Rudder Roll Stabilisation of Ships

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We present a case study of control system design for rudder-based stabilisers of ships using RHC.

The rudder’s main function is to correct the heading of a ship; however, depending on the type of ship, the rudder may also be used to produce, or correct, roll motion.

Rudder roll stabilisation consists of using rudder-induced roll motion to reduce the roll motion induced by waves.

An automatic control system is necessary to provide the rudder command based on measurements of ship motion.
Reduced roll motion is important for it can affect the performance of the ship, as indicated in the following considerations.

- Transverse accelerations that occur due to rolling interrupt tasks performed by the crew. This increases the amount of time required to complete a mission.

- Roll accelerations may produce cargo damage, for example, on soft loads such as fruit.

- Roll motion increases hull resistance.

- Large roll angles limit crew capability to handle equipment on board, and/or to launch and recover systems.
Amongst the different types of stabilisers, rudder-based stabilisation is a very attractive technique. The reasons for this are that almost every ship has a rudder (thus no extra equipment may be necessary), and also this technique can be used in conjunction with other stabilisers.

As we shall see, this design problem is far from trivial.
To describe the motion of a marine vehicle, two reference frames are considered: an *inertial* frame and a *body-fixed* frame.
The motion of a marine vehicle can be considered in six degrees of freedom.

The table shows the notation used to describe the different motion components.

<table>
<thead>
<tr>
<th>translation</th>
<th>surge</th>
<th>sway</th>
<th>heave</th>
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<tr>
<td>position</td>
<td>$x$</td>
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<td>$z$</td>
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<tr>
<td>linear rate</td>
<td>$u$</td>
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<table>
<thead>
<tr>
<th>rotation</th>
<th>roll</th>
<th>pitch</th>
<th>yaw</th>
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<tr>
<td>angle</td>
<td>$\phi$</td>
<td>$\theta$</td>
<td>$\psi$</td>
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<tr>
<td>angular rate</td>
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For marine vehicles position and orientation are described relative to the inertial reference frame, whilst linear and angular velocities are expressed in the body-fixed frame. This choice is convenient since some of these magnitudes are measured on board, and thus, relative to the body-fixed frame.

A convenient abstraction to obtain a mathematical model that captures the different effects that give rise to ship motion, is to separate the motion due to control action (rudder motion) from the motion due to the waves. This abstraction results in two models that can be combined using superposition.
The first part of the model can be obtained using Newton’s laws. This approach yields a nonlinear model that describes the motion components in terms of the forces and moments acting on the hull. By linearising this model, we can obtain a linear state space model that describes the ship response due to the rudder (control) action.

A discrete time version of this model can be expressed as

$$x_{k+1}^c = A_c x_k^c + B_c u_k,$$

where the state $x_k^c$ and control $u_k$, are given by

$$x_k^c = \begin{bmatrix} v_k^c & p_k^c & r_k^c & \phi_k^c & \psi_k^c \end{bmatrix}^T$$

and $u_k = \alpha_k$,

with $\alpha_k$ being the current rudder angle.
The parameters of the model (1), the values of the entries of the matrices $A_c$ and $B_c$, will vary with the forward speed of the vessel. However, this variation is such that constant values can be considered for different speed ranges. System identification techniques and data collected from tests can be used to estimate the parameters for different speed ranges.

The different sets of parameters can then be used in a gain scheduling-like approach to update the model. This helps to minimise model uncertainty.
The second part of the model incorporates the motion induced by the waves.

The sea surface elevation can be described in stochastic terms by its power spectral density, or *sea spectrum*.

The ship motion induced by the waves can be interpreted as a filtering process made by the ship’s hull, which has a selected response to certain frequencies and attenuates others. The frequency response of the hull due to wave excitation is called the *ship response operator*. 
The total effect can be incorporated into our model as a coloured noise output disturbance. The roll motion induced by the waves will thus be modelled with a shaping filter:

\[
\begin{align*}
  x_{k+1}^w &= A_w x_k^w + w_k, \\
  y_k^w &= x_k^w + v_k,
\end{align*}
\]

where

\[
x^w = [p^w \quad \phi^w]^T,
\]

and \(w_k\) and \(v_k\) are sequences of i.i.d. Gaussian vectors with appropriate dimensions.
Motion due to the Waves

For a given hull shape, the filtering characteristics of the hull depend on the forward speed of the ship $U$, and the heading angle relative to the waves: the encounter angle $\chi$. 
Motion due to the Waves

The variations in the characteristics of the motion response due to speed and encounter angle are the consequence of a *Doppler-like effect*. The frequency observed from the ship is called the *encounter frequency* $\omega_e$, and is, in general, different from the *wave frequency* $\omega_w$ (observed from a fixed-reference frame):

$$\omega_e = \omega_w - \frac{\omega_w^2 U}{g} \cos \chi.$$  \hspace{1cm} (4)

The encounter effect produces significant variations in the motion response of the ship even for the same sea state, hereby defined by the sea spectrum. Consequently, the values of the parameters of the model (3) should be updated for changes in different sea states and sailing conditions.
Disturbance Model Parameter Estimation

The model of the disturbance (waves) needs to be updated when the sea state or sailing conditions change. Here, we present a simple approach to estimate the parameters based on the control scheme shown in the figure.
If the stabiliser control loop is open—that is, the rudder is kept to zero angle, and only minor corrections are applied to the rudder to keep the course—we can then use the roll angle and roll rate measurements to estimate the parameters of a second order shaping filter.

Under these conditions, the measurements coincide with the state of the following shaping filter:

\[
\begin{bmatrix}
\phi^w_{k+1} \\
p^w_{k+1}
\end{bmatrix} =
\begin{bmatrix}
\theta_{11} & \theta_{12} \\
\theta_{21} & \theta_{22}
\end{bmatrix}
\begin{bmatrix}
\phi^w_k \\
p^w_k
\end{bmatrix} +
\begin{bmatrix}
w^\phi_k \\
w^p_k
\end{bmatrix}.
\]
By defining the vector

\[ \theta_k = \begin{bmatrix} \theta_{11}(k) & \theta_{12}(k) & \theta_{21}(k) & \theta_{22}(k) \end{bmatrix}^T, \]

we can express the available measurements as

\[ \theta_{k+1} = \theta_k + \theta_{wk}, \]

\[
\begin{bmatrix} \phi^w_k \\ p^w_k \end{bmatrix} = \begin{bmatrix} \phi^w_{k-1} & p^w_{k-1} & 0 & 0 \\ 0 & 0 & \phi^w_{k-1} & p^w_{k-1} \end{bmatrix} \begin{bmatrix} \theta_k \\ \theta_{wk} \end{bmatrix} + \begin{bmatrix} \phi^w_k \\ p^w_k \end{bmatrix}.
\]

The system (5) is in a form that we can apply a Kalman filter to estimate \( \hat{\theta}_{k|k} \) from the measurements \( \phi^w_k \) and \( p^w_k \). The variable \( \theta_{wk} \) represents a small random variable that accounts for unmodelled dynamics.
To assess the proposed method to estimate the parameters of the filter and the quality of the prediction using the above model, a series of simulations were performed for different sea states and sailing conditions.

The parameters to describe the sea state that were used are the average wave period \( T \) and the significant wave height \( H_s \) (average of the highest one third of the wave heights). These parameters are used in the International Towing Tank Conference recommended model for the wave power spectral density, commonly termed \textit{ITTC spectrum} in the marine literature:

\[
S_{\zeta\zeta}(\omega_w) = \frac{172.75H_s^2}{T^4\omega_w^5} \exp\left\{ \frac{-691}{T^4\omega_w^4} \right\} \quad \text{(m}^2\text{sec}/\text{rad}).
\]
Disturbance Model Parameter Estimation

The process for obtaining the ship roll power spectral density is indicated via an example in the figure.
Once the roll power spectral density is obtained, the roll motion realisations (time series) are computed as

$$\phi(t) = \sum_i \bar{\phi}_i \sin(\omega_{ei} t + \theta_i),$$

(7)

where the phases are chosen randomly with a uniform distribution in \([-\pi, \pi]\) and the amplitudes calculated from

$$\bar{\phi}_i^2 = 2S_{\phi\phi}(\omega_{ei})\Delta\omega_i,$$

(8)

where $S_{\phi\phi}(\omega_e)$ represents the roll power spectral density (S-roll in the previous figure).

The number of regular (sinusoidal) components used to simulate the time series is normally between 500 and 1000 to avoid pattern repetition depending on the total simulation time. This procedure for simulating time series for a ship response is a standard practice in naval architecture and marine engineering.
Disturbance Model Parameter Estimation

The measurements taken from one of the realisations were used to estimate the parameters. The figure shows the evolution of the estimates of the parameters. A sampling period of 0.25 sec was adopted based on the value of the roll natural period of the vessel of approx 7 sec.
The figure shows the roll angle and rate predictions for two other different realisations using 5 and 10 step-ahead predictions. This consists of using the measured state as initial condition for model (3) and then running this model forward in time.
From the above example we can see that the filter converges relatively quickly, and the quality of predictions for the behaviour of the ship can be deemed satisfactory.

We note that, for the chosen sailing conditions, the use of a second order disturbance model seems to give good results. For other sea conditions, it may be necessary to resort to higher order models.

This process should be performed before closing the control loop. The proposed control strategy can be considered a quasiadaptive control strategy. That is, if the sailing condition (heading and speed) or the sea state changes, it may be necessary to open the loop and re-estimate the parameters of the disturbance model to avoid significant degradation in the closed loop performance.
Thus, we have:

- A model for the ship motion due to the rudder control action:

\[
    x_{k+1}^c = A_c x_k^c + B_c u_k,
\]

where the state \( x_k^c \) and control \( u_k \), are given by

\[
    x_k^c = \begin{bmatrix} v_k^c & p_k^c & r_k^c & \phi_k^c & \psi_k^c \end{bmatrix}^T \quad \text{and} \quad u_k = \alpha_k,
\]

with \( \alpha_k \) being the rudder angle.

- A model for the roll motion induced by the waves:

\[
    x_{k+1}^w = A_w x_k^w + w_k,
\]

\[
    y_k^w = x_k^w + v_k,
\]

where

\[
    x^w = [p^w \quad \phi^w]^T,
\]

and \( w_k \) and \( v_k \) are sequences of i.i.d. Gaussian vectors.
The complete state space model can then be represented via state augmentation as

$$x_{k+1} = Ax_k + Bu_k + Jw_k,$$
$$y_k = Cx_k + n_k,$$

where $x^T = [x^c_k \ x^w_k]$.

The following measurements are assumed available to implement the control:

$$y_k = \begin{bmatrix} p_k & r_k & \phi_k & \psi_k \end{bmatrix}^T + n_k$$
$$= \begin{bmatrix} (p^c_k + p^w_k) & r^f_k & (\phi^c_k + \phi^w_k) & \psi^f_k \end{bmatrix}^T + n_k,$$

where, $n_k$ is noise introduced by the sensors, and $r^f_k$ and $\psi^f_k$ are the filtered yaw rate and yaw angle respectively.
Note that we have only considered the wave disturbance affecting the roll angle and the roll rate but not the yaw. The reason for this is that, in conventional autopilot design, the yaw is filtered and only low frequency yaw motion is corrected. This is done to avoid the autopilot making corrections to account for the sinusoidal wave-induced yaw motion. Therefore, in the problem we are assuming that the yaw is measured after the yaw wave filter.
The design of a control strategy that uses the rudder for simultaneous course keeping and roll reduction is not a simple task and must be performed so as to best deal with the following issues:

- **Underactuated System.** One control action (rudder force) achieves two control objectives: *roll reduction* and *low heading (yaw) interference*.

A key fact to understand, for the successful application of this technique, is that the dynamics associated with the rudder-induced roll motion are faster than the dynamics associated with the rudder-induced yaw motion. This phenomenon depends on the shape of the hull and the location of the rudder and the centre of gravity of the ship.
Uncertainty. There are three sources of uncertainty:

First, there is incomplete state information available (the necessary sensors can be very expensive).

Second, there are disturbances from the environment (wave-induced motion) that, in principle, cannot be known a priori.

Third, there is uncertainty associated with the accuracy of the model.

Nonminimum Phase System. The response of roll due to rudder action presents nonminimum phase dynamics. This imposes fundamental design limitations and trade-offs.
A Challenging Control Problem

- **Input constraints.** The rudder action demanded by the controller should satisfy *rate* and *magnitude* constraints.

  Rate constraints are associated with safety and reliability. By imposing rate constraints on the rudder command, we ensure an adequate lifespan of the hydraulic actuators.

  Magnitude constraints are associated with performance and economy. Large rudder angles induce flow separation (loss of actuation and poor performance), and a significant increase in drag (resistance). Also, it is desirable to reduce the maximum rudder action at higher speeds to reduce the mechanical loads on the rudder and the steering machinery.
Output constraints. Since the rudder affects the ship heading, it may be necessary to include constraints on the maximum heading deviations allowed when the rudder is used to reduce roll.

Unstable plant. The response of yaw to rudder action is marginally unstable: there is an integrator.

Based on the above considerations, it is evident that the problem of rudder roll stabilisation of ships is a challenging one and, as such, the chosen control strategy plays an important role in achieving high performance.
The basic control objectives for the particular motion control problem being addressed here are as follows:

(i) minimise the roll motion, which includes roll angle and accelerations;

(ii) produce low interference with yaw;

(iii) satisfy input constraints.
In a discrete time framework, all the above objectives are captured in the following optimisation problem.

**Definition (Output Feedback Control with Input Constraints)**

Find the feedback control command $u_k = \mathcal{K}(y_k)$ that minimises the objective function

$$V = \lim_{N \to \infty} \frac{1}{N} E \left\{ \sum_{k=0}^{N} y_k^T Q y_k + (y_{k+1} - y_k)^T S (y_{k+1} - y_k) + u_k^T R u_k \right\}$$

subject to the system equations

$$x_{k+1} = A x_k + B u_k + J w_k,$$
$$y_k = C x_k + n_k,$$

and the input constraints $|u_k| \leq u_{\text{max}}$ and $|u_{k+1} - u_k| \leq \delta u_{\text{max}}$. 

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Output Feedback Control

Choosing the matrices $Q$ and $S$ as:

$$Q = \text{diag}\{Q_p, 0, Q_\phi, Q_\psi\},$$
$$S = \text{diag}\{S_p, 0, 0, 0\},$$

the objective function (10) becomes (assuming no sensor noise)

$$V = \lim_{N \to \infty} \frac{1}{N} E\left\{\sum_{k=0}^{N} \left[ Q_p p_k^2 + Q_\phi \phi_k^2 + S_p (p_{k+1} - p_k)^2 \right] + Q_\psi \psi_k^2 + R \alpha_k^2 \right\}. \quad (11)$$

The objective function (11) incorporates a term that weights the roll accelerations via the difference $p_{k+1} - p_k$. The reason is that both roll angle and roll acceleration affect the performance of the ship; and are directly related to the criteria often used to evaluate ship performance in the marine environment.
Finite Horizon Optimal Control Problem

The problem defined above is not easy to solve due to the presence of constraints. We will approximate its solution using the certainty equivalent solution of an associated finite horizon problem, together with a receding horizon implementation.

Given the initial condition $\tilde{x}_0$, we seek the sequence of control moves

$$\{\tilde{u}_{0}^{\text{opt}}(\tilde{x}_0), \ldots, \tilde{u}_{N-1}^{\text{opt}}(\tilde{x}_0)\}$$

that minimises the objective function

$$V_N \triangleq \frac{1}{2} \tilde{x}_N^T \hat{P} \tilde{x}_N + \sum_{j=0}^{N-1} \frac{1}{2} (\tilde{x}_j^T \hat{Q} \tilde{x}_j + \tilde{u}_j^T \hat{R} \tilde{u}_j + \tilde{u}_j^T \hat{T} \tilde{x}_j + \tilde{x}_j^T \hat{T}^T \tilde{u}_j),$$

subject to

$$\tilde{x}_{j+1} = A \tilde{x}_j + B \tilde{u}_j,$$

$$\tilde{y}_j = C \tilde{x}_j,$$
and the constraints

\[|\dot{u}_j| \leq u_{\text{max}} \quad \text{and} \quad |\dot{u}_{j+1} - \dot{u}_j| \leq \delta |u_{\text{max}}|.\]

The augmented state \(\tilde{x}\) in (14) is given by

\[
\tilde{x} = \begin{bmatrix}
\dot{v}^c \\
\dot{p}^c \\
\dot{r}^c \\
\dot{\phi}^c \\
\dot{\psi}^c \\
\dot{\phi}^w \\
\dot{p}^w
\end{bmatrix}^T.
\]

The matrices in the objective function are

\[
\check{Q} = (A - I)^T(C^TSC)(A - I) + C^TQC,
\]

\[
\check{R} = B^T(C^TSC)B + R, \quad \check{T} = B^T(C^TSC)(A - I).
\]

The matrices \(Q, S\) and \(R\) are the parameters defining the objective function (10), and the matrices \(A, B\) describing the augmented system are

\[
A = \begin{bmatrix}
A^c & 0 \\
0 & A^w
\end{bmatrix}, \quad B = \begin{bmatrix}
B^c \\
0
\end{bmatrix},
\]

(15)

The matrix \(\check{P}\) in (13) is taken as the solution of the corresponding discrete time algebraic Riccati equation.
Posing the Problem as a QP

The QP solution of the above problem is given by

\[ u^{\text{OPT}}(\dot{x}_0) = \arg \min_{u \leq W} \frac{1}{2} u^T(H_1 + H_2)u + u^T(F_1 + F_2)\dot{x}_0, \quad (16) \]

where

\[ H_1 = \Gamma^TQ\Gamma + R, \quad H_2 = \tilde{T}\tilde{\Gamma} + \tilde{\Gamma}^T\tilde{T}, \]
\[ F_1 = \Gamma^TQ\Omega, \quad F_2 = \tilde{T}\tilde{\Omega}, \]

\[ Q = \text{diag}\{\breve{Q}, \ldots, \breve{Q}, \breve{P}\}, \]
\[ R = \text{diag}\{\breve{R}, \ldots, \breve{R}\}, \]
\[ T = \text{diag}\{\breve{T}, \ldots, \breve{T}\}, \]
Converting the Problem to a QP

\[ \Omega = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, \quad \bar{\Omega} = \begin{bmatrix} I \\ A \\ \vdots \\ A^{N-1} \end{bmatrix}, \]

\[ \Gamma = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix}, \quad \bar{\Gamma} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-2}B & A^{N-3}B & \cdots & 0 \end{bmatrix}. \]
Converting the Problem to a QP

The matrices $L$ and $W$ that define the constraint set in (16) are given by

$$L = \begin{bmatrix} I & E \\ -I & -E \end{bmatrix}; \quad W = \begin{bmatrix} \bar{M}_{\text{mag}} \\ \bar{M}_{\text{rate}} \\ \bar{M}_{\text{mag}} \\ \bar{M}_{\text{rate}} \end{bmatrix}$$

where $I$ is the $N \times N$ identity matrix and

$$E = \begin{bmatrix} 1 & \ldots & 0 \\ -1 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & \ldots & -1 & 1 \end{bmatrix}, \quad \bar{M}_{\text{mag}} = \begin{bmatrix} u_{\text{max}} \\ \vdots \\ u_{\text{max}} \end{bmatrix}, \quad \bar{M}_{\text{rate}} = \begin{bmatrix} u_{-1} + \delta u_{\text{max}} \\ \delta u_{\text{max}} \\ \vdots \\ \delta u_{\text{max}} \end{bmatrix}.$$  

The above problem is be solved on line, and the implicit receding horizon feedback control law is implemented, that is,

$$u_k = \mathcal{K}_N(\dot{x}_0(y_k)) = \tilde{u}_0^{\text{OPT}}(\dot{x}_0(y_k)),$$

the first element of the optimal sequence (12).
Using Certainty Equivalence

Using the certainty equivalence principle, the initial condition for solving the above problem is provided by a Kalman filter. That is, at each step \( k \) we take \( \dot{x}_0 = \hat{x}_{k|k} \), where:

\[
\begin{align*}
\text{Prediction:} \\
\hat{x}_{k|k-1} &= A\hat{x}_{k-1|k-1} + Bu_{k-1}, \\
\Sigma_{k|k-1} &= A\Sigma_{k-1|k-1}A^T + R_w.
\end{align*}
\]

\( (17) \)

\[
\begin{align*}
\text{Measurement update:} \\
L_k &= \Sigma_{k|k-1}C^T(C\Sigma_{k|k-1}C^T + R_v)^{-1}, \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + L_k(y_k - C\hat{x}_{k|k-1}), \\
\Sigma_{k|k} &= (I_n - L_kC)\Sigma_{k|k-1},
\end{align*}
\]

\( (18) \)

The predictions \( \dot{x}_j \) used in the finite horizon problem (see (14)) are then \( j \)-step predictions given the measurement \( y_k \) at the time instant \( k \), that is, \( \dot{x}_j = \hat{x}_{k+j|k} \).
To summarise the proposed control strategy, the following steps are envisaged at each sampling instant:

(i) Take measurements, that is, obtain $y_k$ (see (9)) and the previous control action $u_{k-1}$.

(ii) Update the state prediction (17) and estimate the state $\hat{x}_{k|k}$ using (18) and the measured output.

(iii) Using $u_{k-1}$ and the initial condition $\dot{x}_0 = \hat{x}_{k|k}$ solve the QP (16) to obtain the sequence of controls (12).

(iv) Update the control command $u^c_k = \ddot{u}_0^{\text{OPT}}(\dot{x}_0)$. 
Simulation studies were carried out to assess the performance of a rudder-based stabiliser designed according to the proposed strategy.

A speed of 15 kts for the simulations. This is the nominal speed of the vessel chosen and also the speed at which this vessel performs its missions most of the time.

The performance was assessed for the following scenarios:

- **Case A**: Beam seas ($\chi = 90$ deg), $H_s = 2.5$ m, $T = 7.5$ sec;
- **Case B**: Quartering seas ($\chi = 45$ deg), $H_s = 2.5$ m, $T = 7.5$ sec;
- **Case C**: Bow seas ($\chi = 135$ deg), $H_s = 4$ m, $T = 9.5$ sec.
The wave heights \((H_s)\) were chosen to represent moderate and rough conditions under which a vessel the size of this naval vessel can be expected to perform. The particular wave average periods \(T\) are the most probable periods for the chosen wave heights in ocean areas around Australia.

The control action was updated with a sampling rate of 0.25 sec. The maximum rudder angle was limited to 25 deg, and maximum rudder rate was limited to 20 deg/sec.
Constrained Predictive Control of Rudder-Based Stabilisers

The performance was assessed via

(i) percentage of reduction in roll angle variance and RMS value;

(ii) yaw angle RMS value induced by the rudder;

(iii) percentage of reduction of motion induced interruptions [MII].

MII is an index that depends on the roll angle and roll acceleration and yields the number of interruptions per minute that a worker can expect due to tipping and loss of balance.
Constrained Predictive Control of Rudder-Based Stabilisers

The tuning was performed in beam seas, and then the parameters of the controller were fixed for the rest of the simulation scenarios. The only part of the controller that changed with each sailing condition was the model for the disturbance, which was estimated prior to closing the loop.

Different prediction horizons were tested. As expected, for short prediction horizons ($N = 1$ and $N = 2$), the performance was poorer than for longer horizons ($N = 5$ and $N = 10$). For a horizon of 10 samples periods, an improvement of 10% in roll reduction was achieved with respect to that obtained with $N = 1$. For horizons longer than 10 sample periods there was no significant improvement. Therefore, this is the horizon that was adopted so as to limit the size of the QP problem.
Simulation Results. Case A: Beam seas

ITTC spectrum, (Hs:2.5[m], T :7.5 [sec]), Speed: 15 [kt], Enc. Angle:90[deg]

- Roll
- Roll Rate
- Roll Acc
- Yaw
- Rudder

The plot shows the simulation results for a beam sea condition with an ITTC spectrum, speed of 15 knots, and an encounter angle of 90 degrees. The diagrams compare open-loop and closed-loop responses over time. The y-axis represents various parameters such as roll, roll rate, roll acceleration, yaw, and rudder, and the x-axis represents time in seconds.
Simulation Results. Case A: Beam seas

- This case is close to the worst condition that the ship can experience in regard to roll motion for the assumed sea state: beam seas. The wave period in this case is close to the natural roll period, which is approximately 7 sec. Therefore, the roll excitation due to the waves is close to resonance. Notwithstanding this, we can still observe good performance.

- Results obtained from over 20 different realisations indicate roll reductions on the order of 55–60% for roll RMS values. A significant improvement is achieved, however, in regard to MII: 80–90% reduction.

- From the rudder action depicted in the figure, we can see that the controller generates a command that satisfies the magnitude constraints.
Simulation Results. Case B: Quartering seas

ITTC spectrum, (Hs:2.5[m], T :7.5 [sec]), Speed: 15 [kt], Enc. Angle:45[deg]

- Roll
- Roll Rate
- Roll Acc
- Yaw
- Rudder

Graphs showing the comparison between Open loop and Closed loop responses over time.
The performance in quartering seas decreases significantly. This is expected due to the low encounter frequency of the disturbance.

As depicted in the figure, due to the high interference with yaw for sailing conditions having low encounter frequency, it may be necessary to incorporate an output constraint in order to limit the maximum heading deviation. The low encounter frequency, here appearing in quartering seas, may also be present in other sailing conditions if the sea state is given by very large period waves produced in severe storms.
Simulation Results. Case C: Bow seas

ITTC spectrum, (Hs: 4.5[m], T: 9.5 [sec]), Speed: 15 [kt], Enc. Angle: 135[deg]

- Roll
- Roll Rate
- Roll Acc
- Yaw
- Rudder
This case presents the best performance despite the more severe sea state: 4 m waves. If we compare the RMS of roll in open loop with that of case B, we can see that these are similar. However, due to the higher encounter frequency of the disturbances in case C, the roll reduction is significantly better.
Conclusions

- We have presented a case study and control system design for a problem of significant practical importance.

- The simplifying assumptions have been kept to a minimum. Hence, almost all aspects of the design process have been addressed to some degree.

- The RHC formulation offers a unified framework to address many of the difficulties associated with the control system design for this particular problem: multivariable nature, constraints, uncertainty, stochastic disturbance rejection.

- The simulations presented illustrate the performance of RHC and suggest that the methodology should be successful in practical applications.