State Estimation Subject to Random Measurement Delays

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***Abstract*— The state estimation problem is studied for the networked control systems subject to random measurement delays and the measurements without the time-stamps. With the random delay bounded by one step only, a new measurement model is proposed with the out-of-sequence measurements. The estimator form is given based on the mean of all received measurements at each time. The estimator gains can be derived by solving a set of recursive discrete-time Riccati equations. Furthermore, the estimator can be guaranteed to be optimal.**

I. INTRODUCTION

IN the past decade, networked control systems (NCSs) have gained much attention in communication networks, control and state estimation [1]. In a NCSs, data typically travel through the communication networks from sensors to the controller and from controller to the actuators. As a direct consequence of the finite bandwidth for data transmission over networks, random communication delays, out-of-sequence measurements, and packet losses are inevitable in networked systems where a common medium is used for data transfers and should be properly handled in order to achieve satisfactory estimation and control performance [2-4].

In the networked control system, the sensor measures the output of the system at every time and transmits the measurement to a data processing center (the estimator). The estimation problem for NCSs with random delays or packet dropouts has received many results during the past few years [5-9]. As is well known, however, standard Kalman filtering can not be applied to systems with output delays. So time stamping is necessary to reorder the packets when the measurements arrive out of order. Schnato[4] proposed the estimators subject to simultaneous random packet delay and packet dropout, and measurements are time-stamped and can be re-ordered at the estimator site. Zhang and Xie[10] studied the optimal estimation problem for discrete-time systems with time-varying delay in the measurement channel, and the measurements are time-stamped which can only take one value at each time instant. Without time-stamps, Sun[11] investigated the estimation problem for the systems with bounded random measurement delays and packet dropouts, which are described by some binary distributed random variables whose probabilities are only known. But in [11], the measurement model can receive the same measurement with multiple times. Since the network transmission has limited capabilities, one measurement should not be re-received. And so far, the estimation problems with the out-of-sequence measurements are seldom reported without using the time-stamps.

In this paper, a new measurement model is proposed without time-stamps. Due to the time delay, more than one measurement may be received at each time instant in the actual network communication. Then the estimation measurements are adopted by the mean of all the received measurements. For technical simplicity but without alerting the core difficulty, we only consider the case where the maximum time delay is one time step. This paper is mainly organized as follows. Problem formulation is given in Section II; and Section III firstly considers the estimation problem with received multi-measurements with out-of-sequence; the whole estimation solution with the random delay is given in Section IV. Finally, Section V draws some conclusions of this paper.

II. PROBLEM FORMULATION

Consider the following discrete-time linear stochastic system:

$$x_{k+1} = Ax_k + \upsilon_k \tag{1}$$

$$y_k = Cx_k + \omega_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the system state, $y_k \in \mathbb{R}^m$ is the measured output, $\upsilon_k \in \mathbb{R}^n$ and $\omega_k \in \mathbb{R}^m$ are system noise and measure noise respectively. A, C are matrices of the appropriate dimensions. The initial state x_0 and υ_k, ω_k are Gaussian, uncorrelated, white, with mean $(\bar{x}_0, 0, 0)$ and covariance (P_0, Q_k, R_k) respectively. We also assume that the pair (A, C) is observable, and $R > 0$.

In the networked system, the sensor measures the output of the system at every time and transmits the measurement to the estimator. The time-delay is unavoidable by the unreliable network communication. Thus it is impossible to guarantee that all packets are correctly delivered to the destination. In [13], Schenato considered that the measurements are time-stamped, encapsulated into packets, and then transmitted through a digital communication network (DCN). Since the packet delay is random, it is possible that between two

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consecutive sampling periods no packet or multiple packets are delivered. This means packets to arrive in burst or even out of order at the receiver side. Time-stamping of measurements are necessary to reorder packets at the receiver side as they can arrive out of order. It guarantees that all observation packets correctly delivered to the estimator site.

In the paper, we will consider the measurements without time stamping, so we do not know the correct order of received measurements because of the random delays. For simplicity, we only consider the maximum random delay is $N = 1$. That is that the random delay is 0 or 1. Assumed that there is no packet loss and the packets can not be received repeatedly, we give the state transition diagram for time-delay :

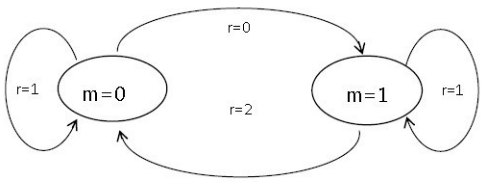

Fig. 1. the state transition for random time-delay.

where m is the time delay, and r is the number of the received packets at time k. Thus there are following cases when the maximum random delay is 1:
Case 1: $m = 0$ it states there are no delays happen, i.e., the measurement y_{k-1} is received on time at k-1 time, then from the Fig. 1, there may be the following two cases by the number of received measurements r:

Case 1.1, when $r = 0$ there no packet arrive, then time delay will happen, it is $m = 1$;

Case 1.2, when $r = 1$ the packet y_k received on time, and $m = 0$ at next time.

Case 2: $m = 1$, i.e. y_{k-1} is not arrived at k-1 time, it is that one time-delay happened at k time, then according to the Fig. 1, there have:

Case 2.1, when $r = 1$ there must be y_{k-1} received at k time, and y_k happens time-delay, it is $m = 1$;

Case 2.2, when $r = 2$ there are y_k, y_{k-1} received simultaneously, and there will be $m = 0$.

Form these four cases, we know that when $r = 0$ or 1, the received measurement can be precisely deduced. The estimator can be presented easily in the first three cases. Under the Case 2.2, it is difficult that we do not know the correct order of the arrival sequence measurements. In the next section, we mainly give the state estimator for Case 2.2.

III. Estimator Design With Out-Of-Sequence Measurements

The problem in Case 2.2 is that: at time k-1,the measurement y_{k-1} does not arrive, but there are two measured outputs $\{y_{k-1}, y_k\}$ arriving at time k. Because of absent time-stamps, we do not know the order of $\{y_{k-1},\ y_k\}$. Then the arrival sequences have two cases at time k:
case a: the packets received at the correct order

$$\tilde{y}_k = \begin{bmatrix} y_{k-1} \\ y_k \end{bmatrix}.$$

case b: the packets received at incorrect order

$$\tilde{y}_k = \begin{bmatrix} y_k \\ y_{k-1} \end{bmatrix}.$$

Thus, the observation processes of the measurements received by the estimator is modeled as:

$$\tilde{y}_k = \begin{bmatrix} y_k^{(1)} \\ y_k^{(2)} \end{bmatrix} \tag{3}$$

with

$$y_k^{(1)} = (1-\gamma_k)y_{k-1}+\gamma_k y_k \ ; \ \ y_k^{(2)} = \gamma_k y_{k-1}+(1-\gamma_k)y_k \tag{4}$$

where γ_k is a scalar quantity taking on values 0 and 1 with

$$p \triangleq Pr\{\gamma_k = 1\};\ \ 1-p \triangleq Pr\{\gamma_k = 0\};\ \ E\gamma_k = E\gamma_k^2 = p \tag{5}$$

and we assume that $0 < p < 1$.
Our goal is to obtain the optimal estimator which the form is chosen to be linear of the received observations as follows:

$$\hat{x}_{k+1} = F_k\hat{x}_{k-1} + [H_{k1} \ \ H_{k2}]\tilde{y}_k \tag{6}$$

It is useful to define the estimator error and error covariance:

$$\begin{aligned} e_{k+1} \quad &\triangleq x_{k+1} - \hat{x}_{k+1} && (7) \\ \bar{P}_{k+1} \quad &\triangleq E_x E_{\gamma_k}[(x_{k+1} - \hat{x}_{k+1})(x_{k+1} - \hat{x}_{k+1})^T] && (8) \end{aligned}$$

where E_x is the expectation with respect to υ, ω and x_0; and E_{γ_k} is expectation with respect to γ_k.
The estimate $\hat{x}_{k+1}$ needs to be optimal in the sense that it minimizes the error covariance, i.e. it is desired to find the estimator to minimize (8). We demand that the estimator is unbiased, i.e. $E_x E_\gamma e_{k+1} = 0$, and we also want the estimation error covariance to be uniformly bounded, as defined below.

Definition 1: the estimation error covariance is called uniformly bounded if there exists a constant $M > 0$ independent of P_0, such that

$$\bar{P}_k \leq M \tag{9}$$

for all $k = 0, 1, 2, \ldots$.

The estimator error e_{k+1} is defined in (7). Substitute (1), (3) and (6) into it, we get

$$\begin{aligned} &e_{k+1} = x_{k+1} - \hat{x}_{k+1} \\ &= A^2x_{k-1} + A\upsilon_{k-1} + \upsilon_k - F_k\hat{x}_{k-1} \\ &- [H_{k1} \ \ H_{k2}] \begin{bmatrix} (1-\gamma_k)C + \gamma_k CA \\ \gamma_k C + (1-\gamma_k)CA \end{bmatrix} x_{k-1} \\ &- [H_{k1} \ \ H_{k2}] \begin{bmatrix} (1-\gamma_k)\omega_{k-1} + \gamma_k C\upsilon_{k-1} + \gamma_k\omega_k \\ \gamma_k\omega_{k-1} + (1-\gamma_k)C\upsilon_{k-1} + (1-\gamma_k)\omega_k \end{bmatrix} \end{aligned} \tag{10}$$

From the unbiased property $E_x E_{\gamma_k} e_{k+1} = 0$, with the property of γ_k in (5) and the mean of noises is 0, we get

$$F_k = A^2 - [H_{k1} \quad H_{k2}] \begin{bmatrix} (1-p)C + pCA \\ pC + (1-p)CA \end{bmatrix} \tag{11}$$

Substituting F_k into (10), the error is rewritten as

$$\begin{aligned} e_{k+1} = & \left[A^2 - [H_{k1} \ H_{k2}] \begin{bmatrix} (1-p)C + pCA \\ pC + (1-p)CA \end{bmatrix}\right] e_{k-1} \\ & - [H_{k1} \ H_{k2}] \begin{bmatrix} (p-\gamma_k)C + (\gamma_k - p)CA \\ (\gamma_k - p)C + (p-\gamma_k)CA \end{bmatrix} x_{k-1} \\ & - [H_{k1} \quad H_{k2}] \begin{bmatrix} (1-\gamma_k)\omega_{k-1} + \gamma_k C\upsilon_{k-1} + \gamma_k \omega_k \\ \gamma_k \omega_{k-1} + (1-\gamma_k)C\upsilon_{k-1} + (1-\gamma_k)\omega_k \end{bmatrix} \\ & + A\upsilon_{k-1} + \upsilon_k \end{aligned} \tag{12}$$

Lemma 1 Consider the estimation error dynamic equation (12), suppose A is unstable, then a necessary condition for the estimation error to be unbiased and error covariance to be uniformly bounded is that $H_{k1} = H_{k2}$ for all k. Consequently, the optimal estimator has the form

$$\hat{x}_{k+1} = F_k \hat{x}_{k-1} + H_k \frac{1}{2}(y_{k-1} + y_k) \tag{13}$$

Proof: the system matrix A is unstable, then as $k \to \infty$, $x_k \to \infty$. From the Definition 1, the error e_{k+1} will be uniformly bounded, then

$$[H_{k1} \ H_{k2}] \begin{bmatrix} (p-\gamma_k)C + (\gamma_k - p)CA \\ (\gamma_k - p)C + (p-\gamma_k)CA \end{bmatrix} = 0 \tag{14}$$

must be satisfied.
From (14), it follows
$H_{k1}[(p-\gamma_k)C+(\gamma_k-p)CA]+H_{k2}[(\gamma_k-p)C+(p-\gamma_k)CA]$
$= (H_{k1}-H_{k2})[(p-\gamma_k)C+(\gamma_k-p)CA] = 0$
since $[(p-\gamma_k)C+(\gamma_k-p)CA] \neq 0$, it must be $H_{k1} = H_{k2}$. We let $H_k = 2H_{k1} = 2H_{k2}$, substituting it into (6), then the estimator (6) can be finally equivalent to (13).
Substituting H_k into (11), it follows that

$$\begin{aligned} F_k &= A^2 - [H_k \quad H_k]\frac{1}{2} \begin{bmatrix} (1-p)C + pCA \\ pC + (1-p)CA \end{bmatrix} \\ &= A^2 - H_k\frac{1}{2}((1-p)C + pCA) + H_k\frac{1}{2}(pC + (1-p)CA) \\ &= A^2 - \frac{1}{2}H_k(C + CA) \end{aligned} \tag{15}$$

Obviously, in (13), it is used the mean of measurements to estimate, and the optimal estimation gain H_k is given in the following theorem.

Theorem 1 For the system (1)-(2), and the estimator form (13), suppose the estimation error covariance $\bar{P}_{k-1}$ is given. Then the estimation gain H_k for

$$\min_{H_k} \bar{P}_{k+1} \tag{16}$$

is given by

$$H_k = \frac{1}{2}(A^2 \bar{P}_{k-1}(C + CA)^T + AQ_{k-1}C^T)M_k^{-1} \tag{17}$$

where $M_k = \frac{1}{4}[(C + CA)\bar{P}_{k-1}(C + CA)^T + CQ_{k-1}C^T + R_{k-1} + R_k]$.
The corresponding solution for $\bar{P}_{k+1}$ is given by

$$\bar{P}_{k+1} = A^2 \bar{P}_{k-1} A^{2T} - H_k M_k H_k^T + AQ_{k-1}A^T + Q_k \tag{18}$$

$$P_0 = Ex_0 x_0^T \tag{19}$$

Proof: from (1), (14), the estimator error is

$$\begin{aligned} e_{k+1} = & (A^2 - \frac{1}{2}H_k(C + CA))e_{k-1} + A\upsilon_{k-1} + \upsilon_k \\ & - \frac{1}{2}H_k(\omega_{k-1} + C\upsilon_{k-1} + \omega_k) \end{aligned} \tag{20}$$

It is obvious that the noise of system $A\upsilon_{k-1} + \upsilon_k$ and the measurement noise $\omega_{k-1} + C\upsilon_{k-1} + \omega_k$ are correlated, thus the estimation error covariance is

$$\begin{aligned} \bar{P}_{k+1} &= E_x E_\gamma [e_{k+1} e_{k+1}^T] \\ &= (A^2 - \frac{1}{2}H_k(C + CA))\bar{P}_{k-1}(A^2 - \frac{1}{2}H_k(C + CA))^T \\ &+ AQ_{k-1}A^T - \frac{1}{2}AQ_{k-1}C^T H_k^T - \frac{1}{2}H_k C Q_{k-1}A^T + Q_k \\ &+ \frac{1}{4}H_k C Q_{k-1} C^T H_k^T + \frac{1}{4}H_k R_{k-1} H_k^T + \frac{1}{4}H_k R_k H_k^T \\ =& (H_k + H_k^*)M_k(H_k + H_k^*)^T - H_k M_k H_k^{*T} - H_k^* M_k H_k^T \\ &- H_k^* M_k H_k^{*T} + A^2 \bar{P}_{k-1} A^{2T} + AQ_{k-1}A^T + Q_k \\ &- \frac{1}{2}H_k C Q_{k-1} A^T - \frac{1}{2}AQ_{k-1}C^T H_k^T \\ &- \frac{1}{2}H_k(C + CA)\bar{P}_{k-1}A^{2T} - \frac{1}{2}A^2\bar{P}_{k-1}(C + CA)^T H_k^T \end{aligned} \tag{21}$$

where $M_k = \frac{1}{4}[(C + CA)\bar{P}_{k-1}(C + CA)^T + CQ_{k-1}C^T + R_{k-1} + R_k]$.
To minimize (16), the H_k^* should be chosen as

$$H_k^* = -\frac{1}{2}(A^2 \bar{P}_{k-1}(C + CA)^T + AQ_{k-1}C^T)M_k^{-1}$$

and $H_k = -H_k^*$, then the estimator gain (17) is obtained. Substituting H_k, H_k^* back to (21), we get (18), with the initial condition is $P_0 = Ex_0 x_0^T$.

Remark 1: when the system noise and the measurement noise are uncorrelated, the error covariance equation is monotone, and the estimation is optimal. How about the case when the noises are correlated between system and measurement? In the following we will give the monotonicity property when the noises are correlated.

From (21), we have

$$\begin{aligned} \bar{P}_{k+1} = & (A^2 - \frac{1}{2}H_k(C + CA))\bar{P}_{k-1}(A^2 - \frac{1}{2}H_k(C + CA))^T \\ & + AQ_{k-1}A^T - \frac{1}{2}AQ_{k-1}C^T H_k^T - \frac{1}{2}H_k C Q_{k-1} A^T \\ & + \frac{1}{4}H_k C Q_{k-1} C^T H_k^T + Q_k + \frac{1}{4}H_k R_{k-1} H_k^T \\ & + \frac{1}{4}H_k R_k H_k^T \end{aligned} \tag{22}$$

and (17)
$H_k \quad = \quad 2(A^2\bar{P}_{k-1}(C \ + \ CA)^T \ + \ AQ_{k-1}C^T)((C \ +$

$$CA)\bar{P}_{k-1}(C+CA)^T + CQ_{k-1}C^T + R_{k-1} + R_k)^{-1} \tag{23}$$

Denote the mapping (22), (23) from $\bar{P}_{k-1}$ to $\bar{P}_{k+1}$ by $\mathcal{F}(\cdot) : S_+^n \to S_+^n$, i.e.,

$$\bar{P}_{k+1} = \mathcal{F}(\bar{P}_{k-1}) \tag{24}$$

Lemma 2 $\mathcal{F}(\cdot)$ is a monotonic function, i.e., if $\bar{P}_{k-1}^{(1)} \geq \bar{P}_{k-1}^{(2)} > 0$, then

$$\mathcal{F}(\bar{P}_{k-1}^{(1)}) \geq \mathcal{F}(\bar{P}_{k-1}^{(2)}) \tag{25}$$

Proof: denote the mapping (22) from $\bar{P}_{k-1}$ and H_k to $\bar{P}_{k+1}$ by $G(\cdot,\cdot) : S_+^n \times R^n \to S_+^n$, then since the solution H_k in (23) is obtained by minimizing (22), that it is

$$H_k = \arg\min_{\tilde{H}_k} G(\bar{P}_{k-1}, \tilde{H}_k) \tag{26}$$

with the suppose $\bar{P}_{k-1}^{(1)} \geq \bar{P}_{k-1}^{(2)}$, let $H_k^{(1)}$ and $H_k^{(2)}$ be the corresponding H_k as obtained in (23) by (26), then

$$\begin{aligned} \bar{P}_{k+1}^{(2)} &= G(\bar{P}_{k-1}^{(2)}, H_k^{(2)}) \\ &\leq G(\bar{P}_{k-1}^{(2)}, H_k^{(1)}) \\ &\leq G(\bar{P}_{k-1}^{(1)}, H_k^{(1)}) \\ &= \bar{P}_{k+1}^{(1)} \end{aligned} \tag{27}$$

Hence, the lemma holds.
In the above, the two equalities follow from (26). The first inequality follows from (26) as well. The second inequality follows from (22), i.e., $G(\bar{P}_{k-1}, \tilde{H}_k)$ is linear in $\bar{P}_{k-1}$ when H_k is fixed.

Remark 2 From lemma 2, we know that the estimator in Theorem 1 is optimal.

IV. Optimal Estimator For Random Delays Bounded $N = 1$

From Theorem 1, we know that when $r = 2$ the measurements adopted by estimator in (13) is the mean of the sequences y_{k-1} and y_k, Then for the all cases of Case 1 and Case 2, the following model for the measurement received by the estimator is adopted as:

$$\mathbf{y}_k = \begin{cases} \frac{1}{r}\sum_{i=1}^{r} y_{k-m+i-1} & r = 1 \ \ or \ \ 2 \\ \Phi & r = 0 \text{ there is no packet.} \end{cases} \tag{28}$$

where Φ is an empty set.

Theorem 2 Considering the system (1),(2), m is the random time delay and r is the number of the received packets at time k, then based on the observation (28) the optimal estimator is given by:

• when there is no packet received, it is $r = 0$, there is no need to update the state estimate, i.e., the most recent state estimate remains at $\hat{x}_{k-1}$ and error covariance at $\bar{P}_{k-1}$.

• when $r > 0$, the optimal estimator is

$$\hat{x}_{k+r-m} = F_k \hat{x}_{k-m} + H_k \mathbf{y}_k \tag{29}$$

where

$$F_k = A^r - \frac{1}{r} H_k \sum_{i=0}^{r-1} CA^i \tag{30}$$

$$\begin{aligned} H_k = r(&A^r \bar{P}_{k-m}(\textstyle\sum_{i=0}^{r-1} CA^i)^T \\ &+ \sum_{i=1}^{r-1}\sum_{j=0}^{i-1} A^i Q_{k-m+r-i-1} A^{jT} C^T) M_k^{-1} \end{aligned} \tag{31}$$

$$\begin{aligned} M_k = &\textstyle\sum_{i=0}^{r-1} CA^i \bar{P}_{k-m}(\sum_{i=0}^{r-1} CA^i)^T + \sum_{i=0}^{r-1} R_{k-m+i} \\ &+ \sum_{i=1}^{r-1}\sum_{j=0}^{i-1} CA^j Q_{k-m+i-j-1} A^{jT} C^T \end{aligned} \tag{32}$$

with the error covariance equation is

$$\begin{aligned} \bar{P}_{k-m+r} = &A^r \bar{P}_{k-m} A^{rT} - H_k M_k H_k^T \\ &+ \sum_{i=0}^{r-1} A^i Q_{k-m+i} A^{iT} \end{aligned} \tag{33}$$

where $m = 0$ or 1, and $r = 0$, 1 or 2, and the initial error covariance is $\bar{P}_0 = E x_0 x_0^T$.

Proof: From (1) (2) (28), the error is :

$$\begin{aligned} e_{k-m+r} &= x_{k-m+r} - \hat{x}_{k-m+r} \\ &= A^r x_{k-m} + \sum_{i=0}^{r-1} A^i \upsilon_{k-m+r-i-1} - F_k \hat{x}_{k-m} \\ &- \frac{1}{r} H_k \sum_{i=0}^{r-1} CA^i x_{k-m} - \frac{1}{r} H_k \sum_{i=1}^{r-1}\sum_{j=0}^{i-1} CA^j \upsilon_{k-m+i-j-1} \\ &- \frac{1}{r} H_k \sum_{i=0}^{r-1} \omega_{k-m+i} \end{aligned} \tag{34}$$

by the estimator's unbiased property, we get (30). Substituting (34) into (8), the estimator error covariance becomes

$$\begin{aligned} \bar{P}_{k-m+r} &= E_x[e_{k-m+r} e_{k-m+r}^T] \\ &= (A^r - \frac{1}{r} H_k \sum_{i=0}^{r-1} CA^i) \bar{P}_{k-m} (A^r - \frac{1}{r} H_k \sum_{i=0}^{r-1} CA^i)^T \\ &+ \sum_{i=0}^{r-1} A^i Q_{k-m+r-i-1} A^{iT} + \frac{1}{r^2} H_k \sum_{i=0}^{r-1} R_{k-m+i} H_k^T \\ &+ \frac{1}{r^2} H_k \sum_{i=1}^{r-1}\sum_{j=0}^{i-1} CA^j Q_{k-m+i-j-1} A^{jT} C^T H_k^T \\ &- \frac{1}{r} \sum_{i=1}^{r-1}\sum_{j=0}^{i-1} A^i Q_{k-m+r-i-1} A^{jT} C^T H_k^T \\ &- \frac{1}{r} H_k \sum_{i=1}^{r-1}\sum_{j=0}^{i-1} CA^j Q_{k-m+r-i-1} A^{iT} \end{aligned}$$

(35)

Similar to Theorem 1, with minimizing the (35), (31)-(33) are obtained, and the similar to Lemma 2, we know that $\bar{P}_{k-m+r}$ is monotonic in $\bar{P}_{k-m}$. Hence, the estimator (29) is the optimal.

V. Conclusion

In this paper, for the networked control systems with bounded random measurement delay of at most one, the optimal estimator is derived without using time stamps. The key to our development in the estimation of the networked control systems is to use the mean of all the received measurements at each instant time. We have shown that the state estimator is optimal in the class of linear estimators with the properties if zero bias and uniformly bounded estimation error covariance. Further work will be intended for longer time delays.

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