Localizability Exploration of Sensor Network

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***Abstract*— Localizability analysis with respect to either the whole network or a single node has been explored in previous work. Usually, a set of nodes and specified connections between nodes jointly construct a graph. During this paper, the localizability exploration of two merging network is characterized to be globally rigidity analysis of the graph. We will give a series of sufficient and necessary condition on the localizability of two merging graph corresponding to several different combinations of nodes in each graph.**

I. INTRODUCTION

LOCALIZATION problem is the fundamental and important issue among the abundant expected application of sensor network [1][2], which include but not limited in the area of wildlife tracking[4], ocean monitoring[5], intelligent factory[6][7], information encryption[15] and the newly appeared carbon sink[8].

Generally, there are two kinds of way for obtaining the location information. First is the distance-based localization scheme and the second is the distance-free scheme. The localization scheme discussed in this paper is based on distance measurement. We want to remark that the localization scheme discussed here is a different definition compared with the range detection technique. The distance detection or ranging technique usually means the technique that used for obtaining the distance measurement. This might be finished by detecting the flight time of radio or ultrasonic signals and therefore obtaining the distance between the signal source and the target [16]. A localization scheme, generally distance-based localization scheme, can use the detected distance information for getting location information.

Localization scheme can be divided into two cases: sequential scheme and concurrent scheme[22]. For a sequential scheme, there are usually at least three anchor nodes in a 2D plane and every node without location information is tested if it has three direct distance measurement with the nodes that known location. If so, its location will be computed by its distance measurement with three location known nodes and then be added into the set of location known nodes. If not, its location could not be computed. The scheme will test all the nodes one by one according this way. For a concurrent scheme, every node will compute its location by using distance measurement within its neighborhood. Computation at each node can exist currently and this scheme will iterate to the correct location value finally.

Both sequential and concurrent scheme can only compute the location of nodes that are 'localizable' or 'easily localizable'. The localizable nodes should fit some conditions, such as 3-connected condition in [18] and convex hull constraints in [8][3].

For a randomly deployed sensor network, it is probably that not all the sensor nodes are localizable since the existence of so called flex and flip ambiguity in the localization problem[18]. The localizability problem is often characterized through the graph theory.

A graph is rigid if it could not be continuously deformed. If every realization of the graph with same distance constraints is identical, then the graph is globally rigid. A graph is globally rigid if and only if it is 3-connected and redundantly rigid. Here, 3-connected means the graph is still connected after removal of any two nodes. Redundantly rigid means the graph is still rigid after removal of any edge.

If a globally rigid graph could not be globally rigid any more after removing any one edge from the graph, it is a minimally globally rigid graph. We give a description of those several concepts in Fig.1.

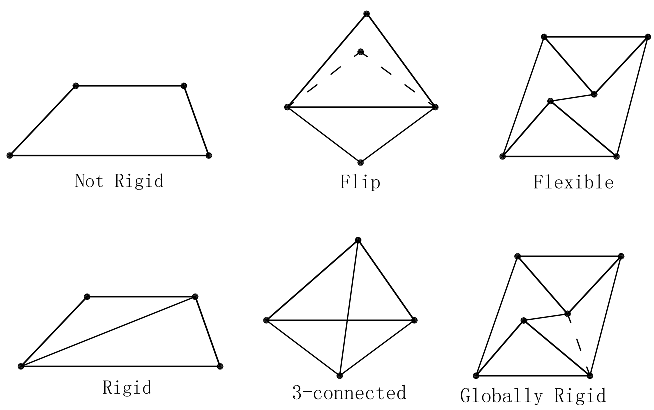

Fig. 1: Several Concepts

The existed work on localizability focused on the localizable conditions of either a whole network[20] or one single node[21]. They proved conditions for the localizability in view of the connectivity and rigidity of the graph. The

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drawback of localizability conditions for a whole network is that they can only judge wether the whole network is localizable but cannot find the localizable nodes from a given network. Practically, a randomly deployed sensor network is hardly to be localizable in the view of the whole network[21]. The drawback of localizability conditions for one single node is that it can test only one node at a time and the condition is not sufficient and necessary. Connectivity and rigidity test for both cases need global information.

Neither for a whole network nor for one single node, our thread is to explore the localizability condition for a set of nodes. Assume there is one set of location known nodes during the whole network. Existed work is to analyze the connectivity and rigidity of every location unknown node one by one. But during this paper, we want to collect each single node with its several neighbors together to build a test set. Different number of neighbors leads to specified test set. Then analyze the localizable conditions between the test set and the location known set.

We characterize this set based thread as a connectivity and rigidity test of two merging graph. The location known set corresponds to a location fixed graph in 2D plane, while each test set corresponds to the other graph. The connectivity and rigidity condition need to answer two questions: how many connections between these two graph is required and where should they be draw.

II. Problem Statement and Related Work

During the localizability exploration, the sensor network is usually treated as a graph $\mathbf{G}(V,E)$. In graph theory, V and E stand for the vertex set and edge set of the graph respectively. Here, V stands for a set of all the nodes and E stands for a set of the existed edges between connected nodes in real network.

For a graph $\mathbf{G}$, it is localizable in 2D plane if and only if the graph is globally rigid and there are three location known nodes in the graph. Given at least 3 location known nodes, the localizable condition for a network can be transformed into a globally rigidity test. During this paper, we want to explore the localizable condition between two set of nodes. This can be characterized as the globally rigidity test of two merging graph, one of which is location-all-known.

It can be mathematically described as below. Given two graph $\mathbf{G_1}(V_1,E_1)$ containing N_1 nodes and $\mathbf{G_2}(V_2,E_2)$ containing N_2 nodes, we are asked to draw fixed number of edges between these two graph to merge them together and obtain a merging minimally globally rigid graph $\mathbf{G}(V,E)$ containing all N_1+N_2 nodes and edges between them. Without loss of generality, we assume all N_1 nodes in $\mathbf{G_1}$ are location known. Then we turn to our question that, merging $\mathbf{G_2}$ onto $\mathbf{G_1}$ to obtain a globally rigid graph $\mathbf{G}$, how many edges is required and how to draw these edges between graph $\mathbf{G_1}$ and $\mathbf{G_2}$. For simplicity of analysis, we assume $\mathbf{G_1}$ has only three nodes here and discuss the cases of $\mathbf{G_2}$ having different number of nodes and variable edges.

A natural question is why we choose to explore conditions for several specified case of $\mathbf{G_1}$ and $\mathbf{G_2}$.

Firstly, two merging graph $\mathbf{G_1}(V_1,E_1)$ and $\mathbf{G_2}(V_2,E_2)$ have variable (V_1,E_1) and (V_2,E_2) and numerous possible combination, if either of them if not globally rigid. It is hard to give a generalized condition for merging $\mathbf{G_1}$ and $\mathbf{G_2}$ and we choose to give a sufficient and necessary condition for two merging graph with specified number of nodes and edges and also give a necessary condition for two merging graph with variable number of nodes and edges.

Secondly, we believe the condition for two merging graph could be much tighter than the condition for one node merging with a graph[21] since the interconnection between nodes in the test graph might eliminate some degree-of-freedom.

Thirdly, for a localization scheme, the main object is to localize nodes as many as possible even if not all nodes of the network are localizable. In other words, we can tolerate an algorithm that cannot find all possible combinations. There are kinds of 'easily localizable' network topology such as 3-connected case in [18] and referred work inside and 'wheel' case in [19]. Both cases can find parts of the localizable nodes through the whole network even if the network is not whole localizable. But the conditions, such as 3-connected, is only a sufficient but not necessary condition. In other words, there are still some localizable nodes that are not included in either case.

Below we show an example of graph in Fig.2, which does not fit 3-connected condition but is still localizable. This example is also mentioned in [19] and its localizability can be analyzed through the localizable condition of one single node in [21]. We will explore its localizability by regarding two location unknown nodes 4 and 5 as a test set. We will prove the connections shown as dashed line in Fig.2 is the only choice for merging this set onto the location known set by introducing minimally connections.

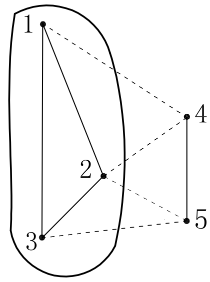

Fig. 2: A Counter Example of 3-connected Condition

The algorithm in [27] based on the 3-connected and redundantly rigid condition above could be utilized for testing the globally rigidity of the network. But this condition is explored in view of the whole network. Utilizing this judgement condition on a given network graph $\mathbf{G}$, we could determine whether the whole network is localizable but still cannot tell which nodes of the network are not localizable. The work of [20] analyzed the localizability of each single node. They give a so far closest pair of mutual independent sufficient condition and necessary condition for one single node's localizability.

Similar like our set based thread, there are also some work giving localizable conditions for the merging of two globally rigid sub-network[25][26]. Those cases can be recognized as a localizability analysis between two globally rigid graph. During this paper, we also give conditions for the globally rigidity of merging graph, but only one of the graph is required to be globally rigid and the other graph's structure is not constrained, which we name as a free graph here. According to our definition of free graph, the globally rigid graph without anchors is free. But the graph without promise of globally rigid is also contained in the range of free graph. This is also our main difference with the existed work such as [25] and [26].

III. Conditions for Localizability of Two Merging Graph

During this paper, we want to give some more conditions for determining the localizability corresponding to several specialized cases of $\mathbf{G_1}$ and $\mathbf{G_2}$. We assume $\mathbf{G_1}$ has three location known nodes inside.

A. A Necessary Condition for Localizability of Two Merging Graph

In a 2D plane, if a graph is fixed on the uniquely position, it has zero degree-of-freedom. But if a graph is globally rigid, it has still 3 degree-of-freedom since the lack of three anchors[20]. These 3 degree-of-freedom correspond to the rotation, translation and reflection of the graph in 2D plane[14]. So, globally rigid is only a necessary condition for uniquely localizable.

Actually, each free node's movement in a 2D plane has 2 degree-of-freedom, so there are $2n$ degree-of-freedom for n nodes. One pair-wise connection between two nodes could eliminate 1 degree-of-freedom. A natural question is how many connections is required at least to guarantee the globally rigidity of a graph. And how many connections is required for our case, during which number of nodes and connections in two graph is specified.

We first give a lemma about the general case:

Lemma 1: If a graph $\mathbf{G}$ with n nodes is globally rigid, it have at least $2n-2$ edges in the graph.

Proof: For a globally rigid graph $\mathbf{G}$ containing n nodes, although relative of each node inside is fixed, it still has 2 degree of freedom in a 2D plane as a whole. If without any edge, the nodes in $\mathbf{G}$ are not connected and each single node in a 2D plane has 2 degree of freedom. So, there should be $2n$ degree of freedom for n nodes if the connections was not built inside $\mathbf{G}$.

Therefore, edges inside $\mathbf{G}$ eliminate $2n-2$ degree of freedom. Considering each edge corresponds to one degree of freedom, we can conclude that there should be at least $2n-2$ edges in $\mathbf{G}$ if it is a globally rigid graph. ■

Then we can obtain the number of required edges between two merging graph:

Corollary 1: For two graph $\mathbf{G_1}$, containing m nodes and p edges inside, and $\mathbf{G_2}$, containing n nodes and q edges inside, if their merging graph $\mathbf{G}$ is globally rigid, there should be at least $2(m+n)-(p+q)-2$ edges connected between $\mathbf{G_1}$ and $\mathbf{G_2}$.

A globally rigid graph $\mathbf{G}$ with n nodes is named to be a minimally globally rigid graph, if it has exactly $2n-2$ edges. The minimally globally rigid graph could be constructed through a sequential way[28]. The construction way of minimally globally rigid graph will be described below.

B. Two Nodes Cases

As shown in Fig.3, graph $\mathbf{G_1}$ contains three anchor nodes and graph $\mathbf{G_2}$ contains two nodes. There is an an interconnected edge between node 4 and node 5 in graph $\mathbf{G_2}$. The reason for utilizing the concept of globally rigidity between graph is to use interconnections inside the graph to weaken the globally rigidity condition. The example shown in Fig.3(b) is almost same with the counter example shown in Fig.2. Difference is the treatment that we package node 4 and node 5 into a set. Though neither of them fit the 3-connected condition, but the graph formed by the all 5 nodes is still globally rigid. We firstly answer the question that how

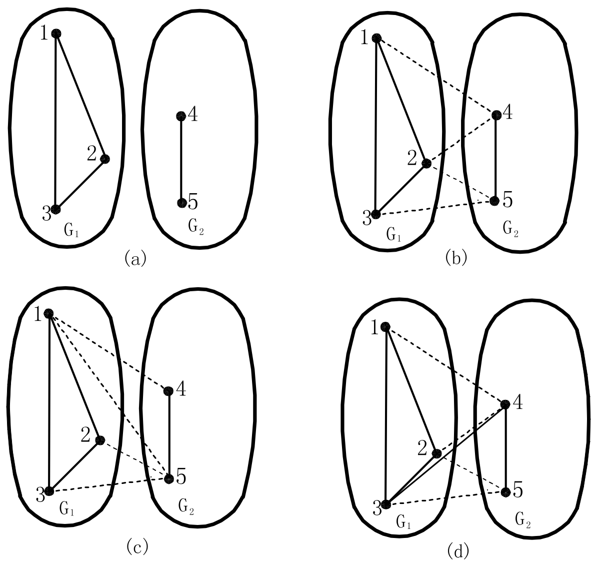

Fig. 3: Two nodes case

many edges between two graph is required at least. Through the above corollary, we can compute there should be at least 4 connections to merge two graph $\mathbf{G_1}$ and $\mathbf{G_2}$ together to be a globally rigid graph.

Then we should explore how to draw these 4 connections between two graph to make the merging graph globally rigid. Since each node in both graph is identical, if the link number is limited at 4, it is easy to exhaust that there are only two possible cases for two graph as shown in Fig.3(a).

Here, we introduce *Jackson and Jordan*'s conclusion about globally rigidity and *Berg and Jordan*'s conclusion about minimally globally rigidity as two lemmas:

Lemma 2 ([23]): A distance graph is globally rigid if and only if the graph is 3-connected and redundantly rigid.

Lemma 3 ([28]): Given a minimally globally rigid graph **G** containing at least 4 nodes, suppose one node is added to it with 3 connections to **G**. Then, the resulting graph is still minimally globally rigid if one edge between two of these 3 nodes in **G** is removed.

Then, we give our condition about the above case:

Theorem 1: Given two graph, one is globally rigid $\mathbf{G_1}$, containing only three anchors inside, and the other one $\mathbf{G_2}$ is free, having two connected nodes inside, the merging of two graph is globally rigid if and only if there is one node in $\mathbf{G_1}$ connected both of two nodes in $\mathbf{G_2}$ and each of the other two nodes in $\mathbf{G_1}$ connects a different node in $\mathbf{G_2}$ with each other.

Proof: First, we give the sufficiency proof. As shown in Fig.3(d), if the edge between node 3 and 4 is added into the graph, node 4 would be 3-connected with three anchors. Then the graph built by $\mathbf{S}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4})$ is a minimally globally rigid graph. After that, add node 5 onto the graph and build connections with node 2, 3 and 4. As the sequential setup process of minimally globally rigid graph described in *Lemma 3*, the edge between node 3 and 4 is eliminated and the constructed graph is minimally globally rigid.

Followed is the necessity proof. There are four edges needed to draw onto three nodes at each side and every node in corresponding graph is identical. So there are only two possible connected cases if the number of links is fixed at 4. From *Lemma 2*, we could determine the merging graph shown in Fig.3(c) is not globally rigid since the graph would be not connected if node 1 and node 5 were removed. Then only one possible connected case is globally rigid. ■

Remark 1: For the case with more than 3 nodes in $\mathbf{G_1}$, the merging graph **G** is globally rigid if there are 4 anchor nodes have connections with the free graph, any three of the anchor nodes is not collinear, and one of the link from the intersected node in $\mathbf{G_1}$ is replaced by a new edge from the 4th node. This remark could be proved through adding a new node into the graph in *Theorem 1* to build a newly minimally globally rigid graph.

Remark 2: For the case with more than 4 links between two graph, it could be recognized as adding redundant links after building the minimally globally rigid graph. The condition we given here is a tight bound for the setup of a merging globally rigid graph. Without the requirement of minimally links, the sufficient condition still works.

C. Three Nodes Cases

As shown in Fig.4(a), both graph $\mathbf{G_1}$ and graph $\mathbf{G_2}$ have 3 nodes inside. The difference is that there are still 3 degree of freedom for $\mathbf{G_2}$. In other words, the coordination of nodes in $\mathbf{G_1}$ is fixed but nodes in $\mathbf{G_2}$ could move in 2D plane if without links between two graph. Here, the nodes in $\mathbf{G_2}$ is pairwise connected and formed a triangle.

First, we analyzed possible connected way between two graph as shown in Fig.4(a). For a graph **G** containing 6 nodes, there should be at least 10 edges to make sure the globally rigidity. Besides 6 edges forming two triangles, there still need 4 more edges between two graph. An extra constraint about these 4 edges is they should be directly connected with 3 anchors in graph $\mathbf{G_1}$.

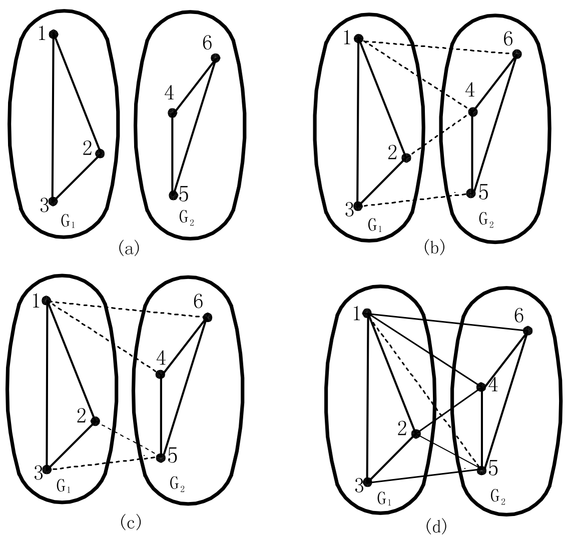

Fig. 4: Three nodes case

Since there are 4 edges directly connected with 3 anchor nodes, one of three anchors in $\mathbf{G_1}$ should have two directly connections with $\mathbf{G_2}$. Without loss of generality, we assume node 1 is the one that has two directly connections with two nodes in graph $\mathbf{G_2}$, say node 4 and 6 as shown in Fig.4. Then there are only two kinds of choice for the left two nodes in graph $\mathbf{G_1}$, either connected two different nodes in $\mathbf{G_2}$ as shown in Fig.4(b) or connected the same node left in $\mathbf{G_2}$ as shown in Fig.4(c).

Now, we prove the previous one is the right choice to make the formed graph globally rigid. We describe this conclusion as a theorem here.

Theorem 2: Given a graph $\mathbf{G_1}$ containing 3 anchor nodes inside, the free graph $\mathbf{G_2}$ could be added on $\mathbf{G_1}$ to construct a minimally globally rigid graph **G** if and only if there are four edges connected between $\mathbf{G_1}$ and $\mathbf{G_2}$, one node of $\mathbf{G_1}$ has direct connections with 2 different anchor nodes in $\mathbf{G_1}$ and two others has single connections with different anchor nodes of $\mathbf{G_1}$.

Proof: First we prove the sufficiency. Since each node in corresponding graph is identical, so we can construct two possible graph and connections, as shown in Fig.4(b), according to the description of the theorem.

Consider the graph **G** containing all 6 nodes as a whole and then decompose graph **G** into two parts: one part is the graph formed by node 1 to 5, the other part is node 6 with its connected edges.

Adding an auxiliary line between node 1 and 5, as shown in Fig.4(d), we could build a minimally globally rigid graph formed by node 1 to 5. The proof is same as *Theorem 1* since node 4 and 5 could be recognized as the two-node-graph case described in *Theorem 1*. The difference is only on the interconnected node choice, i.e., node 2 in Fig.3 and node 1 in Fig.4, in two cases.

After adding node 6 onto the minimally globally graph formed by 1 to 5 in Fig.4(d), we could prove the resulted graph is still minimally globally rigid through *Lemma 3*.

Then we prove the necessity. As the analysis before, there are only two kinds of connections if the number of edges between two graph is limited at 4. One is shown as Fig.4(c). Through *Lemma 2*, the globally rigid graph G should be 3-connected and redundantly rigid. 3-connected means the graph should be connected after removal of any two nodes. But the merging graph formed by all 6 nodes shown in Fig.4(c) is not 3-connected because the merging graph would not be connected after removal of node 1 and 5. So, the one shown as Fig.4(b) is the only choice. ■

D. Four Nodes Cases

So far, we could prove the condition for globally rigidity of two graph merging in above cases. But the free graph appeared in both cases are rigid. Here, we want to discuss the condition for globally rigidity of merging graph when one of the graph is not rigid.

As shown in Fig.4(a), the nodes in graph 2 construct a quadrangle. According to the definition of rigid, a quadrangle is not rigid since it could be deformed. What we want to do is to explore conditions that can make the merging graph formed by all nodes in both graph globally rigid while introducing least edges between two graph. This is an almost identical requirement with building a minimally graph. The difference here is the nodes in graph 1 are anchored and edges in both graph 1 and graph 2 are fixed.

We want to analyze the globally rigidity based on our conclusion in last subsection. As *Lemma 3* described, every minimally globally rigid graph could be constructed sequentially. Once we obtained a minimally graph as shown in Fig.4(b), adding one more node onto it, we could get a new graph like Fig.5(b) or Fig.5(c) according to different choice of connected node. Take Fig.5(b) for example, node 7 is added into graph 2 and connected with node 5 and 6. According to *Lemma 3*, if adding one more edge between node 7 and any one node in graph 1 and eliminating edge between node 5 and 6, then the obtained graph would be a minimally globally graph. The connected case like shown in Fig.5(c) is same as above.

We give our conclusion about the condition to build a globally rigid graph between a globally rigid graph and a non-rigid graph as below.

Theorem 3: Given one graph $\mathbf{G_1}$ with three pairwise connected anchors and another graph $\mathbf{G_2}$ with four nodes whose connection formed a quadrangle, the merging graph of them is globally rigid and there are least connections between those graph if and only if the three conditions below are satisfied simultaneously:

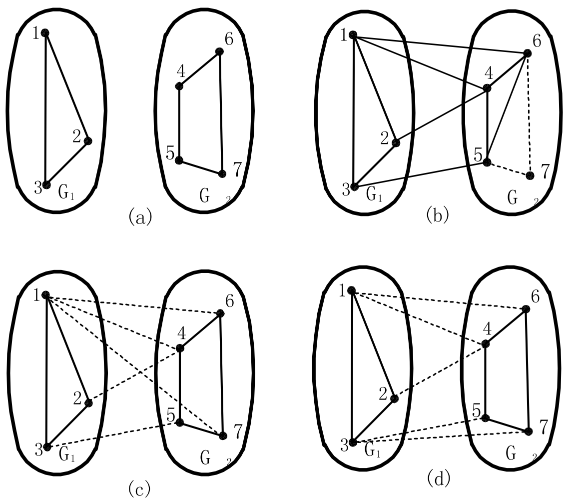

Fig. 5: Four nodes case

1. Each of nodes in $\mathbf{G_1}$ has at least one connection with $\mathbf{G_2}$;
2. Each of nodes in $\mathbf{G_2}$ has at least one connection with $\mathbf{G_1}$;
3. There is only one node in $\mathbf{G_2}$ connects with two nodes in $\mathbf{G_1}$ and one of those nodes in $\mathbf{G_1}$ has connection with another node in $\mathbf{G_2}$.

Proof: First we prove the sufficiency. Consider the graph $\mathbf{G}$ formed by all 7 nodes in Fig.5(a) as a whole. Condition 3 leads to a "Z" shape connection between 4 nodes. Since each node in corresponding graph is identical, we can build the "Z" shape connection between node 1,2,4 and 6. Then we connect the left nodes according to condition 1 and 2, we can obtain two kinds of possible combinations as shown in Fig.5(c) and Fig.5(d).

In both cases, nodes 1 to 6 construct a subgraph $\mathbf{G_a}$, shown as solid line in Fig.5(b). We could notice that $\mathbf{G_a}$ is identical with the graph $\mathbf{G}$ described in *Theorem 2*, i.e., the graph shown in Fig.4(b). The graph shown as $\mathbf{G_a}$ has been proved to be a minimally globally rigid in *Theorem 2*.

Adding one more node, say node 7, onto the graph $\mathbf{G_a}$ shown in Fig.5(b) through connecting node 7 with node 5 and node 6 and eliminating edge between node 5 and node 6, we could obtain a graph shown as Fig.5(d). According to *Lemma 3*, adding and eliminating as described above is a typical process in constructing a minimally globally rigid graph. So, we can conclude that the graph as shown in Fig.5(d) is a minimally globally rigid graph.

And then we prove the necessity of those three conditions one by one.

For condition 1: For nodes in $\mathbf{G_1}$, if any one of them has not direct connection with nodes in $\mathbf{G_2}$, then the resulted graph would be flipped over the edge between the other two nodes. So, each of nodes in $\mathbf{G_1}$ must have at least one connection with $\mathbf{G_2}$.

For condition 2: The nodes in $\mathbf{G_2}$ construct a quadrangle. If any one node of the quadrangle has no connection with $\mathbf{G_1}$, it could be flipped over the diagonal line of the quadrangle. Then the merging graph is definitely un-localizable. So, each of nodes in $\mathbf{G_2}$ needs at least one connection with $\mathbf{G_1}$.

For condition 3: According to corollary in the first subsection, there are still 5 more edges needed to be added between two graph. Since there are 5 more edges needed to be added onto those four nodes of $\mathbf{G_2}$, there must be one node in $\mathbf{G_2}$ that has two connections with nodes of $\mathbf{G_1}$.

Consider the graph shown in Fig.5(d), node 4 in $\mathbf{G_2}$ has two connections with node 1 and 2 of $\mathbf{G_1}$ respectively. At the same time, node 1 in $\mathbf{G_1}$ has another connection besides node 4 in $\mathbf{G_2}$. If not so, such as connecting node 6 with node 3 rather than node 1, the resulted graph is not 3-connected. This is caused by the fact that the graph would not be connected after removal of node 3 and 4. According to *Lemma 2*, it cannot be globally rigid then. ■

IV. Remark on Sequential Localization Scheme

The traditional sequential localization scheme is based on 3-connected condition. Every node's coordination is computed by its three neighbors if those neighbors' coordination was known. The nodes fit 3-connected condition is computed one by one. This process can be finished in polynomial steps. But as we pointed before, only small part of the network is localizable according to this way. The case shown in Fig.2 is a counter example.

Compared with sequential way, a concurrent localization scheme could use information of neighborhood only. Each node of the network can compute its coordination by using information from its three neighbors. But the iteration process will not be converged if some tight constrains are fit. Besides the constrains limitation, another problem for the concurrent way is its complexity. The iteration will converge in infinite steps, which is hardly to realize in practice.

We choose a sequential way, but we consider a set of nodes rather than one single node at each step. This will improve the efficient of the sequential way and reduce the chance that miss localizable nodes during the network. Compared with the concurrent way in [3], we utilize a more reasonable assumption about the network deployment.

V. Conclusion

During this paper, we discussed the localizability of two merging graph of nodes. The localizability is explored through graph rigidity theory. Different from existed work, the analysis object is neither the whole network nor one single node, but two merging graph of nodes. And the graph are not required to be all globally rigid. We give localizable conditions for several specified cases of the graph. Compared traditional trilateration thread, our set based thread could reduce the risk of missing localizable nodes. Compared the conditions of merging one node on a graph, our conditions is much tighter and more efficient. The analysis of these condition between graph can guide the localization algorithm in the following work. For example, if there are some nodes lying outside the communication range of anchor graph, we could amplify the communication power of specified nodes according to the conditions given in this paper.

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