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Optimal Trajectories for the Preview Control of Dual-Stage Actuators

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Abstract— **Preview Control for Dual-Stage Actuators (DSA) consists in allowing movement of the slow actuator while maintaining the system output at the reference point. This strategy is possible only if the fast actuator is able to compensate the movement of the slow one. The advantages of this control scheme are related to an improvement on the settling time of the output, a consequence of the fact that the slow actuator is allowed to move ahead of the reference transition time. This paper will discuss fundamental limitations that constrain the trajectories of the primary actuator to a feasible set, i.e., a set whose trajectories the secondary actuator is able to effectively compensate. From this initial discussion optimal trajectories will be devised via quadratic programming. Experimental results show the effectiveness of the proposed design.**

I. Introduction

The concept of Dual-Stage Actuators seems to have arisen independently in two seminal works in the field of robotic manipulators: the first work to use the name of Macro/Micro Manipulators was by Sharon, et. al. [1] as early as 1983, while a similar concept was proposed by Kanai, et. al. [2] in the same year. But it was only after the dual-stage concept was picked up by the Hard Disk Drive (HDD) industry, in the early 1990's, that a large development in the area was seen [3]. In fact, until today the label "track seeking" is used to mention movement from one reference point (or HDD track) to another, and "track following" to mention the accuracy in which a given reference is followed under the influence of disturbances. Motivated by the pressing necessity of market-driven improvements for faster and denser hard disks, the HDD industry saw in DSA a strong ally to the solution of disturbance rejection and settling time performance. It was the problem of track following that was of greatest interest in those years because there are complicated broad band disturbance and noise in HDD servo systems, requiring advanced control technology to achieve nanoscale positioning accuracy. The work in [4], for example, presents $\mathcal{H}_2$ and LQG methods for combining both actuators. Immediately after followed $\mathcal{H}_\infty$ methods such as the one in [5], and later on mixed $\mathcal{H}_2/\mathcal{H}_\infty$ robust strategies such as [6]. Up to date this area remains an active topic of research with [7] proposing an automatically tunned controller. As time went on and advances progressed steadily, the dual-stage concept also moved to other applications such as advanced machining tools [8], [9], wafer alignment on microlithography [10], scanning probe microscopy [11], optical drives [12] and large scale rotary actuators [13].

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Such interest in Dual-Stage Actuators (DSA) is of no surprise since they were specifically designed to improve the inherent limitations of single-stage actuators. The dual-stage structure is comprised of a long-range actuator in what is called the "first stage," connected in parallel with a fast dynamics actuator, called the "second stage." The fast-dynamics (or micro) actuator affects the output by providing small additive movements to the first (or macro) stage. By coupling the slow long-range actuator with a fast short-range one, it is expected that the overall system performance is improved because the defects of one actuator may be compensated by the merits of the other.

While the track following task was promptly improved by DSA, it took some time till the research community expressed interest in the track seeking problem. At first, different techniques originally designed for a single stage actuator were implemented to the DSA. In [14], a form of Composite Nonlinear Feedback (CNF) is presented following the original ideas of [15]. Later on, a pioneer work was given by [16] where the actuators were finally combined in a form of control specifically designed for DSA: the primary actuator is designed to overshoot a limited and controlled amount, this overshoot is compensated by the secondary actuator providing faster rise times and, consequently, faster settling times. Soon after it was suggested that a similar idea could be applied before the output transition time and came to be known as Preview Control. This strategy relies on future information in order to improve the performance of the system [17]. It is especially suited for dual-stage systems because it allows the primary actuator to move ahead of the output transition time [18]. During this movement, the secondary actuator compensates the error generated by the primary by moving towards the opposite direction. The overall output is thus secured at the reference while both actuators are in motion. The first work that considered some form of pre-actuation came on [19] which projected optimal trajectories in the Time/Energy sense. This work, however, did not consider the hard constraint given by the saturation

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of the secondary actuator. Such limitation was partially treated in [20], where a practical controller was proposed and implemented. This approach is still not complete in the sense that it considered only the limited range of the secondary actuator, and not its *dynamic* limitations, i.e., it assumed the secondary actuator to be infinitely fast. The current paper shall dispose this assumption and consider a realistic model of the secondary actuator. Given this scenario, the main contribution of this paper is the development of optimal trajectories for DSA during pre-actuation in order to improve the track seeking performance of the system. The trajectories here devised will also respect the constraints imposed by track following, i.e., the dynamic limitations of the secondary actuator.

The paper is organized as follows: Section II describes in detail the fundamental concepts of dual-stage actuators and preview control; the guidelines for the design of feasible trajectories are given in Section III; a form of calculating these trajectories according to the guidelines is presented Section IV with the aid of quadratic programming; Section V exposes the implementation of the proposed design in the experimental set up of a DSA; and Section VI concludes the paper.

II. Preliminaries

Comments on Dual Stage Actuators (DSA) and on the ideas that comprise the Preview Control strategy will be presented in this section so that the paper may be appreciated by readers from different backgrounds.

A. Dual-Stage Actuators

As the name suggests, DSA are comprised of two different actuators which perform the same motion but specialize in different tasks: the primary actuator is usually of large range - hence, it is also called the macro, or coarse actuator - but of moderate speed; the secondary actuator is very fast but constrained by a small range - hence, also called the micro, or fine actuator. Ideally one would prefer to have a fast actuator that possesses sufficiently large range (what is meant by *sufficiently large*, obviously, depends on the application), but a trade off between size and speed is inevitable. In order to solve this problem DSA attempt to get the best of two worlds: an overall actuator that possesses a large range and fast dynamics.

Commonly, the structure that is able to accommodate these specifications involves a traditional form of actuation on the primary stage, such as linear motors, voice-coil motors, etc., which are able to provide for the large range. The second stage, on the other hand, is comprised of a piezoelectric actuator because they are are notoriously light and fast. Such structure may be represented by a double integrator as the primary actuator working in parallel with a mass-spring-damper as the secondary one. Note that usually a pre-compensator is put in place so that nonlinearities such as the friction on the linear motor and the hysteresis of the piezo actuator are sufficiently attenuated, justifying these simplified models [16]. The DSA model is then described as,

$$\begin{aligned} \Sigma_1 &: \dot{x}_1 = A_1 x_1 + B_1 u_1,\ x_1(0) = 0,\ |u_1| \leq \bar{u}_1 \\ \Sigma_2 &: \dot{x}_2 = A_2 x_2 + B_2 u_2,\ x_2(0) = 0,\ |u_2| \leq \bar{u}_2 \\ y &= y_1 + y_2 = C_1 x_1 + C_2 x_2, \end{aligned} \tag{1}$$

where $x_1 = [y_1 \ \ \dot{y}_1]^\intercal$ is associated with the primary actuator and $x_2 = [y_2 \ \ \dot{y}_2]^\intercal$ with the secondary one, and $\bar{u}_i$ is the control saturation level for u_i. Furthermore,

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ b_1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}. \end{aligned} \tag{2}$$

Notice that there is no interaction between actuators apart from their individual contribution to the system output. The no-interaction assumption is valid when the actuators have a large difference in mass (so that the secondary actuator does not affect the primary), and in bandwidth (so that the motion of the primary actuator is easily compensated by the secondary). Hence the obvious choice of piezoelectric actuators.

As described by (1), the overall system is Dual-Input Single-Output (DISO), which provides the control designer with (too much) freedom because there are several combinations of trajectories in the form $y_1 + y_2$ that lead to the same output trajectory y. One form of taking advantage of this freedom is discussed in what follows under the name of Preview Control.

B. Preview Control

The Preview Control concept exploits the redundancy of actuators in order to reduce the transition time from one reference level to another. From the output equation of system (1) one notices that it is not necessary that the primary actuator output y_1 tracks the reference. In fact, it only needs to be close enough to the reference so that the secondary actuator may reach it, i.e.,

$$y_1 \approx r_0 \rightarrow y_1 + y_2 = r_0. \tag{3}$$

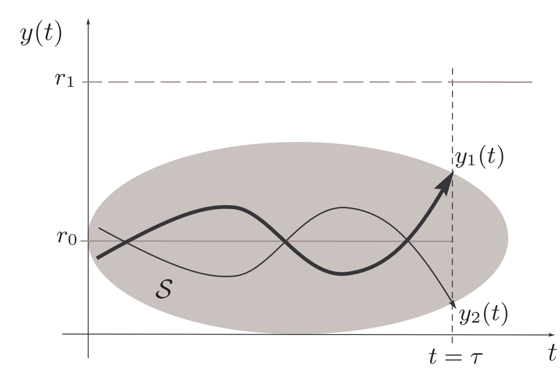

Fig. 1. Schematic representation of the Preview Control strategy. During $t \leq \tau$ the primary actuator (thick line) may move from r_0 in the direction of r_1 as long as it does not leave the region where the secondary actuator (thin line) is able to reach the reference achieving $y = y_1 + y_2 = r_0$.

Which means that around the reference the primary actuator has some freedom of movement. The designer may use this freedom to allow this actuator to move ahead of time, speeding up the process, provided, of course, that the information of the future reference level r_1 and transition instant $t = \tau$ are available. This scenario is exemplified in Fig. 1 where the actuators are depicted in what is called the "pre-actuation interval" ($t \leq \tau$).

During this pre-actuation interval the trajectory of y_1 will deviate from the reference r_0, generating an error that must be compensated by the secondary actuator as described in (3). Obviously, this error cannot be unbounded. This is suggested in Fig. 1 where the region $\mathcal{S}$ depicts the set where the primary actuator must lie so that this error may be compensated. The problem addressed in this paper relates to this set $\mathcal{S}$. How may one define "around the reference" in terms of $\mathcal{S}$? What is the shape of $\mathcal{S}$? And, more importantly, how does one guarantee that $x_1 \in \mathcal{S}$ during Preview Control?

III. Feasible Trajectories

The Preview Control strategy uses the primary actuator to improve the performance of the system and the secondary one to guarantee the tracking of r_0. From Fig. 1 it is clear that the secondary actuator must track the difference between y_1 and r_0, which is nothing but a "mirrored" version of the y_1 trajectory, as depicted in figure:

$$y_2 = r_0 - y_1 \rightarrow y = y_1 + y_2 = r_0. \tag{4}$$

Therefore, when in motion the primary actuator must respect the limitations of the secondary actuator.

Consider the secondary actuator model described as:

$$\epsilon_1 \ddot{y}_2 = y_2 + \epsilon_2 \dot{y}_2 + \beta u_2 \tag{5}$$

where $\epsilon_1 := 1/a_1$, $\epsilon_2 := a_2/a_1$ and $\beta := b_2/a_1$. In our previous works we have assumed the secondary stage is infinitely fast, that is, $\epsilon_1 = 0$ and $\epsilon_2 = 0$ [20]. Then, the model (5) may be reduced to a static one: $y_2 = -\beta u_2$, with u_2 bounded by $\bar{u}_2$. In this senario, when designing a trajectory $z(t)$ for the primary actuator to follow, it is only necessary that $z(t)$ satisfies,

$$|z(t)| \leq \beta \bar{u}_2.$$

Hence, in this previous formulation $|r_0 - y_1(t)| \leq \delta$ for $\delta := \beta \bar{u}_2$, which defines the set $\mathcal{S}$ in a static manner.

In the present work we consider a practical case with nontrivial ϵ_1 and ϵ_2. The motivation for this is the fact that a bounded input will generate bounded states, which in turn implies that the secondary actuator may not track *any* trajectory within its range. The constants ϵ_1 and ϵ_2 must be considered such that limits on $\dot{z}(t)$ and $\ddot{z}(t)$ may be imposed, and the *dynamic* limitations of the secondary actuator may be satisfied. This is achieved in the lemma below.

Lemma 3.1: Given the dual-stage system in (1) with constants a_1, $a_2 < 0$ and $b_2 > 0$, and a desired trajectory $z(t)$, the controller

$$u_2 = \kappa(y_2, \dot{y}_2, z, \dot{z}, \ddot{z}) = (-z - \epsilon_2 \dot{z} + \epsilon_1 \ddot{z})/\beta + \gamma(z, y_2) \tag{6}$$

with $\epsilon_1 := 1/a_1$, $\epsilon_2 := a_2/a_1$, $\beta := b_2/a_1$, and,

$$\gamma(y_2, \dot{y}_2, z, \dot{z}) = [h_1 \;\; h_2] \begin{bmatrix} y_2 - z \\ \dot{y}_2 - \dot{z} \end{bmatrix} \tag{7}$$

for $h_1, \; h_2 > 0$ achieves,

$$\lim_{t\to\infty} |y_2(t) - z(t)| = 0 \quad if \quad |\kappa(y_2, \dot{y}_2, z, \dot{z}, \ddot{z})| \leq \bar{u}_2. \tag{8}$$

In particular, if $y_2(0) - z(0) = 0$ and $\dot{y}_2(0) - \dot{z}(0) = 0$, then

$$y_2(t) = z(t), \;\; \forall t \geq 0.$$

∎

Proof: The proof follows from a direct substitution of (6) in Σ_2, which, disconsidering $\gamma(\cdot)$, results in,

$$\begin{aligned} \epsilon_1 \ddot{y}_2 &= y_2 + \epsilon_2 \dot{y}_2 + \beta \mathrm{sat}(u_2) \\ &= (y_2 - z) + \epsilon_2(\dot{y}_2 - \dot{z}) + \epsilon_1 \ddot{z} \end{aligned} \tag{9}$$

One may now define the trajectory error as $e_t(t) := y_2(t) - z(t)$,

$$\ddot{e}_t = a_1 e_t + a_2 \dot{e}_t$$

By noticing that $a_1, \; a_2 < 0$ it is obvious that $e(t) \rightarrow 0$ as $t \rightarrow \infty$. Moreover, given the zero initial conditions $e_t(0) = 0$ and $\dot{e}_t(0) = 0$, then, $y_2(t) = z(t), \;\; \forall t \geq 0$. Furthermore, the term $\gamma(z, y_2)$ compensates deviations from the desired trajectory due to uncertainty or disturbances, adding robustness to the system. That is, tracking is achieved. Q.E.D.

The Lemma above states that for a given trajectory $z(t)$ to be trackable by the secondary actuator it must satisfy condition (8). But, from (4) one notices that $z(t) = r_0 - y_1(t)$, i.e., it is the primary actuator that defines the trajectory $z(t)$. Hence, it is the primary actuator that must satisfy the condition given above. This is formally expressed in the next Lemma.

Lemma 3.2: Consider the dual-stage system described by equations (1), let r_0 be the reference level to be tracked during pre-actuation, let $y_1(t)$, $\dot{y}_1(t)$ and $\ddot{y}_1(t)$ satisfy

$$|y_1(t) - r_0 + \epsilon_2 \dot{y}_1(t) - \epsilon_1 \ddot{y}_1(t)| \leq \beta \bar{u}_2, \tag{10}$$

and let $u_2(t)$ be given by

$$u_2(t) = (y_1 - r_0 + \epsilon_2 \dot{y}_1 - \epsilon_1 \ddot{y}_1)/\beta. \tag{11}$$

Then,

$$\lim_{t\to\infty} |y_1(t) + y_2(t) - r_0| = \lim_{t\to\infty} |y(t) - r_0| = 0. \tag{12}$$

In particular, if $y(0) = r_0$ and $\dot{y}(0) = 0$, then $y(t) = r_0, \; \forall t \geq 0$. ∎

Proof: Since $y = y_1 + y_2$ and $\dot{y} = \dot{y}_1 + \dot{y}_2$, it follows that, for the DSA in (1) with the secondary actuator under control law (11),

$$\begin{aligned} \ddot{y} &= \ddot{y}_1 + (y_2 + \epsilon_2 \dot{y}_2 + \beta u_2)/\epsilon_1, \\ &= a_1(y_1 + y_2 - r_0) + a_2(\dot{y}_1 + \dot{y}_2), \\ &= a_1(y - r_0) + a_2 \dot{y}. \end{aligned} \tag{13}$$

Which satisfies (12). Furthermore, from the assumption on the initial conditions, it is clear that $y(t) = r_0, \;\; \forall t \geq 0$. Q.E.D.

Remark 3.1: Note that as $\gamma(\cdot)$ is a small term which compensates deviations from the desired trajectory, it can be neglected in calculating $\kappa(\cdot)$. As a result, the condition in (8) reduces to $|z + \epsilon_2 \dot{z} - \epsilon_1 \ddot{z}| \leq \beta \bar{u}_2$, which further implies (10). Moreover, in the subsequent analysis, we can leave the $\gamma(\cdot)$ term out of the calculation for convenience.

If the conditions given in Lemma 3.2 are satisfied, the system output will be maintained at the reference r_0 during pre-actuation ($t \leq \tau$). It is now possible to design Preview Control trajectories that are trackable by the secondary actuator.

IV. Quadratic Programming Solution

We may now turn our attention into finding a solution for the trajectories of $y_1(t)$, $0 < t \leq \tau$, that satisfies the conditions given in Lemma 3.2. During Preview Control it is required that the primary actuator: 1) respects constraint (10) so that the total output y stays at the reference r_0; and 2) improves the performance of the system by being closer to and moving towards r_1 at the switching time $t = \tau$. Neither of these tasks is easily solved. In practice, different trajectories may be designed via a trial-and-error approach where a compromise between tasks 1) and 2) is achieved. With the results given in the last section however, something better can be achieved by framing these problems in a Quadratic Programming (QP) form. In order to do so, it is first necessary to discretize the system. Henceforth, the following notation will be used to describe the primary actuator in discrete form:

$$\xi(k+1) \quad = \quad A_d \xi(k) + B_d u_1(k), \tag{14}$$

where $\xi(k) \in \mathbb{R}^2$ are the discrete equivalent of the primary actuator states and $u_1(k) \in \mathbb{R}$ is its input. The matrices A_d and B_d are obtained from matrices A_1 and B_1 in (1) via some discretization method with sampling time T and condition (10) takes the following discrete form:

$$|a_1 \xi_1(k) + a_2 \xi_2(k) - b_1 u_1(k)| \leq b_2 \bar{u}_2, \tag{15}$$

where it was assumed without loss of generality that $r_0 = 0$. The solution sought should come in the form of discrete inputs to be applied to the primary actuator,

$$\mathbf{u_1} = [\hat{u}_1(1)\ \hat{u}_1(2)\ \ldots \hat{u}_1(N)]^\intercal, \tag{16}$$

In order to obtain $\mathbf{u_1}$, consider the following cost function,

$$V \quad := \quad \frac{1}{2}(\xi(N) - x_s)^\intercal P(\xi(N) - x_s) + \sum_{k=0}^{N-1} \Lambda_k \tag{17}$$

where,

$$\Lambda_k := \frac{1}{2}\left(\xi(k)^\intercal Q \xi(k) + u_1(k)^\intercal R u_1(k)\right) \tag{18}$$

P, $Q \geq 0$ and $R > 0$ are free weight matrices, N is the so-called prediction horizon (must be such that $TN = \tau$) and x_s is the desired steady state response at sample N. The discrete variables $\xi(k)$ and $u_1(k)$ are the prediction of the states of the primary actuator during pre-actuation and the variables of interest, respectively. With this cost function and an appropriate choice of the matrices P, Q, R and x_s, it is possible to design a trajectory that minimizes the distance $|r_1 - y_1|$ and maximizes the speed in which y_1 approaches r_1 at the final instant of the trajectory ($t = \tau$).

Recall that the problem of minimizing the cost function (17) is equivalent to finding the vector $\mathbf{u_1}(x) \in \mathbb{R}^N$ given by,

$$\mathbf{u_1} = \arg \min_{L\mathbf{u_1} \leq Z} \ \frac{1}{2} \mathbf{u_1}^\intercal \mathcal{H} \mathbf{u_1} + \mathbf{u_1}^\intercal F(\xi - x_s). \tag{19}$$

where, $\mathcal{H} = \Gamma^\intercal \mathbf{Q} \Gamma + \mathbf{R}$ is the so-called $\mathcal{H}$*essian* of the quadratic program and L and Z describe the constrains that must be respected. The matrices are defined as follows,

$$F = \Gamma^\intercal \mathbf{Q} \Omega,$$

$$\Gamma := \begin{bmatrix} B_d & 0 & \ldots & 0 & 0 \\ A_d B_d & B_d & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_d^{N-1} B_d & A_d^{N-2} B_d & \ldots & \ddots & B_d \end{bmatrix}, \tag{20}$$

with,

$$\begin{array}{rcl} \mathbf{Q} & := & \text{diag}\{C^\intercal Q C, \ldots, C^\intercal Q C, P\}, \\ \mathbf{R} & := & \text{diag}\{R, \ldots, R\}. \\ \Omega & := & \begin{bmatrix} A_d & A_d^2 & \ldots & A_d^N \end{bmatrix}^\intercal. \end{array} \tag{21}$$

Via matrices L and Z it is possible to add the necessary constraints on the input $|u_1| \leq \bar{u}_1$ and constraint (10). This is formally stated in the following Theorem.

Theorem 4.1: Consider system (1) with the secondary actuator control law u_2 as in (11). Let the primary actuator follow a trajectory given by the solution of (19) through (21) during pre-actuation, and define the matrices L and Z as follows,

$$L = \begin{bmatrix} I_N \\ \Psi \\ -I_N \\ -\Psi \end{bmatrix}, \quad Z = \begin{bmatrix} \mathbf{u}_{\max}^1 \\ b_2 \mathbf{u}_{\max}^2 \\ \mathbf{u}_{\max}^1 \\ b_2 \mathbf{u}_{\max}^2 \end{bmatrix} \tag{22}$$

where I_N is an $N \times N$ identity matrix,

$$\Psi = \begin{bmatrix} \psi & 0 & 0 & \ldots & 0 \\ 0 & \psi & 0 & \ldots & 0 \\ 0 & 0 & \psi & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \psi \end{bmatrix} \Gamma - b_1 \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}, \tag{23}$$

and,

$$\begin{array}{rcl} \psi & := & [a_1\ a_2], \\ \mathbf{u}_{\max}^i & := & \begin{bmatrix} \bar{u}_i & \ldots & \bar{u}_i \end{bmatrix}^\intercal. \end{array} \tag{24}$$

Then, given $y(0) = r_0$ and $\dot{y}(0) = 0$,

$$y(t) = r_0, \ 0 \leq t \leq \tau.$$

That is, the tracking of r_0 is achieved during pre-actuation. ∎

Proof: From Lemma 3.2 it follows that if inequality (10) is satisfied, then $y(t) = r_0$ during pre-actuation. In order to satisfy (10) note that the multiplication of the first row of Ψ by $\mathbf{u_1}$ will result in,

$$\begin{aligned} \psi B_d \hat{u}_1(0) - b_1\hat{u}_1(1) \leq & \quad b_2\bar{u}_2 \\ a_1\xi_1(1) + a_2\xi_2(1) + \psi\xi(0) - b_1\hat{u}_1(1) \leq & \quad b_2\bar{u}_2. \end{aligned} \tag{25}$$

Since $\xi(0) = 0$, it follows that

$$a_1\xi_1(1) + a_2\xi_2(1) - b_1\hat{u}_1(1) \leq b_2\bar{u}_2,$$

wich satisfies condition (15) for $k = 0$. By induction it is straightforward to assert that all points in the trajectory are thus satisfied. Furthermore, the constraints on the control effort are obviously satisfied from $I_N\mathbf{u_1} \leq \mathbf{u}^1_{\max}$, resulting in a feasible trajectory both for the primary and secondary actuator. Q.E.D.

Remark 4.1: From the solution (16) a reference trajectory $\hat{x}_1 = [\hat{y}_1 \;\; \dot{\hat{y}}_1]^{\intercal}$ may be computed and tracked via some classical controller such as a simple PD:

$$u_1(k) = \hat{u}_1(k) + W(x_1(k) - \hat{x}_1(k)).$$

The combined use of (16) with a PD controller adds some robustness to the system since small deviations of y_1 from $\hat{z}$ may be compensated.

V. Implementation

The techniques so far presented will be implemented on an experimental set up of a Dual-Stage Actuator providing results that validate what was proposed.

The system in hand is comprised by a linear motor (LM) as the first stage and a piezoelectric actuator (piezo) as the second one. While the LM has a travel range of 0.5 m and a 1 μm resolution glass scale encoder, the piezo actuator's range is limited in ± 15 μm and has an integrated capacitive position sensor with 0.2 nm resolution. The capacitive sensor is used to measure the incremental displacement provided by the piezo to the LM. The DSA in hand is fully described by equation (1), in this particular case the system constants are given by:

$$\begin{aligned} a_1 &= -10^6, & b_1 &= 1.7\times 10^7 & \bar{u}_1 &= 1 \\ a_2 &= -1810, & b_2 &= 3\times 10^6 & \bar{u}_2 &= 5 \end{aligned} \tag{26}$$

A. The Secondary Actuator Control Law

The secondary actuator controller is given in (6) and will be maintained both during and after pre-actuation. The difference being that an extra term ($\gamma(\cdot)$) will be added for improved performance so the error may converge to zero faster. The applied control law becomes,

$$\begin{aligned} u_2 &= \tfrac{1}{b_2}((y_1 - r_0)a_1 + \dot{y}_1 a_2 - \ddot{y}_1 + \gamma(\cdot)), \\ \gamma(x_1, x_2, r) &= H\begin{bmatrix} y_2 - (y_1 - r) \\ \dot{y}_2 - \dot{y}_1 \end{bmatrix}, \end{aligned} \tag{27}$$

with $H = [h_1 \;\; h_2]$ a linear gain that may be designed via classical methods of control systems' theory. In this case,

$$H = [83.85 \;\; 0.036] \times 10^{-2}. \tag{28}$$

B. The Primary Actuator Control Law

The primary actuator will be subject to two different controllers, one during the Preview Control interval, and another one after it.

For $0 < t \leq \tau$ the controller presented in Section III and calculated in Section IV is used. This is now a "responsible" form of Preview Control that achieves a faster performance without compromising the trackability of r_0. For its design it was used $T = 0.1$ ms, $N = 200$, $R = 1.5\times 10^4$, $Q = 10^{-3}$ and

$$P = \begin{bmatrix} 10^3 & 0 \\ 0 & 10^{-1} \end{bmatrix}.$$

When $t > \tau$ the preview control is over and the system output y must now track r_1 instead r_0. While the secondary actuator controller may remain unchanged, the primary control law must switch to some form of fast tracking control. There are several forms of advanced controllers for fast tracking servomechanisms, in this study the traditional Proximate Time-Optimal Servomechanism (PTOS) will be used. This strategy is an improved approximation of the Time-Optimal Control (TOC) in the sense that it does not suffer from chattering and, hence, is a practical controller. The PTOS control law is given by:

$$u_1(t) = k_2(-f_{ptos}(e_1) - \dot{y}_1),$$

with,

$$f_{ptos}(e) = \begin{cases} (k_1/k_2)e_1, & \text{for } |e_1| \leq y_l, \\ \text{sgn}(e)(\sqrt{2b_1\alpha\bar{u}_1|e_1|} - \bar{u}_1/k_2), & \text{for } |e_1| > y_l. \end{cases}$$

A stability condition requires that $0 < \alpha < 1$, and the following constraints guarantee the continuity of the controller,

$$y_l = \frac{\bar{u}_1}{k_1}, \quad k_2 = \sqrt{\frac{2k_1}{b_1\alpha}}.$$

This controller has several interesting properties whose detailed descriptions are outside the scope of this paper. The interested reader is referred to [21]. Here, its ability to overshoot a controlled amount is explored. As proposed in [16] one may allow this controller to overshoot without surpassing the limitation of the secondary actuator. Since this overshoot is compensated by the second stage, the total output rise time is faster. This will be clear during the discussion on the obtained results. The controller is tuned as follows,

$$\alpha = 0.7, \quad k_1 = 2.09, \quad k_2 = 0.019. \tag{29}$$

Notice, however, that only α and k_1 are free to be tuned as the designer wishes.

C. Experimental Results

The traditional form of DSA control is shown in Fig. 2. The output transition instant is given at $t = \tau = 0.02$ s. This figure shows the advantage of allowing the primary actuator to overshoot by providing a faster rise time. As soon as this actuator is *around* r_1 the secondary one reaches the reference

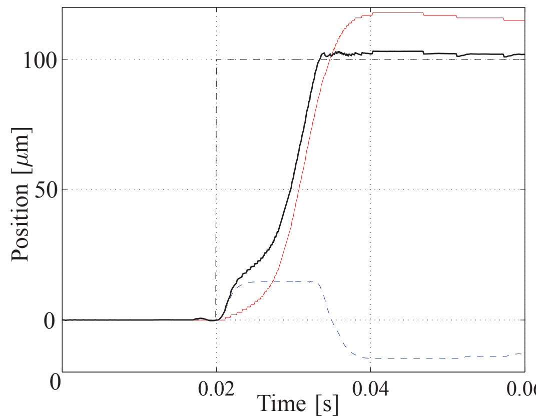

Fig. 2. Traditional form of DSA high performance control. The primary actuator (thin line) overshoots a desired amount, the secondary actuator (dashed line) compensates for the overshoot and the settling time of the total input (thick line) is reduced [16].

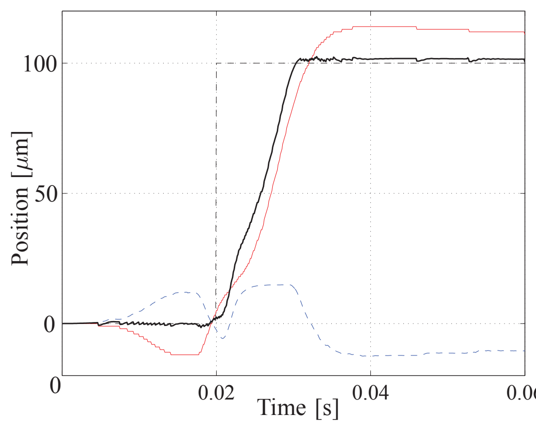

Fig. 3. Proposed controller with the Preview Control strategy. An improvement of 30 % is achieved due to Preview Control. Curves are the same as in Fig 1. All the plots come from experimental results.

and $y = r_1$ within a small tolerance (in this case ± 2 μm). The resulting settling time is 13 ms.

The next plot, shown in Fig. 3, shows the Preview Control strategy working with the preview trajectory designed according to Theorem 3.2. Note that, unexpectedly, the primary actuator moves away from the future reference r_1 in the first moments of pre-actuation. This is so that it may have more time and room inside $\mathcal{S}$ to accelerate towards r_1 when the switching instant $t = \tau$ comes. The resulting improvement is of about 30 % since the settling time with Preview Control is 10 ms.

VI. Conclusion

This paper has advanced the study on the Preview Control strategy for Dual-Stage Systems (DSA). It was of particular interest the limitations that the secondary actuator imposes to the trajectory during pre-actuation. In order to identify these limitations a set $\mathcal{S}$ was defined where the primary actuator should stay so that the error generated by it might be compensated by the secondary. In addition, an optimal form of Preview Control trajectory design was proposed providing guarantees that the primary actuator will lie inside $\mathcal{S}$ during the required time interval. Experimental results have both validated the proposed approach and promoted the benefits of Preview Control.

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