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Consensus Control for High Order Continuous-time Agents with Communication Delays

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Abstract— **This paper is concerned with the consensus control problem for a network of high order linear continuous-time agents subject to communication delays between neighbouring nodes in the network. Departing from the common practice which firstly designs the consensus control under the delay-free assumption and then computes the maximally tolerated communication delay, we propose a new simple technique which allows us to achieve consensus for any communication delay for marginally stable agents. Our approach also achieves consensus for unstable agents, provided the time delay is within a certain range. Our consensus algorithm is designed under the usual assumptions of undirected connectivity for the network. The proposed consensus control law requires communication between neighboring agents only at certain sampling points, rather than at all times. The control law is also unique in the sense that it is nonlinear in the continuous time domain but linear when the agents are viewed in the sampled-data domain. The proposed technique is expected to pave a new way for new theoretical studies on network properties required for consensus control.**

I. INTRODUCTION

Consensus control is one of the fundamental problems in understanding of many biological and social group behaviours [1] and in distributed control of multi-agent systems. Early work on control of multi-agent systems focused on the so-called consensus problem where a networked set of agents want to merge to a common state [2]–[5]. Consensus control research then quickly finds relevance in many other related problems including synchronization, formation, swarming, flocking, and rendezvous [6]–[8]. Consensus algorithms also find wide applications in many disciplines, including smart grid [9], [10], sensor networks [11], [12] and distributed parameter estimation [13].

The seminal work [3] solved the average consensus problem by studying first-order integrator networks with and without time delay. In the work of [4], consensus protocols were designed for the first-order integrator multi-agent systems (MASs) in both the continuous-time setting and discrete-time setting. The work of Ma and Zhang [14] considered the consensusability of linear MASs without delay and showed that the consensusability of MASs depends on the dynamic structure of each agent and the communication network topology among agents. Reference [15] studied the consensus conditions of first-order integrator systems under both directed and undirected communication network topologies. Consensus using quantized information has also been considered in [16]. Other works on consensus for first-order integrator networks or networks without delay can be found in [11], [17].

Communication delays arise naturally in information exchange between neighbors [18]. This can be due to a combination of transmission delays, measurement delays and computation delays. Control problems for time-delay systems have attracted a lot of attention in the past decades; see, e.g., [19]–[22]. For example, [21] utilizes receding horizon control to stabilize input-delay systems and a sufficient condition for the asymptotical stability of the closed-loop system is presented in terms of a linear matrix inequality. The work of [19] is most pertinent to our proposed approach. This paper makes use of the reduction approach [23], which can transform a delay system into a delay-free system, to investigate the stabilization problem for linear systems with both state delay and input delay.

Several recent research results have considered the problem of consensus control design in the presence of communication delays. The work of [24] studies consensus among identical agents that are at most critically unstable and coupled through networks with uniform constant communication delay and an upper bound for delay tolerance is obtained which explicitly depends on agent dynamics and network topology. In [25] a similar consensus problem is studied with the goal to find, among all standard static protocols that achieve the consensus for the multi-agent system under no input delay, the maximum input delay such that the system remains consensusable under the same protocol. The work of [26] considers the consensus problem for a network of high-order unstable agents using delayed relative state measurements and the maximum permissible time delay is computed via parametric algebraic Riccati equations. In [27], a linear matrix inequality solution is given to the consensus problem by relating the maximum permissible time delay to the eigenvalues of the Laplacian matrix of the communication network. Also given in [27] is a frequency domain solution for first-order integrator agents. In all the above works, the common thread is to start with the linear consensus control for the delay-free case and then seek for the maximum time delay without losing consensusability. By doing so, the maximum permissible time delay is limited. An alternative view of the shortcoming of these approaches is that the time delay information is not fully utilized in the design of the control protocol.

In this paper, we consider the consensus control problem

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for high order continuous-time agents with communication delays. We first study discrete-time agents with one-step communication delay between agents and show that consensus can be achieved by using local linear feedback of delayed state and control information provided that the time delay is within a certain range. We then apply this result to continuous-time agents by considering the sampled-data models with sampling period equal to the communication delay. Finally, we show that the consensus control law for the sampled-data agents can indeed guarantee the consensus for the original continuous-time agents. The proposed consensus control law is unique in the sense that it is nonlinear in the continuous time domain but linear when the agents are viewed in the sampled-data domain. We note that our underlying idea of using both the delayed state information and delayed control information in the neighborhood to design local controllers is similar to that in [28], yet our controller is in a sample-and-hold form, requiring communications between neighbours only at sampling points, rather than at all times.

The rest of the paper is organized as follows. The problem under consideration is described in Section II. Consensus control problem for discrete-time agents with one-step delay is studied in Section III. Main results are presented in Section IV. Simulation examples are given in Section V. Conclusions are provided in Section VI.

II. Problem Formulation

Let the directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ denotes the communication topology between multi-agents with the set of vertices $\mathcal{V} = \{1, 2, \ldots, N\}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The ith vertex represents the ith agent and the edge (i, j) denotes that the agent j obtains information from the agent i. $\mathcal{E} \subset \{(i, j) : i, j \in \mathcal{V}\}$ is the edge set. The set of neighbors of the ith agent is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ is called the weighted adjacency matrix of $\mathcal{G}$ with nonnegative elements and $a_{ij} > 0$ if and only if $i \in \mathcal{N}_i$. The in-degree of the ith vertex is denoted by $d_i = \sum_{j\in\mathcal{N}_i} a_{ij} = \sum_{j=1}^{N} a_{ij}$ and the in-degree matrix $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$. The Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Note that $a_{ij} = a_{ji}$, $\forall i, j \in \mathcal{V}$ if and only if $\mathcal{G}$ is an undirected graph [29]. Obviously, for an undirected graph, $\mathcal{L}$ is a symmetric, positive semi-definite matrix and all its eigenvalues λ_i are non-negative. For a connected graph, we have $0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_N$. Note that $\mathcal{L}\mathbf{1}_N = \mathbf{0}_N$.

In this paper, we will consider the consensus control for a network of continuous-time high-order linear time-invariant agents with the following dynamics

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i(t) \in R^n$ and $u_i(t) \in R^m$ are the state and control input of the ith agent, respectively. $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are constant matrices and the initial state is denoted by $x_i(0)$.

Definition 1 (consensus) The agents in the network achieve consensus if

$$\lim_{t\to\infty} \|x_j(t) - x_i(t)\| = 0, \quad \forall i, j \in \{1, \ldots, N\}, \tag{2}$$

for any initial value $x_i(0)$.

Assumption 1: The network topology $\mathcal{G}$ is an undirected connected graph.

Assumption 2: All the eigenvalues of A lie in the closed right half plane and B has full column rank.

Assumption 3: (A, B) is controllable.

Assumption 4: Communication between neighbouring nodes is subject to delay of $\tau > 0$.

Remark 1: This paper only considers undirected graphs due to pages limitation. Our approach is also valid for directed graphs.

Remark 2: If some eigenvalues of A lie in the open left-half plane, it is a standard practice to decompose the system (1) into two sub-systems, one asymptotically stable which requires no consensus control action, and one with eigenvalues in the closed right half plane, which is considered under Assumption 2. Thus, Assumption 2 does not lose generality. Under this assumption, the controllability of (A, B) is equivalent to the stabilizability of (A, B). It is well known that even in the delay-free case, the stabilizable condition is necessary for the consensusability of continuous-time multi-agent systems [14]. Hence, Assumption 3 is reasonable.

Remark 3: In the presence of the communication delay, a commonly used consensus protocol for the ith agent is given by

$$u_i(t) = K \sum_{j=1}^{N} a_{ij}(x_j(t-\tau) - x_i(t-\tau)) \tag{3}$$

with some constant gain matrix $K \in R^{m\times n}$; see, e.g., [24]–[27]. A major disadvantage of this protocol is that a lot of useful information about the neighboring agents is not utilized. This is due to the well-known fact that, for time-delay systems, the delayed state $x(t-\tau)$ does not contain the full information about the system at time t.

Our consensus control problem is to design, for each agent i, the controller $u_i(t)$ using local information from agent i and its neighboring agents such that the closed-loop network of agents will achieve consensus. More specifically, $u_i(t)$ has access to the information of $\{x_j(s) - x_i(s), u_j(s), u_i(s)\}$ where $s \leq t - \tau$ and $j \in \mathcal{N}_i$, and our goal is to achieve (2).

III. Consensus for Discrete-time Agents with One-step Delay

In this section, we consider the discrete-time multi-agent system

$$\tilde{x}_i(k+1) = \tilde{A}\tilde{x}_i(k) + \tilde{B}\tilde{u}_i(k), i = 1, \ldots, N, \tag{4}$$

where $\tilde{x}_i(k) \in R^n$ and $\tilde{u}_i(k) \in R^m$ are the state and control input of the ith agent, respectively. Suppose the communication between neighbors is with a delay $d = 1$.

Assumption 5: All the eigenvalues of $\tilde{A}$ lie on or outside the unit circle.

Lemma 1: Suppose $(\tilde{A}, \tilde{B})$ is controllable, $\tilde{B}$ has full column-rank and

$$0 < \frac{\lambda_N - \lambda_2}{\lambda_N + \lambda_2} < \frac{1}{\prod_i |\lambda_i^u(\tilde{A})|}, \tag{5}$$

where λ_2 and λ_N are respectively the smallest and the largest positive eigenvalues of the Laplacian matrix $\mathcal{L}$ and $\lambda_i^u(\tilde{A})$ is an unstable eigenvalue of $\tilde{A}$.

Select any δ such that

$$\frac{\lambda_N - \lambda_2}{\lambda_N + \lambda_2} \le \delta < \frac{1}{\prod_i |\lambda_i^u(\tilde{A})|}. \tag{6}$$

Let

$$K = \omega(\tilde{B}'Q\tilde{B})^{-1}\tilde{B}'Q\tilde{A}^2, \tag{7}$$

where $\omega = 2(\lambda_2+\lambda_N)^{-1}$ and Q is a positive definite solution to the modified Riccati inequality

$$Q - \tilde{A}'Q\tilde{A} + (1-\delta^2)\tilde{A}'Q\tilde{B}(\tilde{B}'Q\tilde{B})^{-1}\tilde{B}'Q\tilde{A} > 0. \tag{8}$$

Then the controller

$$\begin{aligned}\tilde{u}_i(k) = K\sum_{j=1}^{N} a_{ij}[&(\tilde{x}_j(k-1) - \tilde{x}_i(k-1)) \\ &+\tilde{A}^{-1}\tilde{B}(\tilde{u}_j(k-1) - \tilde{u}_i(k-1))]\end{aligned} \tag{9}$$

renders consensus for system (4) with delay $d = 1$.

Proof: Note that the controller (9) uses a one-step delay information, which renders the closed-loop system (4) a time-delay system with one-step delay in the control input. To deal with this time delay, we deploy a well-known reduction technique from [20] to transform the closed-loop system to a delay-free system. To this end, we define

$$y_i(k) = \tilde{x}_i(k) + \tilde{A}^{-1}\tilde{B}\tilde{u}_i(k),\ i = 1, \dots, N. \tag{10}$$

Then, the transformed system dynamics becomes

$$\begin{aligned} y_i(k+1) &= \tilde{x}_i(k+1) + \tilde{A}^{-1}\tilde{B}\tilde{u}_i(k+1) \\ &= \tilde{A}\tilde{x}_i(k) + \tilde{B}\tilde{u}_i(k) + \tilde{A}^{-1}\tilde{B}\tilde{u}_i(k+1) \\ &= \tilde{A}y_i(k) + \tilde{A}^{-1}\tilde{B}\tilde{u}_i(k+1). \end{aligned} \tag{11}$$

In addition, (9) can be rewritten as

$$\tilde{u}_i(k+1) = K\sum_{j=1}^{N} a_{ij}[y_j(k) - y_i(k)]. \tag{12}$$

That is, the transformed closed-loop system (11)-(12) is free of delay. According to the results on delay-free consensus problem for discrete-time systems [29], [30], if $(\tilde{A}, \tilde{A}^{-1}\tilde{B})$ is controllable, $\tilde{A}^{-1}\tilde{B}$ has full column-rank and (5) holds

$$0 < \frac{\lambda_N - \lambda_2}{\lambda_N + \lambda_2} < \frac{1}{\prod_i |\lambda_i^u(\tilde{A})|}, \tag{13}$$

where $\lambda_i^u(\tilde{A})$ is an unstable eigenvalue of $\tilde{A}$, then system (11) achieves consensus under the control (12) where K can be designed in the following procedure. Select a δ such that

$$0 < \frac{\lambda_N - \lambda_2}{\lambda_N + \lambda_2} \le \delta < \frac{1}{\prod_i |\lambda_i^u(\tilde{A})|}. \tag{14}$$

Let P be a positive definite solution to the modified Riccati inequality

$$\begin{aligned} 0<&P - \tilde{A}'P\tilde{A} \\ &+(1-\delta^2)\tilde{A}'P\tilde{A}^{-1}\tilde{B}(\tilde{B}'\tilde{A}'^{-1}P\tilde{A}^{-1}\tilde{B})^{-1}\tilde{B}'\tilde{A}'^{-1}P\tilde{A} \end{aligned} \tag{15}$$

and

$$K = \omega(\tilde{B}'\tilde{A}'^{-1}P\tilde{A}^{-1}\tilde{B})^{-1}\tilde{B}'\tilde{A}'^{-1}P\tilde{A},\ \omega = \frac{2}{\lambda_2 + \lambda_N}. \tag{16}$$

Since $(\tilde{A}, \tilde{B})$ is controllable and $\tilde{B}$ has full column-rank, $(\tilde{A}, \tilde{A}^{-1}\tilde{B})$ is controllable and $\tilde{A}^{-1}\tilde{B}$ has full column-rank. (5) and (6) are respectively (13) and (14). In addition, the inequality (15) is equivalent to (8) via $P = \tilde{A}'Q\tilde{A}$. Accordingly, (16) becomes (7). Hence, system (11) achieves consensus under (12), which implies

$$\lim_{k\to\infty}[y_i(k) - y_j(k)] = 0, ,\quad \lim_{k\to\infty}\tilde{u}_i(k) = 0,$$

and further

$$\begin{aligned} &\lim_{k\to\infty}[\tilde{x}_i(k) - \tilde{x}_j(k)] \\ =\ &\lim_{k\to\infty}[y_i(k) - \tilde{A}^{-1}\tilde{B}\tilde{u}_i(k) - y_j(k) + \tilde{A}^{-1}\tilde{B}\tilde{u}_j(k)] \\ =\ &0. \end{aligned}$$

Hence, system (4) reaches consensus under control (9). ■

Lemma 1 presents a solution to the consensus control problem for discrete-time multi-agent systems (4) subject to one-step communication delay. In the next section, it will be used to solve the continuous-time consensus problem formulated in Section II by discretizing the continuous-time multi-agent system (1).

IV. Consensus for Continuous-time Agents with Arbitrary Delay

Consider the continuous-time multi-agent system (1). We take an extra assumption about the communication delay.

Assumption 6: Any two distinct eigenvalues of A, denoted by μ_1 and μ_2, satisfy

$$\mathrm{Im}(\mu_1\tau - \mu_2\tau) \ne 2q\pi, \forall q = \pm1, \pm2, \dots \tag{17}$$

whenever $\mathrm{Re}(\mu_1) = \mathrm{Re}(\mu_2)$ ($\mathrm{Im}(\cdot)$ and $\mathrm{Re}(\cdot)$ stand for the imaginary part and the real part of a complex number, respectively).

In the sequel, we let

$$\tilde{A} = e^{A\tau}, \tilde{B} = \int_0^\tau e^{A(\tau-s)}dsB.$$

Theorem 1: Suppose (A, B) is controllable, B has full column-rank and

$$0 < \frac{\lambda_N - \lambda_2}{\lambda_N + \lambda_2} < \frac{1}{\prod_i |\lambda_i^u(\tilde{A})|}. \tag{18}$$

Select a δ to satisfy (6) and let K be given by (7) and (8). Then system (1) achieves consensus under the following control protocol

$$u_i(t) = K\sum_{j=1}^{N} a_{ij}[(x_j(k\tau-\tau)-x_i(k\tau-\tau)) + \tilde{A}^{-1}\tilde{B}(u_j(k\tau-\tau)-u_i(k\tau-\tau))],\quad \forall t\in[k\tau,(k+1)\tau). \tag{19}$$

Proof: The proof takes 3 steps.

Step 1: Let the sample period for the continuous-time system (1) be $T=\tau$ and the sample instants be $k\tau, k=0,1,\ldots$. Note that $u_i(t)$ given by (19) is constant during the sample period $t\in[k\tau,(k+1)\tau)$, i.e.

$$u_i(t)\equiv u_i(k\tau), t\in[k\tau,(k+1)\tau). \tag{20}$$

Then

$$\begin{aligned} & x_i((k+1)\tau) \\ = \ & e^{A\tau}x_i(k\tau)+\int_{k\tau}^{(k+1)\tau} e^{A((k+1)\tau-\sigma)}Bu_i(\sigma)d\sigma \\ = \ & e^{A\tau}x_i(k\tau)+\int_{k\tau}^{(k+1)\tau} e^{A((k+1)\tau-\sigma)}Bd\sigma u_i(k\tau) \\ = \ & \tilde{A}x_i(k\tau)+\tilde{B}u_i(k\tau). \end{aligned}$$

Denoting

$$\tilde{x}_i(k)=x_i(k\tau), \tilde{u}_i(k)=u_i(k\tau),$$

then

$$\tilde{x}_i(k+1)=\tilde{A}\tilde{x}_i(k)+\tilde{B}\tilde{u}_i(k). \tag{21}$$

Step 2: According to [31], the condition (17) and the controllability of (A,B) can ensure that $(\hat{A},\hat{B})$ is controllable. Under Assumption 6, the matrix $\int_0^\tau e^{A(\tau-s)}ds$ is nonsingular, thus $\hat{B}$ has full column-rank since B has full column-rank. Also note that the control (19) reduces to the control (9) at sample points. Therefore, according to Lemma 1, we have

$$\begin{aligned} &\lim_{k\to\infty}[\tilde{x}_i(k)-\tilde{x}_j(k)]=0, \\ &\lim_{k\to\infty}\tilde{u}_i(k)=0,\ \forall i,j\in\{1,\ldots,N\}, \end{aligned}$$

which implies

$$\lim_{t\to\infty}[x_i(k\tau)-x_j(k\tau)]=0, \lim_{t\to\infty}u_i(k\tau)=0. \tag{22}$$

Step 3: For any $t\geq 0$, let $f(t)$ represent the unique integer such that $t\in[f(t)\tau, f(t)\tau+\tau)$. Note

$$x_i(t)=e^{A(t-f(t)\tau)}x_i(f(t)\tau)+\int_{f(t)\tau}^{t} e^{A(t-s)}Bu_i(s)ds.$$

For $s\in[f(t)\tau,t]$, there holds $u_i(s)=u_i(f(t)\tau)$. Hence,

$$\begin{aligned} & x_i(t) \\ = \ & e^{A(t-f(t)\tau)}x_i(f(t)\tau)+\int_{f(t)\tau}^{t} e^{A(t-s)}dsBu_i(f(t)\tau) \\ = \ & e^{A(t-f(t)\tau)}x_i(f(t)\tau) \\ & +\int_0^{t-f(t)\tau} e^{A(t-f(t)\tau-\sigma)}d\sigma Bu_i(f(t)\tau) \\ = \ & e^{A(t-f(t)\tau)}x_i(f(t)\tau) \\ & +e^{A(t-f(t)\tau)}\int_0^{t-f(t)\tau} e^{-A\sigma}d\sigma Bu_i(f(t)\tau). \end{aligned}$$

Denote

$$g(t)=t-f(t)\tau.$$

Then we have

$$\begin{aligned} x_i(t) \ = \ & e^{Ag(t)}x_i(f(t)\tau)+e^{Ag(t)} \\ & \times\int_0^{g(t)} e^{-A\sigma}d\sigma Bu_i(f(t)\tau), \end{aligned} \tag{23}$$

and

$$\begin{aligned} & x_i(t)-x_j(t) \\ = \ & e^{Ag(t)}[x_i(f(t)\tau)-x_j(f(t)\tau)] \\ & +e^{Ag(t)}\int_0^{g(t)} e^{-A\sigma}d\sigma B[u_i(f(t)\tau)-u_j(f(t)\tau)]. \end{aligned}$$

Since $g(t)\in[0,\tau)$, $e^{Ag(t)}$ and $\int_0^{g(t)} e^{-A\sigma}d\sigma$ are bounded. Together with (22), it can be derived that

$$\lim_{t\to\infty}[x_i(t)-x_j(t)]=0.$$

Hence, system (1) achieves consensus under (19). This ends the proof. ■

Remark 4: The above discussion indicates that the consensus result for discrete-time multi-agent system with one-step communication delay has been used to solve the continuous-time consensus control problem successfully. This is done by establishing the relation between the states at non-sampling points and those at sampling points, i.e., (23).

Remark 5: Comments on the condition (18) are in order. Note that $\lambda_i^u(\tilde{A})=(\exp(\lambda_i^u(A)))^\tau$. For marginally stable agents, A has purely imaginary eigenvalues, which means that the right hand side of (18) equals to 1. In this case, (18) always holds. That is, consensus can always be achieved for any time delay. If A has strictly unstable eigenvalues, then the right hand side will shrink exponentially with respect to τ. In this case, (18) gives a good characterization about the tolerable time delay. More specifically, the tolerable time delay is closely related to the connectivity (λ_2) and synchronizability (λ_N) of the network as well as the growth rate of the agents' states ($\lambda_i^u(A)$).

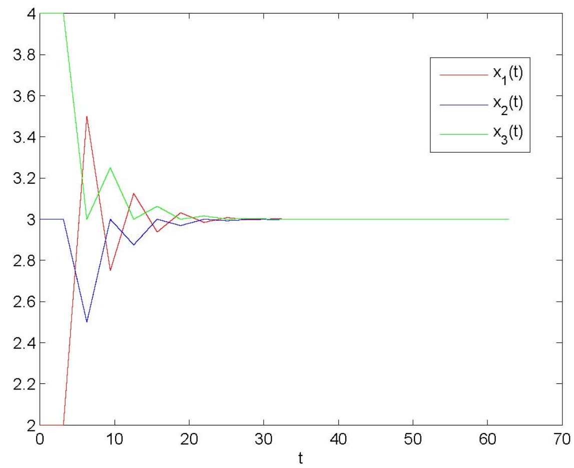

Fig. 1. Asymptotical behavior of the states

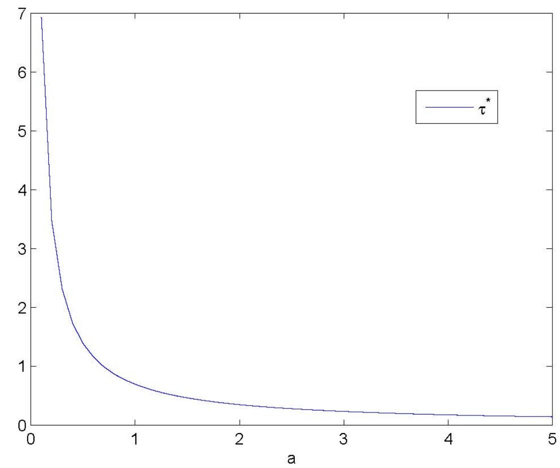

Fig. 2. Consensuable delay bound

V. Simulation Examples

A. Integrator System

Consider the following integrator multi-agent system

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \ldots, N,$$

where $x_i(t) \in R$, $u_i(t) \in R$ and the initial value is

$$x_1(0) = 2, x_2(0) = 3, x_3(0) = 4.$$

The communication topology is given by

$$2 \text{ ——— } 1 \text{ ——— } 3 \ .$$

The associated Laplacian matrix is

$$\mathcal{L} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix},$$

and its eigenvalues are

$$\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 3.$$

The method given in [32] restricts $\tau \in (0, \frac{\pi}{6})$ to guarantee the consensusability of the above integrator system. But using our method, the system can reach consensus for any large τ. Take $\tau = \pi$ for example. According to Theorem 1, the consensus control can be chosen to be

$$\begin{aligned} u_i(t) = \frac{1}{2\tau} \sum_{j=1}^{N} a_{ij} [(x_j(k\tau - \tau) - x_i(k\tau - \tau)) \\ + \tau(u_j(k\tau - \tau) - u_i(k\tau - \tau))], \\ \forall t \in [k\tau, (k+1)\tau). \end{aligned}$$

In this case, the asymptotical behavior of the state for each node is given in Fig. 1. It can be observed that the state of each node converges to 3, which is the average of the initial value.

B. Non-integrator System

Consider the non-integrator system

$$\dot{x}_i(t) = a x_i(t) + u_i(t), \quad i = 1, \ldots, N,$$

where $a > 0$. Using the condition (18), we can derive a delay bound $\tau^\star$ for the consensusability of the above system under a control in the form of (19); see Fig. 2.

VI. Conclusions

In this paper, we have proposed a novel and simple technique for consensus control of a network of continuous-time agents with any communication time delay between neighboring agents. High order dynamic models are allowed for each agent. Our consensus algorithm requires communication between neighbouring agents only at sampling instants, not at all times. Our work is preliminary in the sense that a time-invariant undirected connected communication graph is assumed. Our next natural step is to generalize the result to time-varying and directed graphs. Other extensions include developing parallel results for discrete-time agents and discovering more advanced control protocols to optimizing the consensus convergence rate.

References

[1] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, “Flocking in fixed and switching networks,” *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.
[2] A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile agents using nearest neighbor rules,” *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.
[3] R. Olfati-Saber and R. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.
[4] W. Ren and R. W. Beard, “Consensus seeking in multiagents systems under dynamically changing interaction topologies,” *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.
[5] R. Olfati-Saber, “Flocking for multi-agent dynamics systems algorithms and theory,” *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.
[6] V. Gazi and K. M. Passino, “Stability analysis of swarms,” *IEEE Transactions on Automatic Control*, vol. 48, no. 4, pp. 692–697, 2003.

[7] H. Su, X. F. Wang, and Z. Lin, "Flocking of multi-agents with a virtual leader," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 293–307, Dec. 2009.

[8] Z. Chen and H. Zhang, "No-beacon collective circular motion of jointly connected multi-agents," *Automatica*, vol. 47, pp. 1929–1937, 2011.

[9] Y. Mou, H. Xing, Z. Lin, and M. Fu, "Decentralized optimal demand-side-management for phev charging in a smart grid," *IEEE Transactions on Smart Grid*, vol. 6, no. 2, pp. 726–736, 2015.

[10] H. Xing, Y. Mou, Z. Lin, and M. Fu, "Distributed bisection method for economic power dispatch in smart grid," *IEEE Transactions on Power Systems*, vol. 30, no. 6, pp. 3024–3035, 2014.

[11] S. Kar and J. M. F. Moura, "Sensor networks with random links: topology design for distributed consensus," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3315–3326, 2008.

[12] ——, "Distributed consensus algorithms in sensor networks: quantized data and random link failures," *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1383–1400, 2010.

[13] S. Kar, J. M. F. Moura, and K. Ramanan, "Distributed parameter estimation in sensor networks: Nonlinear observation models and imperfect communication," *IEEE Transactions on Information Theory*, vol. 58, no. 6, pp. 3575–3605, 2012.

[14] C. Q. Ma and J. F. Zhang, "Necessary and sufficient conditions for consensusability of linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1263–1268, 2010.

[15] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[16] T. Li, M. Fu, L. Xie, and J. Zhang, "Distributed consensus with limited communication data rate," *IEEE Transactions on Automatic Control*, vol. 56, pp. 279–292, 2011.

[17] K. Avrachenkov and M. El Chamie and G. Neglia, "A local average consensus algorithm for wireless sensor networks," *Distributed Computing in Sensor Systems and Workshops (DCOSS)*, 2011.

[18] J. P. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667–1694, 2003.

[19] Y. A. Fiagbedzi and A. E. Pearson, "Feedback stabilization of linear autonomous time lag systems," *IEEE Transactions on Automatic Control*, vol. AC-31, no. 9, pp. 847–855, 1986.

[20] W. H. Kwon and A. E. Pearson, "Feedback stabilization of linear systems with delayed control," *IEEE Transactions on Automatic Control*, vol. 25, no. 2, pp. 266–269, 1980.

[21] J. H. Park, H. W. Yoo, S. Han, and W. H. Kwon, "Receding horizon controls for input-delayed systems," *IEEE Transactions on Automatic Control*, vol. 53, no. 7, pp. 1746–1752, 2008.

[22] F. Mazenc, S.-I. Niculescu, and M. Krstic, "Lyapunov-krasovskii functionals and application to input delay compensation for linear time-invariant systems," *Automatica*, vol. 48, no. 7, pp. 1317–1323, 2012.

[23] Z. Artstein, "Linear systems with delayed controls: a reduction," *IEEE Transactions on Automatic Control*, vol. AC-27, no. 4, pp. 869–879, 1982.

[24] X. Wang, A. Saberi, A. A. Stoorvogel, H. F. Grio, and T. Yang, "Consensus in the network with uniform constant communication delay," *Automatica*, vol. 49, no. 8, pp. 2461–2467, 2013.

[25] J. Xu, H. Zhang, and L. Xie, "Input delay margin for consensusability of multi-agent systems," *Automatica*, vol. 49, no. 6, pp. 1816–1820, 2013.

[26] Z. Wang and H. Zhang and M. Fu, "Consensus problems in networks of agents with communication delays," *IEEE Multi-conference on Systems and Control*, 2015.

[27] W. Hou and M. Fu and H. Zhang, "Consensusability of multi-agent systems with constant communication delay," *Chinese Control Conference*, 2015.

[28] B. Zhou and Z. Lin, "Consensus of high order multi-agent systems with large input and communication delays," *Automatica*, vol. 50, no. 2, pp. 452–464, 2014.

[29] K. You and L. Xie, "Network topology and communication data rate for consensusability of discrete-time multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2262–2275, 2011.

[30] K. You and L. Xie, "Consensusability of discrete-time multi-agent systems over directed graphs," *Chinese Control Conference*, 2011.

[31] G. J. Michael, "Controllability of linear time-invariant sampled-data systems with nonconstant shaping functions," *IEEE Transactions on Automatic Control*, vol. 15, no. 5, pp. 594–595, 1970.

[32] R. O. Saber and R. M. Murray, "Consensus protocols for networks of dynamic agents," *Proceedings of the American Control Conference*, pp. 951–956, 2003.