{if } \omega_i = \omega_i, \\ 0, & \text{if } \omega_i = \omega_i, \end{cases}
\]

\[
f_0(s) = p_0(s) + \sum_{l=1}^{m} \left( l_i + j\omega_i \right) p_i(s)
\]

and, for any $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{2m})^T$ and $1 \leq l \leq 2m$

\[
f_l(s, \alpha) = f_0(s) + \sum_{k=1}^{l} \alpha_k g_k(s).
\]

Note that

\[
f_l(s, \alpha) = f_{l-1}(s, \alpha) + \alpha_l g_l(s)
\]

and that any polynomial in $P$ can be expressed by $f_{2m}(s, \alpha)$ for some $\alpha$ with $0 \leq \alpha_i \leq 1$, $k = 1, 2, \ldots, 2m$. From the decreasing phase property of $P$, we know that, for any nth-order D-stable polynomial $f(s)$, arg $g_k(s)/f(s)$ is monotonically decreasing on $\Delta D$, $k = 1, 2, \ldots, m$.

Given an arbitrary polynomial $f_{2m}(s, \alpha) \in P$, we need to prove that $f_{2m}(s, \alpha)$ is D-stable by reductio ad absurdum. That is, we assume $f_{2m}(s, \alpha)$ is not D-stable and show that there exists some vertex polynomial of $P$ which is also not D-stable. Indeed, according to Lemma 1, $f_{2m}(s, \alpha)$ being not D-stable implies that either $f_{2m-1}(s, \alpha)$ or $f_{2m-2}(s, \alpha)$ is not D-stable. Without loss of generality, we may assume that $f_{2m-1}(s, \alpha)$ is not D-stable. Using Lemma 1 again, we further obtain that either $f_{2m-2}(s, \alpha)$ or $f_{2m-3}(s, \alpha) + \alpha_{2m-1} g_{2m-1}(s)$ is not D-stable. Continuing with this process repeatedly, we will eventually have either $f_0(s)$ or another vertex polynomial of $P$ to be not D-stable. This conclusion contradicts the assumption that $V_P$ is D-stable. Therefore, $f_{2m}(s, \alpha)$ must be D-stable. Since $f_{2m}(s, \alpha)$ is an arbitrary polynomial in $P$, $D$ must be a weak Kharitonov region with respect to $p.$

REFERENCES


