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Abstract	Switching adaptive control is one of the advanced approaches to adaptive control. By employing an array of simple candidate controllers, a properly designed monitoring function and switching law, this approach is capable to search in real time for a correct candidate controller to achieve the given control objective such as stabilization and set-point regulation. This approach can deal with large parameter uncertainties and offers good robustness against unmodelled dynamics. This article offers a brief introduction to switching adaptive control, including some historical background, basic concepts, key design components, and technical issues.	
Keywords (separated by "-")	Adaptive control - Supervisory control - Hybrid systems - Uncertain systems - Multiple models - Switching logic	

Encyclopedia of Systems and Control
DOI 10.1007/978-1-4471-5102-9_119-1

Switching Adaptive Control

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Abstract

Switching adaptive control is one of the advanced approaches to adaptive control. By employing an array of simple candidate controllers, a properly designed monitoring function and switching law, this approach is capable to search in real time for a correct candidate controller to achieve the given control objective such as stabilization and set-point regulation. This approach can deal with large parameter uncertainties and offers good robustness against unmodelled dynamics. This article offers a brief introduction to switching adaptive control, including some historical background, basic concepts, key design components, and technical issues.

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Introduction

Switching adaptive control, also known as *switched adaptive control* or *multiple model adaptive control,* refers to an *adaptive control* technique which deploys a set of controllers and a switching law to achieve a given control objective. The concept of switching adaptive control is generalized from the traditional *gain scheduling* technique (Leith and Leithead 2000). As in the standard adaptive control setting, the model for the controlled plant is assumed to contain uncertain parameters, and the control objective is to stablize the system and, in many cases, to deliver certain performance using real-time information in the measured output. What differentiates switching adaptive control from gain scheduling is that the uncertain parameters are not directly measured and the switching is determined by the system response. This seemingly minor difference is very important because parameter estimation may not be possible due to the lack of persistent excitation; moreover, the sensitivity of the measured output is often suppressed by the feedback control which makes closed-loop identification of the uncertain parameters difficult. Compared with classical adaptive control, switching adaptive control has better inherent robustness against parameter uncertainties and unmodelled dynamics.

By early 1980s, the classical adaptive control theory for linear systems had been well established under a set of so-called classical assumptions, which include:

- Known order of the plant (or known maximum order of the plant)
- Known relative degree of the plant
- Minimum phase dynamics
- Known sign of the high-frequency gain (which is the gain of the plant when the input is high-frequency sinusodial signal)

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Encyclopedia of Systems and Control
DOI 10.1007/978-1-4471-5102-9_119-1

At the same time, it was recognized that the classical adaptive control approach has inherent robustness problems against even miniature unmodelled dynamics (Rohrs et al. 1985). While this generated a wave of research aiming at robustification of the classical adaptive control theory (see, e.g., Ioannou and Sun 1996), a new line of research took place aiming at relaxing the classical assumptions. Nussbaum (1983) paved the way by showing that knowledge of the sign of the high-frequency gain can be avoided for a first order linear system. Morse (1985) developed a "universal controller" which can adaptively stablize any strictly proper, minimum-phase system with relative degree not exceeding two. Martensson (1985) gave a very surprising result by showing that asymptotic stabilization can be achieved adaptively by simply assuming that there exists a finite order stabilizer. But Martensson's controller is impractical due to the need for exhaustive online search of the stabilizer and subsequent excessively high overshoots. Switching adaptive control was then introduced in Fu and Barmish (1986), aiming at achieving adaptive stabilization with minimal assumptions and a guarantee of exponential convergence rate for the state. In contrast to the work of Martensson, a compactness requirement is made on the set of possible plants and an upper bound on the order of the plant is assumed. These assumptions allow a set of possible plants to be partitioned into a finite number of subsets, with each stabilizable by a single controller. A monitoring function and a switching law are then designed to sequentially eliminate incorrect candidate controllers until an appropriate controller is found. Due to the fact that the number of candidate controllers may be large, many follow-up works on switching adaptive control focused on speeding up the switching process by eliminating incorrect candidate controllers without trying them (Zhivoglyadov et al. 2000, 2001). These results can also deal with slowly time-varying parameters and infrequent parameter jumps.

Another major breakthrough came from the works of Morse (1996, 1997) under the term of *supervisory control*. His work considers set-point regulation for uncertain linear systems. A different compactness requirement is used to allow unmodelled dynamics in the system. More specifically, the given uncertain linear system is assumed to belong to a union of sub-families of systems, with each sub-family having a linear controller capable to achieve set-point regulation. Suitably defined output-squared estimation errors are used as monitoring functions and a candidate controller is selected whose corresponding performance signal is the smallest. The major advantages of this switching law are that the "correct" controller can usually be quickly identified without cycling through all possible candidate controllers, leading to a good closed-loop performance.

More recent research on switching adaptive control focuses on more systematic and alternative approaches to the design of candidate controllers and switching laws; see, e.g., Anderson et al. (2000), Hespanha et al. (2001), and Morse (2004). Generalizations to nonlinear systems are also found Battistelli et al. (2012).

Design of Switching Adaptive Control

A switching adaptive controller consists of the following key ingredients:

- Design of control covering
- Design of monitoring function
- Selection of dwell time

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DOI 10.1007/978-1-4471-5102-9_119-1

For illustrative purposes, we consider an adaptive stabilization problem where the system has the following model:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$ 79

with state $x(t) \in R^n$ for some $1 \leq n \leq n_{\max}$ and the measured output $y(t) \in R^r$. The given set of uncertain plants Σ consits of triplets (A, B, C) and we use the notation $\Sigma^{(n)}$ to denote the subset of Σ consisting of those plants having order n. It is assumed that every possible plant $(A, B, C) \in \Sigma$ is a minimal realization (i.e., both controllable and observable) and that every $\Sigma^{(n)}$ is a compact set (i.e., it is closed and bounded). The control objective is to design an adaptive controller to drive the state to zero asymptotically, i.e., $x(t) \to 0$ as $t \to \infty$. It is clear that each possible plant in Σ admits a linear dynamic stabilizer. An alternative description of the uncertain plant is introduced in Morse (1996, 1997) where its transfer function is a member of a continuously parameterized set of admissible transfer functions of the form

$$\Sigma \subset \bigcup_{p \in \mathcal{P}} \{\nu_p + \delta : \|\delta\| \leq \varepsilon_p\}$$ 89

In the above, $\mathcal{P}$ is a compact set in a finite dimensional space, ν_p is a nominal transfer function with its coefficients depending continuously on p, δ is the transfer function of some unmodelled dynamics, $\|\delta\|$ represents a shifted H_∞ norm (obtained by first shifting the poles of δ slightly to the right and then computing its H_∞ norm), and ε_p is sufficiently small so that each set of plants $\{\nu_p + \delta : |\delta| \leq \varepsilon\}$ is stabilizable by a single controller for all $p \in \mathcal{P}$.

Control covering: The purpose is to decompose the given set of plants into a union of subsets such that each subset P_i admits a single controller K_i (called candidate controller) to achieve the given control objective. This is typically done using two properties: inherent robustness of linear controllers and the existence of a finite cover for any compact set. More specifically, if a candidate controller renders a desired control objective for a given plant, then the same objective is maintained when the plant is perturbed slightly. For example, Fu and Barmish (1986) uses the fact that if a given plant is stabilized by a controller then the same controller stabilizes all the plants with sufficiently small parameter perturbations. Similarly, Morse (1996, 1997) uses the fact that the same controller achieves set-point regulation for a small neighborhood of plants. Combining this property with the finite covering property yields

$$\Sigma = \bigcup_{i=1}^{N} \Sigma_i$$ 105

such that each subset Σ_i admits a single controller K_i.

Monitoring Function: The generation of the adaptive switching controller is accomplished using a *switching law* or *switching logic* whose task is to determine, at each time instant, which candidate controller is to be applied. The core of the switching law is a monitoring function. Its very basic role is to be able to detect whether the applied candidate controller is consistent with the corresponding plant subset so that wrong candidate controllers can be eliminated one by one until an appropriate controller is found. A major difficulty for switching adaptive control design is that persistent

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DOI 10.1007/978-1-4471-5102-9_119-1

excitation is not assumed. Consequently, it is not always possible to detect the correct plant subset using the measured output. The key idea is to check which plant subsets are consistent with the generated output.

One simple monitoring function uses a finite-time L_2 norm of the measured output:

$$V(t,\tau) = \int_{t-\tau}^{t} \|y(s)\|^2 \, ds$$

where τ is the so-called *dwell time*. It turns out that for some properly chosen dwell time, a correctly applied candidate controller is able to guarantee some decay property for the monitoring function, i.e., $V(t,\tau) \leq e^{-\lambda\tau} V(t-\tau,\tau)$ for some $\lambda > 0$. This property is sufficient to allow a wrong candidate controller to be eliminated. However, much smarter monitoring functions can be designed so that infeasible candidate controllers (those not corresponding to the true plant) can be eliminated without even being applied. This can be done using the *falsification* approach in parameter estimation where the basic idea is to eliminate all plant subsets Σ_i inconsistent with the measured output signal. For example, consider the following discrete-time model:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) + b_1 u(t-1) + b_2 u(t-2) + w(t)$$

where a_i and b_i are uncertain parameters and w(t) is a bounded disturbance, i.e., $|w(t)| \leq \delta$ for some δ. For this example, we may eliminate all the uncertain parameter subsets which violate the following constraint (Zhivoglyadov et al. 2000):

$$|y(t) + a_1 y(t-1) + a_2 y(t-2) - b_1 u(t-1) - b_2 u(t-2)| \leq \delta$$

More generally, one can use the so-called multi-estimator (Morse 1996, 1997) which involves an array of estimators, one for each plant subset Σi using its nominal model. The output estimation error $e,(l)$ for each such estimator is then used to construct a monitoring function, e.g.,

$$V_i(t,\tau) = \int_{t-\tau}^{t} e^{-2\lambda(t-s)} \|e_i(s)\|^2 \, ds$$

where τ is the dwell time as before and $\lambda > 0$ is an exponential weighting parameter used to guarantee the decay rate of the monitoring function as before. Instead of using the monitoring functions to eliminate infeasible candidate controllers, the candidate controller corresponding to the least estimation error, as measured by the least monitoring function, is selected. The main advantage of the multi-estimator based monitoring functions is that falsification of candidate controllers is done implicitly and a "correct" controller can be quickly reached, leading to good performance.

Dwell Time: The dwell time τ as defined above is a critical component in switching adaptive control. Serving in the monitoring function, this is the minimum nonzero amount of time for a candidate controller to be applied before switching. That is, this provides a sufficient time lag to build the monitoring function so that its exponential decay property is detected when a correct candidate controller is applied. This will allow detection of infeasible plant subsets and selection of a "correct" controller. The use of a dwell time also avoids arbitrarily fast switching, thus gauranteeing the solvability of the system dynamics.

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DOI 10.1007/978-1-4471-5102-9_119-1

The dwell time can be selected a priori by using the fact that if a matrix A is stable, then there exist some positive values λ and τ such that $\left\| e^{At} \right\| \leq e^{-\lambda \tau}$ for all $i > \tau$. This leads to the desired exponential decaying property

$$V\,(t, \tau) \leq e^{-\lambda \tau V}\,(t - \tau, \tau)$$

for the aforementioned monitoring function for adaptive stabilization.

Alternatively, the dwell time can be chosen implicitly. Hespanha et al. (2001) suggest a *hysteresis switching logic* method. This method employs a hysteresis parameter $h > 0$. Suppose the candidate controller K_j is applied at time t_i, then K_j is kept until the next switching time t_{i+1} which is the minimum $t \leq t_i$, such that

$$(1 + h) \min_{1 \leq k \leq N} V_k\,(t, t - t_i) \leq V_j\,(t, t - t_i)$$

Because h $>$ 0, the time difference $t_{l+1} - t_i > 0$ is lower bounded, which implies the existence of a dwell time.

Summary and Future Directions

Switching adaptive control is a conceptually simple control technique capable to deal with large parameter uncertainties. The use of simple candidate controllers (typically linear) imply good closed-loop behavior and good robustness against unmodelled dynamics. Although the discussion above assumes that the number of plant subsets is finite, this assumption is not essential; see Anderson et al. (2000).

Switching adaptive control renders the closed-loop system a switched system or hybrid system, for which a wide range of tools are available to aid the analysis of such a system; see, e.g., Liberzon (2003). However, unique features of such a system arise from the fact that the switching mechanism is chosen by the designer, rather than being a part of the given plant. How to best design the switching mechanism is an interesting issue.

Future works for switching adaptive control include:

1. How to simplify the design of candidate controllers. Finite covering based design often yields a large number of plant subsets, hence a large number of candidate controllers. Since most of the candidate controllers do not need to apply (which is the case when falsification based switching logic is used, for example), smarter ways are needed for the design of candidate controllers.
2. Wider applications. Most of the research so far focuses on stabilization and set-point regulation (which is essentially a stabilization problem). How to incorporate general performance criteria is an essential and yet challenging issue.
3. Better design of monitoring functions and the corresponding switching logic. Most existing monitoring functions use a finite-time L_2 norm of the output (or regulation error), with the key feature that some exponential decay property is guaranteed when the candidate controller is "correct." Note that the key purpose of the monitoring function and the corresponding switching logic is to allow fast falsification of infeasible candidate controllers. Thus, a much wider range of monitoring functions can possibly be used. In particular, how to incorporate set membership identification techniques (Milanese and Taragna 2005) may be of particular interest.

Encyclopedia of Systems and Control
DOI 10.1007/978-1-4471-5102-9_119-1

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DOI 10.1007/978-1-4471-5102-9_119-1

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