

Decentralized Control of Aggregated Loads for Demand Response

Di Guo, Wei Zhang, Gangfeng Yan, Zhiyun Lin, and Minyue Fu

Abstract This paper focuses on the aggregated control of a large number of residential responsive loads for various demand response applications. We propose a general hybrid system model which can capture the dynamics of both Thermostatically Controlled Loads (TCLs) such as air conditioners and water heaters, as well as deferrable loads such as washers, dryers, and Plug-in Hybrid Electric Vehicles (PHEVs). Based on the hybrid system model, the aggregated control problem is formulated as a large scale optimal control problem that determines the energy use plans for a heterogeneous population of hybrid systems. A decentralized cooperative control algorithm is proposed to solve the aggregated control problem. Convergence of the proposed algorithm is proved using potential game theory. The simulation results indicate that the aggregated power response can accurately track a reference trajectory and effectively reduce the peak power consumption.

I. INTRODUCTION

Demand response has the potential to shift and sculpt energy use, and help maintain the supply-demand balance of the grid. It has attracted considerable research attention in recent years. In [1]–[3], various control and scheduling algorithms are developed based on different real time pricing schemes. The aggregated modeling and control for a large load population has been studied extensively recently [4]–[6] which mainly focuses on first-order Thermostatically Controlled Loads (TCLs), and try to design their aggregated power response through centralized control.

This paper focuses on the aggregated control of a heterogeneous load population, including both TCLs as well as deferrable loads. Our goal is to develop a practically sound control strategy to coordinate the loads to achieve a desired aggregated power response. We propose a hybrid system model which is suitable for many responsive loads, e.g., TCLs, PHEVs, etc. We then formulate a decentralized control framework for which each responsive load solves a simple local optimal control problem coordinated through the central aggregator. Different from [7], we assume that there is no direct communication channel among responsive loads. To accomplish the decentralized control, we introduce

a coordination signal generated and broadcasted by the aggregator. This signal reveals system-level information to each responsive load. The responsive loads can thus improve system performance cooperatively by choosing their own optimal operations. Convergence of the decentralized control algorithm is proved using the potential game theory. As long as each responsive load has a finite number of operation modes, the existence of Nash Equilibrium (NE) holds. Although the NE is only a local minimum of the overall aggregated control problem, it typically leads to rather impressive performance as indicated in our simulations. Considering the costly centralized method, this tradeoff is acceptable. Different from many other game-theory based works [7], [8], the payoff (cost) in our work does not represent the explicit welfare received from the market. Instead, it is regarded by each responsive load as a local indicator reflecting the fulfillment of the system-level objective.

Compared with the previous studies, our work has the following advantages. Due to its decentralized nature, our method can handle load heterogeneity in terms of both load types and load parameters. In many cases, load dynamics are time varying due to environmental changes. For example, model parameters of HVACs are highly dependent on the outdoor temperature [9]. The proposed framework can deal with this time varying effect as well. In addition, the proposed decentralized control framework allows for simple incorporation of operational constraints of individual loads. For example, HVAC units have the so-called “lockout” effect, namely, the compressor cannot be immediately turned back on after being turned off to avoid short-cycling issues.

The proposed decentralized aggregated control strategy is validated through realistic simulations. In the simulations, the load population involves both HVACs and PHEVs with heterogeneous load parameters. The “lockout” effect and the dependence on outdoor temperature of HVACs are also taken into account in the simulations. Under this realistic and challenging setup, our approach successfully solves several aggregated control problems for reference tracking and peak reduction applications.

II. PROBLEM FORMULATION

A. General Hybrid System Model for Responsive Loads

Consider a large number of heterogeneous responsive loads in the grid. Denote \mathcal{M} as the responsive load set with $M = |\mathcal{M}|$ being the number of the loads. Many responsive loads have multiple discrete operation modes. Transitions among these modes are governed by certain switching logic rules that depend on the evolution of some continuous state

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variables or exogenous controls. Such dynamics can be described by a hybrid system model, i.e., $\forall i \in \mathcal{M}$

$$\begin{cases} x^i(k+1) = f_{v^i(k)}^i(x^i(k); \theta^i(k)) \\ y^i(k) = h^i(v^i(k)), \quad k = 0, \dots, N-1 \end{cases} \quad (1)$$

where $[0, \dots, N-1]$ is the time horizon, $x^i(k) \in \mathcal{X}^i$ is the continuous state at discrete time instant k with a known initial state $x^i(0)$, $v^i(k) \in \mathcal{V}^i = \{1, \dots, m\}$ represents the discrete operation mode of the system. For each discrete mode $\nu \in \mathcal{V}^i$, the function $f_{\nu}^i(\cdot; \theta) : \mathcal{X}^i \rightarrow \mathbb{R}^n$ is the state update equation that governs the state evolution in mode ν , and $h^i(\nu)$ denotes the output that is assumed to be independent of the continuous state. Here, $\theta^i(k) \in \mathbb{R}^{n_{\theta^i}}$ is the model parameter which is possibly time varying due to environmental changes. Denote by $\mathbf{v}^i = [v^i(0) \dots v^i(N-1)]^T \in \mathcal{Q}^i$ an admissible mode sequence. Define \mathcal{Q}^i as the set of all feasible mode sequences for load i starting from initial state $x^i(0)$, i.e., $\mathcal{Q}^i(x^i(0)) \triangleq \{\mathbf{v}^i \in (\mathcal{V}^i)^N | x^i(k) \in \mathcal{X}^i\}$. Furthermore, define $\mathcal{Q} \triangleq \prod_{i=1}^M \mathcal{Q}^i$ and let $\mathbf{v} = (\mathbf{v}^1, \dots, \mathbf{v}^M) \in \mathcal{Q}$ be an admissible mode sequence profile. We may write $\mathbf{v}^{-i} = (\mathbf{v}^1, \dots, \mathbf{v}^{i-1}, \mathbf{v}^{i+1}, \dots, \mathbf{v}^M)$, where \mathbf{v}^{-i} is an admissible mode sequence profile without load i . Denote $\mathcal{Q}^{-i} = \prod_{j \neq i} \mathcal{Q}^j$. Denote $\mathbf{y}^i = [y^i(0) \dots y^i(N-1)]^T$, we can thus write

$$\mathbf{y}^i = \mathbf{h}^i(\mathbf{v}^i) \quad (2)$$

which is consistent with (1).

Before stating the aggregated control problem, we will first use HVACs and PHEVs as examples to illustrate the application of the hybrid system load model described above.

B. Example for HVAC

HVACs are the most important type of TCLs for demand response. We adopt a second-order Equivalent Thermal Parameter (ETP) model that describes the coupled dynamics of the air and mass temperatures as follows:

$$\begin{cases} \dot{x}_a(t) = \frac{1}{C_a} [x_m(t)H_m - (U_a + H_m)x_a(t) + Q_a + T_o U_a] \\ \dot{x}_m(t) = \frac{1}{C_m} [H_m(x_a(t) - x_m(t)) + Q_m] \end{cases} \quad (3)$$

Here, x_a is the indoor air temperature, x_m is the inner mass temperature. The readers are referred to [9] for a detailed description of these physical parameters and their relations to the ETP model parameters. Depending on the power state of the unit, the heat flux Q_a could take the following two values:

$$Q_a^{\text{on}} = Q_i + Q_s + Q_h \text{ and } Q_a^{\text{off}} = Q_i + Q_s.$$

The coupled ODEs in (3) can be written in a state-space form

$$\frac{d}{dt}x(t) = \hat{A}(t)x(t) + \hat{B}_{v(t)}(t), \quad (4)$$

where $x(t) = [x_a(t) \ x_m(t)]^T$, and $v(t)$ is the discrete mode taking two values with 0 representing the ‘‘Off’’ state and 1 representing the ‘‘On’’ state. Note that the system matrices $\hat{A}(t)$, $\hat{B}_0(t)$ and $\hat{B}_1(t)$ are possibly time-dependent due to

the varying outdoor temperature. Discretizing model (4) with sampling time T results in

$$x(k+1) = A(k)x(k) + B_{v(k)}(k) \quad (5)$$

where $k = 0, 1, \dots, N-1$, $A(k) = e^{\hat{A}(kT)T}$, $B_0(k) = \int_0^T e^{\hat{A}(kT)t} dt \hat{B}_0(kT)$ and $B_1(k) = \int_0^T e^{\hat{A}(kT)t} dt \hat{B}_1(kT)$.

The feasible mode sequence set is determined by two state constraints. First, a suitable air temperature range is desired by each house, i.e., $x_a(k) \in [T_{a,\min} \ T_{a,\max}]$, $k = 0, \dots, N-1$. Second, once the HVAC unit is turned off, we must wait for T_w minutes before turning it back on again to prevent short cycling. We call T_w the minimum off-time. An extra state is introduced in order to cope with this constraint. Denote x_3 as the steps since the last turning-off time. We have

$$x_3(k+1) = \begin{cases} x_3(k) + 1 & \text{in mode 0,} \\ 0 & \text{in mode 1.} \end{cases}$$

Now we introduce the feasible mode sequence set of HVACs as follows: $\mathbf{v} = [v(0) \dots v(N-1)]^T \in \mathcal{Q}(x(0), x_3(0))$ if $\forall k \in \{0, \dots, N-1\}$,

$$v(k) \in \begin{cases} \{0\} & \text{if } 0 < x_3(k) < \lceil \frac{T_w}{T} \rceil \\ \{0, 1\} & \text{otherwise} \end{cases} \quad (6)$$

and $x_a(k+1) \in [T_{a,\min} \ T_{a,\max}]$ with dynamics (5).

Here, $\lceil x \rceil$ denotes the smallest integer not less than x . Equation (6) represents the constraint of short cycling problem, i.e., HVAC units can not be turned on again if the off-time is less than T_w ($0 < x_3(k) < \lceil \frac{T_w}{T} \rceil$).

C. Example for PHEV Charging

A PHEV charging job can be characterized by a tuple (t_0, τ, t_f) , where the three entries represent the arrival time, the size of the job, and the deadline to complete the job, respectively. The job size $\tau = \frac{E}{r}$, where E is the total energy needed to finish the charging and r is charging rate. The timing dynamics of a charging job can be described as a hybrid system with two discrete modes $\{0, 1\}$, representing that the load is waiting to be processed, and actively running respectively. The continuous state space is two-dimensional with $x_1(k)$ representing the remaining time to finish the load if it is running, and $x_2(k)$ representing the time before the deadline. The state update equation in each mode is given by

$$\begin{cases} f_0(x(k)) = \begin{pmatrix} x_1(k) \\ x_2(k) - T \end{pmatrix} \\ f_1(x(k)) = \begin{pmatrix} x_1(k) - T \\ x_2(k) - T \end{pmatrix} \end{cases} \quad (7)$$

Here, T is the sampling time. The feasible mode sequence set $\mathcal{Q} = \{\mathbf{v} \in \mathcal{Q} | x_1(k) \leq x_2(k) \text{ with dynamic (7)}\}$.

D. Aggregated Control Problem

We consider the scenario where the aggregator has signed contracts with participating residential customers to directly control some of their loads. The aggregator could be part of a

Load Serving Entity (LSE) or a Curtailment Service Provider (CSP). It has two-way communication channels with each individual load. Many objectives of the aggregator can be formulated as the following general centralized optimal control problem.

Problem 1 (Aggregated Control Problem):

$$\min_{\mathbf{v}^i \in \mathcal{Q}^i, i=1, \dots, M} J(\mathbf{v}^1, \dots, \mathbf{v}^M)$$

$$\text{where } J(\mathbf{v}^1, \dots, \mathbf{v}^M) \triangleq \sum_{k=0}^{N-1} F_k(\mathbf{x}(k), \mathbf{v}(k)).$$

Here, $\mathbf{x}(k)$ and $\mathbf{v}(k)$ are the overall state and mode vectors for the participating loads at time instant k , i.e.,

$$\mathbf{x}(k) = [x^1(k)^T \quad x^2(k)^T \quad \dots \quad x^M(k)^T]^T,$$

$$\mathbf{v}(k) = [v^1(k) \quad v^2(k) \quad \dots \quad v^M(k)]^T.$$

The function $F_k(\cdot, \cdot) : \prod_{i=1}^M \mathcal{X}^i \times \prod_{i=1}^M \mathcal{Y}^i \rightarrow \mathbb{R}$ is the running cost function of the entire load population.

This optimal control problem can be solved by Dynamic Programming. However, it is often computationally intractable if the number of the loads M is large due to the curses of dimensionality. More importantly, even if such a solution is available, its application in smart grid would be rather limited because the aggregator requires the knowledge of the model parameters and states of all the loads, which is rather costly or even unrealistic.

III. DECENTRALIZED SOLUTION

A. General Solution

We decompose Problem 1 into decentralized subproblems. Suppose load $i \in \mathcal{M}$ has the following objective.

$$\min_{\mathbf{v}^i \in \mathcal{Q}^i} J^i(\mathbf{v}^i, \gamma)$$

$$\text{where } J^i(\mathbf{v}^i, \gamma) \triangleq \sum_{k=0}^{N-1} F_k^i(x^i(k), v^i(k), \gamma(k)). \quad (8)$$

Here, $\gamma = [\gamma(0) \quad \gamma(1) \quad \dots \quad \gamma(N-1)]^T \in \mathbb{R}^N$ denotes the coordination signal received from the aggregator, $F_k^i(\cdot, \cdot) : \mathcal{X}^i \times \mathcal{Y}^i \rightarrow \mathbb{R}$ is the running cost of load i at time instant k . After collecting all the outputs of the loads, the aggregator can generate a coordination signal according to the following equation,

$$\gamma = L(\mathbf{y}^1, \dots, \mathbf{y}^M) = \sum_{i=1}^M w^i(\mathbf{y}^i) + \zeta. \quad (9)$$

where $w^i : \mathbb{R}^N \rightarrow \mathbb{R}^N$ is a function evaluating the output of load i , $\zeta \in \mathbb{R}^N$ is a system metric parameter. Notice that both the states and outputs depend on the mode operations, i.e., \mathbf{v}^i . Substituting (9) and (2) to (8), we can rewrite

$$J^i(\mathbf{v}^i, \gamma) \triangleq \hat{J}^i(\mathbf{v}^1, \dots, \mathbf{v}^M) = \hat{J}^i(\mathbf{v}^i, \mathbf{v}^{-i}), i = 1, \dots, M.$$

The process of deciding \mathbf{v}^i can be regarded as a game played by each load with the corresponding cost function $\hat{J}^i(\cdot)$. We call it mode decision game denoted by $\Gamma = (\mathcal{M}, \mathcal{Q}, \mathcal{J})$, where $\mathcal{J} = (\hat{J}^1, \dots, \hat{J}^M)$.

Definition 1: A mode sequence profile $\mathbf{v}^* = (\mathbf{v}^{1*}, \mathbf{v}^{-1*})$ is a Nash Equilibrium (NE) if $\forall i$ and $\forall \mathbf{v}^i \in \mathcal{Q}^i$

$$\hat{J}^i(\mathbf{v}^*) \leq \hat{J}^i(\mathbf{v}^i, \mathbf{v}^{-i*}).$$

Assumption 1: $\forall i, \forall \mathbf{v}^{-i} \in \mathcal{Q}^{-i}$ and for all $\mathbf{a}^i, \mathbf{b}^i \in \mathcal{Q}^i$

$$\hat{J}^i(\mathbf{a}^i, \mathbf{v}^{-i}) < \hat{J}^i(\mathbf{b}^i, \mathbf{v}^{-i}) \Leftrightarrow J(\mathbf{a}^i, \mathbf{v}^{-i}) < J(\mathbf{b}^i, \mathbf{v}^{-i})$$

Assumption 1 requires the global cost that exhibits the same “directional” behavior, when the individual load unilaterally deviates. Under this assumption, the mode decision game Γ becomes a potential game with the global cost function $J(\cdot)$ the potential function. A potential game with finite action set are known to possess at least one NE in pure strategies [10]. Although the NE is not guaranteed to be the global optimal solution of Problem 1, we will show in the following sections that the performance of the NE is acceptable in real applications.

There are a wide range of distributed learning algorithms that converge to an NE in potential games. However, most methods depend on sufficient information exchange between all the players, which is not practically feasible or rather costly for demand response applications. We thus introduce the coordination signal γ to reduce the communication burden.

Now we give our learning algorithm which is illustrated in Fig. 1 and described by Algorithm 1. The basic idea here is to obtain a better reply for each responsive load at every iteration.

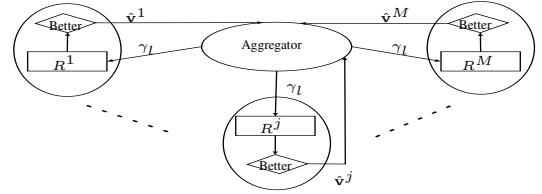


Fig. 1. Learning algorithm in smart grid, where R^i represents responsive load i

Algorithm 1 Learning Algorithm

Input: Cost functions, initial states and the dynamic of each responsive load

Output: $\mathbf{v}^{1*}, \mathbf{v}^{2*}, \dots, \mathbf{v}^{M*}$

- 1: $l = 1$
 - 2: Each load randomly chooses an admissible mode sequence, e.g., \mathbf{v}_0^i and sends it to the aggregator
 - 3: **while** There exists better reply **do**
 - 4: The aggregator generates γ_l and broadcasts it
 - 5: Each load simultaneously solves Problem 2 and obtains a new mode sequence $\hat{\mathbf{v}}^i, i = 1, \dots, M$
 - 6: **if** $\hat{\mathbf{v}}^i$ is a better reply **then**
 - 7: $\hat{\mathbf{v}}^i$ is transmitted to the aggregator
 - 8: **end if**
 - 9: The aggregator randomly accepts one better reply and updates the mode sequences according to (10)
 - 10: $l = l + 1$
 - 11: **end while**
-

At iteration l , load i receives γ_l and solves the following optimal control problem with only local information v_{l-1}^i at the last iteration and the current coordination signal γ_l .

Problem 2 (Decentralized Control for Load i):

$$\min_{\hat{\mathbf{v}}^i \in \mathcal{Q}^i} J^i(\hat{\mathbf{v}}^i, \phi)$$

$$\text{where } \phi = \gamma_l - w^i(\mathbf{h}^i(\mathbf{v}_{l-1}^i)) + w^i(\mathbf{h}^i(\hat{\mathbf{v}}^i))$$

In Algorithm 1, $\hat{\mathbf{v}}^i \in \mathcal{Q}^i$ is a (strictly) better reply at iteration l if

$$\hat{J}^i(\hat{\mathbf{v}}^i, \mathbf{v}_{l-1}^{-i}) < \hat{J}^i(\mathbf{v}_{l-1}^i, \mathbf{v}_{l-1}^{-i}).$$

The mode sequences are updated as follows:

$$\mathbf{v}_l^i = \begin{cases} \hat{\mathbf{v}}^i & \hat{\mathbf{v}}^i \text{ is a better reply and is accepted,} \\ \mathbf{v}_{l-1}^i & \text{otherwise.} \end{cases} \quad (10)$$

Notice that the information required for load i to solve the above optimization problem includes γ_l , \mathbf{v}_{l-1}^i , $\mathbf{h}^i(\cdot)$ and $w^i(\cdot)$. They are all available since γ_l can be received from the aggregator, and the last three are all local.

Theorem 1: The solution proposed here will reach an NE.

Proof: Algorithm 1 can generate a finite improvement path. It is proved in [10] that every maximal improvement path must terminate in an NE in potential games. ■

B. Reference Tracking Problem

We now consider an important class of the general aggregated control problem, which tries to cooperatively control the loads to track a given aggregated power reference. Let $\mathbf{y}_{ref} = [r(0) \ r(1) \ \dots \ r(N-1)]^T \in \mathbb{R}_+^N$ denote a reference power trajectory. Here, the output of each load is the power consumption. Different operation modes correspond to different power consumptions. For both HVACs and PHEVs, $h^i(0) = 0, h^i(1) = p^i \Rightarrow h^i(v^i) = v^i p^i$, where p^i is the power consumption when the HVAC is ‘‘On’’ or the PHEV is actively charging. Hence, the tracking problem can be formulated as an aggregated control problem with the following cost function

$$J(\mathbf{v}^1, \dots, \mathbf{v}^M) = \|\mathbf{y}_{ref} - \sum_{i=1}^M \mathbf{v}^i p^i\|.$$

We design a specific coordination signal

$$\gamma = \mathbf{df} = \mathbf{y}_{ref} - \sum_{i=1}^M \mathbf{v}^i p^i \in \mathbb{R}^N. \quad (11)$$

Here, \mathbf{df} is called the difference vector and $\|\mathbf{df}\|$ is the error between the reference and the aggregated response. We design J^i as

$$J^i(\mathbf{v}^i, \gamma) = J^i(\mathbf{df}) = \|\mathbf{df}\|.$$

Each load has the following objective:

$$\min_{\mathbf{v}^i \in \mathcal{Q}^i} J^i(\mathbf{v}^i, \gamma).$$

Notice that $J = J^1 = \dots = J^M$. Assumption 1 is thus satisfied which makes the the process a potential game.

We can directly use Algorithm 1 to get an NE. However in Algorithm 1 each load randomly chooses an admissible mode sequence initially, which may result in a slow convergence rate. Hence, we introduce Algorithm 2 as illustrated in Fig. 2 to obtain better initial mode sequences. Responsive loads are divided into G groups. Every load is associated with a unique group say $i \in g_j$. Each load solves the following problem.

Problem 3 (Initialization of Load i):

$$\min_{\mathbf{v}^i \in \mathcal{Q}^i} \|\lambda_j^T \mathbf{v}^i p^i\|, \quad i \in g_j$$

Here $\lambda_j \in \mathbb{R}^N$ is the coordination signal at this stage with the k th element $\lambda_j(k)$ determined by

$$r(k) = \sum_{i \in \text{determined } g_m} v^i(k) p^i. \quad (12)$$

Group g_m is determined if all the loads belong to g_m obtain mode sequences and send them to the aggregator.

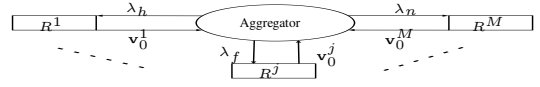


Fig. 2. Initialization stage optimization ($1 \in g_h, j \in g_f$, and $M \in g_n$)

Algorithm 2 Initialization Stage Optimization

Input: \mathbf{y}_{ref} , initial states and the dynamic of each responsive load

Output: $\mathbf{v}_0^1, \mathbf{v}_0^2, \dots, \mathbf{v}_0^M$

- 1: The aggregator randomly divides loads into G groups, i.e., $\exists j \in \{1, \dots, G\}, \forall i$, load $i \in g_j$
- 2: The aggregator obtains λ_1 as follows
- 3: **for** $k = 0$ to $N - 1$ **do**
- 4: $\lambda_1(k) \propto 1/r(k)$
- 5: **end for**
- 6: **for** $j = 1$ to G **do**
- 7: The aggregator transmits λ_j to every load belong to g_j
- 8: Load $i \in g_j$ solves Problem 3, obtains \mathbf{v}_0^i and transmits it to the aggregator
- 9: **if** $j < G$ **then**
- 10: The aggregator updates λ_{j+1} as follows
- 11: **for** $k = 0$ to $N - 1$ **do**
- 12: $\lambda_{j+1}(k) \propto 1/(12)$
- 13: **end for**
- 14: **end if**
- 15: **end for**

C. Online Implementation of Reference Tracking

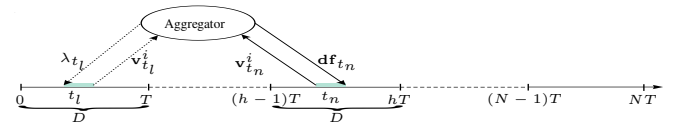


Fig. 3. Online optimization for responsive load i , the process of dashed line is only for HVACs

The solution proposed in the previous subsection is actually an off-line planning. In real power systems, an online method is necessary to track the reference in a decentralized manner. Hence, we revise the above solution and obtain such online method. For simplicity, only the reference tracking problem is considered. Other applications can be obtained similarly.

The solution is depicted in Fig. 3. Suppose there are HVACs and PHEVs in the grid with sampling time T_h and T_v respectively. We assume that $T_h = T_v = T$. One can easily extend our algorithm to the case when $T_h \neq T_v$. Denote \mathcal{M}_h and \mathcal{M}_v the HVAC set and PHEV set respectively. The continuous time horizon is discretized by T into N instants, i.e., $[0, 1, \dots, N - 1]$. We know that HVACs are running all the time. PHEV i however is operated during $[t_o^i, t_f^i]$. The aggregator calculates and sends D times of coordination signals every T time. The aggregator divides HVAC units into $\lceil \frac{M_h}{D} \rceil$ groups. The time interval of $[0, T]$ is also divided into $\lceil \frac{M_h}{D} \rceil$ slots. Each slot is assigned to one group of HVACs to finish the similar procedure described by Algorithm 2. Notice that T is the sampling time of HVACs. So with this division the initialization stage will be finished within one instant. PHEVs are arriving randomly during the whole time horizon. Assume that in $[0, T]$ there is no PHEV arrival. This assumption is realistic because this interval is rather small compared with the whole horizon.

Let $\mathbf{v}_t^i(h :)$ denote a vector

$$[v_t^i(h) \quad v_t^i(h+1) \dots v_t^i(N-1)]^T.$$

Similarly,

$$\lambda_{t_l}(1 :) = [\lambda_{t_l}(1) \quad \lambda_{t_l}(2) \dots \lambda_{t_l}(N-1)]^T,$$

$$\mathbf{df}_{t_n}(h :) = [\mathbf{df}_{t_n}(h) \quad \mathbf{df}_{t_n}(h+1) \dots \mathbf{df}_{t_n}(N-1)]^T.$$

The optimization problems are similar to the previous subsection. At the initialization stage (between 0 and T), each HVAC solves the following optimization problem to find the future modes after instant 0.

Problem 4 (Initialization of HVAC Load i):

$$\min_{\mathbf{v}_{t_l}^i(1:), \mathbf{v}_{t_l}^i \in \mathcal{Q}^i} \|\lambda_{t_l}(1 :)^T \mathbf{v}_{t_l}^i(1 :) p^i\|$$

The first instant mode $v^i(0)$ of HVAC i is randomly determined. Thus the above optimization problem can only determine the modes after instant 0. PHEVs are not participating in the initialization stage since there is no PHEV arrival at the time horizon $[0, T]$.

Between time instant $h - 1$ and h , all the loads including HVACs and the already arrived PHEVs solve the following optimization problem to obtain a better reply (the modes after instant $h - 1$).

Problem 5 (Decentralized Control For Each Load i):

$$\min_{\mathbf{v}_{t_n}^i(h:), \mathbf{v}_{t_n}^i \in \mathcal{Q}^i} \|\mathbf{df}_{t_n}(h :) + (\mathbf{v}_{t_{n-1}}^i(h :) - \mathbf{v}_{t_n}^i(h :)) p^i\|$$

Now we summarize the online implementation as follows.

- Each HVAC unit i randomly choose an admissible $v_0^i(0)$ and evolves according to its dynamic at $k = 0$.

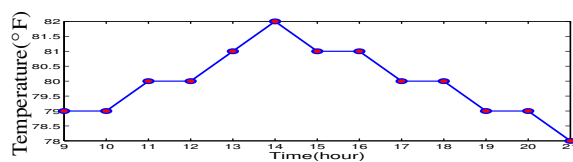


Fig. 4. Hour by hour outdoor temperature of August 15 in Columbus, Ohio (from 9am to 9pm) [11]

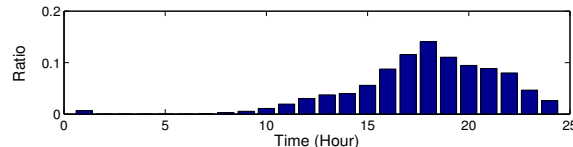


Fig. 5. Last trip ending time distribution [12]

- The following procedure is similar to Algorithm 2. HVACs are divided into $\lceil \frac{M_h}{D} \rceil$ groups. Group of HVACs solve Problem 4 sequentially. When $t = T$, houses evolve individually.
- Next stage is similar to Algorithm 1. At time t_n the aggregator sends \mathbf{df}_{t_n} to all the loads who solve Problem 5, where $(h-1)T \leq t_n < hT$. If $\mathbf{v}_{t_n}^i$ is a better reply, then it is sent to the aggregator who accepts one reply such that the error will be reduced. Responsive loads evolve according to their dynamics.
- If there is no more better reply at time t_e , then the computation stops. The random arrival of PHEVs may restart the computation.

IV. SIMULATION RESULTS

A. Reference Tracking Example

Fig. 6 shows the aggregated response of 1000 HVAC units and 500 PHEVs with the objective of tracking the reference (also depicted in Fig. 6) during a day time.

The ETP model parameters used in the simulation were generated using GridLAB-D [9]. Their power ranges from 4kw to 5kw. The outdoor temperature depicted in Fig. 4 was generated according to the figure in [11] which shows the average weather on August 15 for Columbus, Ohio, USA. We assume that every participating HVAC unit has the same air temperature control deadband $[73^\circ\text{F}, 75^\circ\text{F}]$.

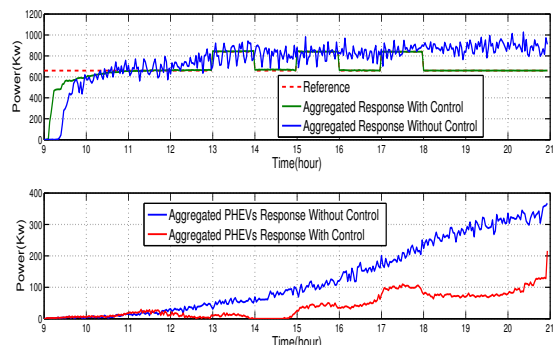


Fig. 6. Reference tracking

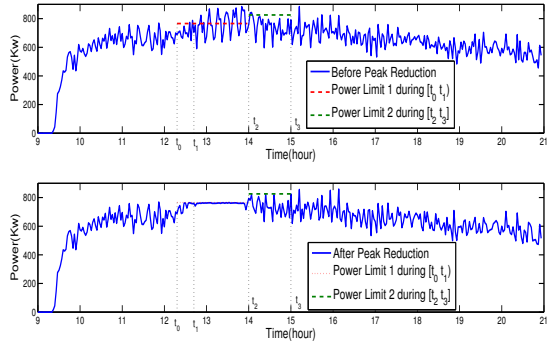


Fig. 7. Peak reduction

Type #	Electric Range	Voltage	Current	Charging Time
1	10 miles	110 (v)	15 (A)	2.1 (h)
2	20 miles	110 (v)	15 (A)	4.2 (h)
3	30 miles	110 (v)	20 (A)	4.8 (h)
4	40 miles	220 (v)	20 (A)	3.3 (h)

TABLE I

4 TYPES OF PHEVs USED IN OUR SIMULATION. DATA COURTESY [13]

Each PHEV charging job is described in Section II-C. Arrival times are generated randomly according to the last trip ending time distribution over one day (See Fig. 5) provided in the 2001 National Household Travel Survey [12]. The size of the charging jobs depends on the State of Charge (SOC) when the vehicle plugs in, as well as the battery capacity of the vehicle. We consider 4 representative types of PHEVs as described in Table I. Each charging job i is randomly associated with one of the four categories and assigned with a random initial SOC uniformly distributed between 0.1 to 0.8. In this way, the job size τ^i can be determined. The deadline time t_f^i is also generated randomly with mean $t_0^i + 10$ and a 2-hour standard deviation.

Without aggregated control, the HVACs will switch on when the air temperature hits the upper boundary of the control deadband, and will switch off when hitting the lower boundary. For the charging of PHEVs, they can randomly determine to run or wait as long as not violating the constraint, i.e., $x_1^i(k) \leq x_2^i(k)$. We can see that the aggregated response tracks the reference accurately with a few exceptions around the beginning time.

B. Peak Reduction Example

Here, we use online implementation algorithm proposed in Section III-C to reduce peak power consumption. Since the highest outdoor temperature is mostly at afternoon as depicted in Fig. 4, peak power consumption occurs around the same time. We consider 1000 HVAC units to illustrate the peak reduction with a minor modification of the optimization objective described in Section III-C, i.e.,

$$\min_{\mathbf{v} \in \mathcal{Q}} J(\mathbf{d}\mathbf{f}) = \min_{\mathbf{v} \in \mathcal{Q}} \sum_{k=0}^{N-1} s(\mathbf{d}\mathbf{f}(k))$$

$$\text{where } s(x) = \begin{cases} x^2 & x \leq 0, \\ 0 & \text{otherwise.} \end{cases}$$

The coordination signal is the same as described in equation (11). Our main purpose is keeping the aggregated response below power Limit 1 in the horizon $[t_0, t_2]$ (at t_2 the temperature is highest). In addition, between t_2 and t_3 , we impose another limit, Limit 2, to prevent large rebounds. The result is given in Fig. 7.

V. CONCLUSION

This work proposes a general framework of decentralized demand side control. The decomposition of the global intractable problem is used such that each individual problem is tractable for each responsive load. The formulation of the mode decision game is employed to obtain a local optimal convergent solution. In the scenario of reference tracking, we design an initialization optimization stage where each responsive load obtains a better initial mode sequence such that the algorithm will converge faster. An online algorithm is also given. Future research will focus on the receding horizon control and its analysis.

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