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–Brief Paper–

DISTRIBUTED CONSENSUS OF THIRD-ORDER MULTI-AGENT SYSTEMS WITH COMMUNICATION DELAY

Wenying Hou, Minyue Fu, and Huanshui Zhang

ABSTRACT

This paper studies the consensus problem for a class of general third-order multi-agent systems on an undirected connected network. By employing a variables transformation, the consensus control problem can be turned into a asymptotical stability problem. Then we present a necessary and sufficient condition for guaranteeing consensus by using Routh-Hurwitz stability criterion. And this result can be applied to a special case of third-order integrator systems. Also we will present a tolerable communication time delay for third-order integrator systems under the assumption that multi-agent systems can reach consensus without communication delay.

Key Words: Multi-agent system, Routh-Hurwitz stability criterion, third-order consensus, time delay.

I. INTRODUCTION

With the development of the network, consensus control problems become a hotspot. It was derived from distributed computation [1], and then it started to have wide applications in areas, including multi-agent coordination (such as vehicle formations [2] and flocking [3]), biological group behavioral analysis [4]. Network consensus problem is that, each agent needs to update its state on the basis of its local neighbours' state information such that all the agents' states agree upon a common value. The key to the consensus problem is how to design an appropriate consensus control protocol such that multi-agent systems (MASs) reach consensus or how to choose some parameters in a fixed form of consensus protocol.

The pioneering work of Olfati-Saber and Murray [5] solved an average consensus problem for first-order integrator system by using the algebraic graph theory and frequency-domain analysis. Since then, there has been a large number of results on consensus, *e.g.*, [6–10]. Most of these results focus on the first-order or second-order systems, and the consensus conditions listed in these papers are general only sufficient conditions.

However, some problems, in reality, can be modeled by third-order differential equation, such as [11]: Consider a group of N vehicles ($i = 1, 2, \dots, N$) and a leader moving along a single lane. The longitudinal dynamics can be modeled according to the following approximated drivetrain model: $\dot{r}_i(t) = v_i(t)$, $\dot{v}_i(t) = a_i(t)$, $\dot{a}_i(t) = -\frac{1}{T_i}a_i(t) + \frac{1}{T_i}u_i(t)$, where r_i, v_i and a_i are the ith vehicle position, velocity and acceleration, respectively, T_i is the time constant of the drivetrain (typically, the time constant of the drive train $T_i > 0$ depends upon specific vehicle features), and u_i is the desired acceleration to be imposed to the ith vehicle within the platoon. Furthermore, from [12–14] we know that chaotic jerk circuit is with third-order dynamics, and consensus of this system and its application in secure communication has received increasing attention. So it is significant to consider the consensus problem of general third-order dynamics, since it can represent a class of the physical system.

The conditions given by reference [15] are only focused on third-order integrator system without time delay. In the actual network, systems are always subjected to time delay as a result of some constraints, such as the limitation of bandwidth. However, few authors considered the influence of the delay on the result of consensus for third-order systems.

In this paper, we consider a consensus problem for a general third-order linear dynamic model in an undirected connected network. Like the method used in [16], we can turn the consensus problem into a synchronous stability problem. The first contribution of this paper is that it presents a necessary and sufficient consensus condition for general third-order consensus problem by analyzing the roots' distribution for its

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Wenying Hou is with the School of Mathematical Sciences, Shandong Normal University, Ji'nan 250014, China.

Huanshui Zhang is with the School of Control Science and Engineering, Shandong University, Ji'nan 250061, China.

Minyue Fu (corresponding author, e-mail: minyue.fu@newcastle.edu.au) is with the School of Electrical Engineering and Computer Science, University of Newcastle, Callaghan, NSW 2308, Australia and School of Automation, Guangdong University of Technology, Guangdong Key Laboratory of IoT Information Processing, Guangzhou 51006, China.

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corresponding closed-loop systems' characteristic equation. The second contribution of this paper is that it provides a method of computing tolerable communication time delay for third-order integrator systems with the assumption that MASs can reach consensus without communication delay.

II. PRELIMINARIES

A multi-agent network is assumed to have N agents. The communication topology between agents is denoted by an undirected graph $G = \{V, E, A\}$, where $V = \{1, 2, \cdots, N\}$ is the set of agents, $E \subset \{(i,j) : i,j \in V\}$ is the edge set, and $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the so-called weighted adjacency matrix (or adjacency matrix, for short). Each edge (i,j) denotes that agent j obtains information from agent i. And if $(i,j) \in \mathcal{E}$, then $(j,i) \in E$ in undirected graph G. The neighbouring set of agent i is denoted by $N_i = \{j \in V : (i,j) \in E\}$. $a_{ij} > 0$ if and only if $j \in N_i$. The degree of agent i is denoted by $d_i = \sum_{j\in N_i} a_{ij} = \sum_{j=1}^{N} a_{ij}$ and the degree matrix $D = diag\{d_1, d_2, \ldots, d_N\}$. The Laplacian matrix L of G is defined by $L = D - A$. Note that A is a symmetric matrix if G is an undirected graph. It is well known [17] that for an undirected graph, L is a symmetric, positive semi-definite matrix and all of its eigenvalues are non-negative. Note that $L\mathbf{1}_N = \mathbf{0}_N$. The eigenvalues of L are denoted by $\lambda_i, i = 1, 2, \ldots, N$. For an undirected graph G, $0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_N$ if and only if G is connected.

Notation. We use the following notations and conventions in this paper: $\mathbb{R}$ denotes the real number field; $\mathbf{1}_m$ denotes the m-dimensional column vector with all components 1; I_m denotes the m-dimensional identity matrix; $\mathbf{0}$ denotes the zero matrix of appropriate dimension.

According to Routh-Hurwitz stability criterion, we present a stability criterion without giving proof.

Lemma 1. For a third-order real polynomial $f(s) = s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3$, $f(s)$ is Hurwitz stable if and only if $\alpha_i > 0$, $i = 1, 2, 3$, $\alpha_1\alpha_2 > \alpha_3$.

We consider the following general third-order linear dynamic model for each agent $i \in V$ in an undirected connected graph G:

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \\ \dot{a}_i(t) &= ca_i(t) + u_i(t). \end{aligned} \tag{1}$$

Here $x_i(t) \in \mathbb{R}$, $y_i(t) \in \mathbb{R}$, $z_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ denote the position, velocity and accelerated velocity, control input of agent i, respectively. And $c \in \mathbb{R}$ is a real constant.

Next, we give the definition of third-order consensus.

Definition 1 (Third-order consensus)**.** A multi-agent system with agent model (1) is said to achieve third-order consensus if, for any initial conditions and $i \neq j$, $i,j \in V$, $\lim_{t\to\infty}(s_i(t) - s_j(t)) = 0$, $\lim_{t\to\infty}(v_i(t) - v_j(t)) = 0$, $\lim_{t\to\infty}(a_i(t) - a_j(t)) = 0$.

III. CONSENSUS CONDITIONS ANALYSIS WITHOUT TIME DELAY

For each agent i, we deploy a consensus control protocol without communication delay given as the following form:

$$\begin{aligned} u_i(t) = k_1 &\sum_{j=1}^{N} a_{ij}\left[s_j(t) - s_i(t)\right] \\ &+ k_2 \sum_{j=1}^{N} a_{ij}\left[v_j(t) - v_i(t)\right] \\ &+ k_3 \sum_{j=1}^{N} a_{ij}\left[a_j(t) - a_i(t)\right], \end{aligned} \tag{2}$$

where $k_l \in \mathbb{R}$, $l = 1, 2, 3$ are gain coefficients to be designed. Set $\hat{s}_i(t) = s_i(t) - s_1(t)$, $\hat{v}_i(t) = v_i(t) - v_1(t)$, $\hat{a}_i(t) = a_i(t) - a_1(t)$, $i = 2, 3, \ldots, N$, and the state error vector as $\hat{s}(t) = [\hat{s}_2(t), \hat{s}_3(t), \ldots, \hat{s}_N(t)]^T$, $\hat{v}(t) = [\hat{v}_2(t), \hat{v}_3(t), \ldots, \hat{v}_N(t)]^T$, $\hat{a}(t) = [\hat{a}_2(t), \hat{a}_3(t), \ldots, \hat{a}_N(t)]^T$, and $\hat{M}(t) = [\hat{s}^T(t), \hat{v}^T(t), \hat{a}^T(t)]^T$. We obtain the following error dynamics:

$$\dot{\hat{M}}(t) = \hat{\Phi}\hat{M}(t), \tag{3}$$

where

$$\hat{\Phi} = \begin{bmatrix} 0_{N-1} & I_{N-1} & 0_{N-1} \\ 0_{N-1} & 0_{N-1} & I_{N-1} \\ -k_1\hat{L} & -k_2\hat{L} & -k_3\hat{L} + cI_{N-1} \end{bmatrix},$$

with $\hat{L} = L_{22} + \mathbf{1}_{N-1}\alpha^T$, $\alpha = [a_{12}, a_{13}, \ldots, a_{1N}]^T$,

$$L_{22} = \begin{bmatrix} d_2 & -a_{23} & \cdots & -a_{2N} \\ -a_{32} & d_3 & \cdots & -a_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N2} & -a_{N3} & \cdots & d_N \end{bmatrix}.$$

Apparently, system (1)–(2) achieves consensus if and only if the error system (3) is asymptotically stable, in other words, the eigenvalues of $\hat{\Phi}$ are all in the open left half plane. From [14] we know that the eigenvalues of

$\hat{L}$ are $\lambda_2, \lambda_3, \dots, \lambda_N$. And there exists an invertible matrix T such that $T^{-1}\hat{L}T = J = \mathrm{diag}(J_1, J_2, \dots, J_s)$, where J_k, $k = 1, 2, \dots, s$ are upper triangular Jordan blocks, whose principal diagonal elements consist of λ_i, $i = 2, 3, \dots, N$. Note that

$$(T^{-1} \otimes I_3)\hat{\Phi}(T \otimes I_3) = \begin{bmatrix} 0_{N-1} & I_{N-1} & 0_{N-1} \\ 0_{N-1} & 0_{N-1} & I_{N-1} \\ -k_1 J & -k_2 J & -k_3 J + cI_{N-1} \end{bmatrix},$$

which implies that the eigenvalues of $\hat{\Phi}$ are given as the roots of $\prod_{i=2}^{N} f_i(s) = 0$, where

$$f_i(s) = s^3 + (k_3\lambda_i - c)s^2 + k_2\lambda_i s + k_1\lambda_i. \tag{4}$$

Thus based on the above analysis, we can obtain the following result.

Lemma 2. The control protocol (2) makes system (1) achieve consensus if and only if all $f_i(s)$, $i = 2, 3, \dots, N$, defined by (4), are all Hurwitz stable.

By using Lemma 1 to the stability analysis of $f_i(s)$, we can obtain a consensus condition as follows.

Theorem 1. The control protocol (2) makes system (1) achieve consensus if and only if

$$k_1 > 0, \quad k_2 > 0, \quad k_3 > \left(\frac{k_1}{k_2} + c\right)\frac{1}{\lambda_2}. \tag{5}$$

Proof. From Lemma 1 we know that $f_i(s)$ is Hurwitz stable if and only if $k_3\lambda_i - c > 0$, $k_2\lambda_i > 0$, $k_1\lambda_i > 0$, $(k_3\lambda_i - c)k_2\lambda_i > k_1\lambda_i$, i.e, $k_1 > 0, k_2 > 0, k_3 > \left(\frac{k_1}{k_2} + c\right)\frac{1}{\lambda_i}$, Thus, (5) holds.

Remark 1. Theorem 1 gives a necessary and sufficient consensus condition for a class of general third-order MASs.

If we take $c = 0$, then system (1) becomes the following third-order integrator model:

$$\dot{s}_i(t) = v_i(t),\ \dot{v}_i(t) = a_i(t),\ \dot{a}_i(t) = u_i(t). \tag{6}$$

Then from Theorem 1 we can obtain the following consensus conditions for system (6) under the same consensus protocol (2).

Corollary 1. The control protocol (2) makes the third-order integrator system (6) achieve consensus if and only if

$$k_1 > 0, \quad k_2 > 0, \quad k_3 > \frac{k_1}{k_2\lambda_2}. \tag{7}$$

Proof. From Theorem 1 we know that system (6) with protocol (2) can achieve consensus if and only if (7) holds by taking $c = 0$.

Remark 2. The result of Corollary 1 is consistent with [15].

IV. CONSENSUS CONDITIONS ANALYSIS WITH TIME DELAY

In this section, for system (6), we consider the following control protocol with constant communication delay τ:

$$\begin{aligned} u_i(t) = k_1 \sum_{j=1}^{N} a_{ij}\left[s_j(t-\tau) - s_i(t-\tau)\right] \\ + k_2 \sum_{j=1}^{N} a_{ij}\left[v_j(t-\tau) - v_i(t-\tau)\right] \\ + k_3 \sum_{j=1}^{N} a_{ij}\left[a_j(t-\tau) - a_i(t-\tau)\right]. \end{aligned} \tag{8}$$

Similarly, we have the following error dynamics

$$\begin{aligned} \dot{\hat{M}}(t) = \begin{bmatrix} 0_{N-1} & I_{N-1} & 0_{N-1} \\ 0_{N-1} & 0_{N-1} & I_{N-1} \\ 0_{N-1} & 0_{N-1} & 0_{N-1} \end{bmatrix}\hat{M}(t) \\ - \begin{bmatrix} 0_{N-1} & 0_{N-1} & 0_{N-1} \\ 0_{N-1} & 0_{N-1} & 0_{N-1} \\ -k_1\hat{L} & -k_2\hat{L} & -k_3\hat{L} \end{bmatrix}\hat{M}(t-\tau). \end{aligned} \tag{9}$$

It is clear that the control protocol (8) can makes system (6) achieve consensus if and only if the error system (9) is asymptotically stable.

Similar to the delay-free case, we need to analyze the characteristic equation for the error system (9). To do this, we take the Laplace transform on (9) and obtain its characteristic equation given by $\prod_{i=2}^{N} f_i(s,\tau) = 0$, where $f_i(s,\tau)$, $i = 2, 3, \dots, N$ are quasi-polynomials given by

$$\begin{aligned} f_i(s,\tau) = s^3 + k_3\lambda_i e^{-\tau s}s^2 + k_2\lambda_i e^{-\tau s}s \\ + k_1\lambda_i e^{-\tau s}. \end{aligned} \tag{10}$$

As we all know that a time-delay system is asymptotically stable if and only if the roots of its characteristic equation are all in the left-open half plane. So we can obtain a bound of time delay by analyzing the distribution of characteristic roots of the closed-loop system, then we can present the following result of delay margin for this consensus control problem.

Theorem 2. Suppose that condition (7) holds. For $r \in \{2, 3, \dots, N\}$, let $\mu_r > \frac{k_1}{k_3}$ be the root of the following equation:

$$\mu_r^3 - k_3^2\lambda_r^2\mu_r^2 + (2k_1k_3 - k_2^2)\lambda_r^2\mu_r - k_1^2\lambda_r^2 = 0.$$

Take $\tau_r = \frac{\arctan \Psi_r}{\sqrt{\mu_r}}$, where $\Psi_r = \frac{k_3\mu_r - k_1}{k_2\sqrt{\mu_r}}$. Set $\tau^* = \min_r \tau_r$, then the control protocol (8) makes (6) achieve consensus if and only if $\tau \in [0, \tau^*)$.

Proof. From Corollary 2.4 of [18] we know that for a quasi-polynomial of the form $f(s, e^{-\tau s}) = f_0(s) + f_1(s)e^{-\tau s}$ with $f_0(s) = s^n + a_1 s^{n-1} + \dots + a_n, f_1(s) = b_1 s^{n-1} + \dots + b_n$, $a_i \in R, b_i \in R, i = 1, 2, \dots, n$, if $f(s, e^{-\tau s})$ is Hurwitz stable for $\tau = 0$ and $f(s, e^{-\tau s})$ is unstable for some $\tau > 0$, then there must exist some $\tau^* \in (0, \tau)$ such that $f(s, e^{-\tau^* s})$ has a root on the imaginary axis, and $f(s, e^{-\tau^0 s})$ is stable for all $\tau^0 \in [0, \tau^*)$.

Here since the condition (7) holds, so (8) can makes system (6) achieve consensus when $\tau = 0$, thus the corresponding characteristic polynomial, $s^3 + k_3\lambda_i s^2 + k_2\lambda_i s + k_1\lambda_i$, is Hurwitz stable. According to the continuous dependence of root to time delay, we know that τ^* is delay margin of consensus if and only if all the roots of (10) will still be in the open left half-plane for all $\tau \in (0, \tau^*)$ and at least one of the quasi-polynomials $f_i(s, \tau^*)$, $i \in \{2, 3, \dots, N\}$ has an imaginary root.

Next, we will only need to examine the imaginary roots of the quasi-polynomials of (10) for $\tau = \tau^*$. Let $s_r = \imath\omega_r$, $\omega_r \in R$, $\omega_r \neq 0$, $r \in \{2, 3, \dots, N\}$. Then $f_r(s_r, \tau) = 0$ means both of its real and imaginary parts are zero, which are given by $m_r(\omega_r) = -k_3\lambda_r\cos(\omega_r\tau)\omega_r^2 + k_2\lambda_r\sin(\omega_r\tau)\omega_r + k_1\lambda_r\cos(\omega_r\tau) = 0$, $n_r(\omega_r) = -\omega_r^3 + k_3\lambda_r\sin(\omega_r\tau)\omega_r^2 + k_2\lambda_r\cos(\omega_r\tau)\omega_r - k_1\lambda_r\sin(\omega_r\tau) = 0$. Re-arranging the above equations gives

$$\begin{bmatrix} \sin(\tau_r\omega_r) \\ \cos(\tau_r\omega_r) \end{bmatrix} = \frac{\omega_r^3 \begin{bmatrix} k_3\omega_r^2 - k_1 \\ k_2\omega_r \end{bmatrix}}{\left[k_2^2\omega_r^2 + \left(k_3\omega_r^2 - k_1\right)^2\right]\lambda_r}.$$

From $\sin^2(\tau_r\omega_r) + \cos^2(\tau_r\omega_r) = 1$ we can obtain that

$$\omega_r^6 - \lambda_r^2\left[k_2^2\omega_r^2 + \left(k_3\omega_r^2 - k_1\right)^2\right] = 0. \qquad (11)$$

Also we can obtain that $\tan(\tau_r\omega_r) = \Psi_r = \frac{k_3\omega_r^2 - k_1}{k_2\omega_r}$, which yields $\tau_r = \frac{\arctan \Psi_r + k\pi}{\omega_r}$, where $k \in \{0, 1\}$ is an minimum integer such that $\tau_r > 0$.

It is obvious that if ω is a root of (11), then $-\omega$ is also a root of (11), and vice versa. Let $\omega_{r1} > 0$ and $\omega_{r2} = -\omega_{r1}$ be the roots of (11), then we have $\Psi_{r1} = -\Psi_{r2}$. Thus, $\tan(\tau_{r1}\omega_{r1}) = -\tan(\tau_{r2}\omega_{r2})$, and we can simply take $\tau_{r1} = \tau_{r2}$. That is to say, we only need to consider the corresponding time delay τ_r for $\omega_r > 0$. Finally, the minimum value of τ is thus given by $\tau^* = \min_r \tau_r$ over all possible roots $\omega_r > 0$ and $r \in 2, 3, \dots, N$.

Set $\mu_r = \omega_r^2$, then (11) turns to

$$\mu_r^3 - k_3^2\lambda_r^2\mu_r^2 + (2k_1k_3 - k_2^2)\lambda_r^2\mu_r - k_1^2\lambda_r^2 = 0. \qquad (12)$$

Apparently, equation (12) is a cubic equation about μ_r, so it has three roots, and we only need to consider the case of $\mu_r > 0$. Then we will analyze of the monotonicity of τ on λ_r.

(i) If $\mu_{r1} < \mu_{r2}$ are two positive roots of equation (12) for the same λ_r, and we assume that $k_3\mu_{r1} - k_1 < 0$, $k_3\mu_{r2} - k_1 > 0$, so $\Psi_{r1} < 0$, $\Psi_{r2} > 0$, thus we have $\tau_{r1} = \frac{\arctan \Psi_{r1} + \pi}{\omega_{r1}} > \frac{\pi}{2\omega_{r1}}$, $\tau_{r2} = \frac{\arctan \Psi_{r2}}{\omega_{r2}} < \frac{\pi}{2\omega_{r2}} < \frac{\pi}{2\omega_{r1}}$, that is to say, $\tau_{r1} > \tau_{r2}$.

(ii) If μ_m, μ_n are two positive roots of equation (12) for $r = m$ and $r = n$, respectively. And we also assume that $k_3\mu_m - k_1 < 0$, $k_3\mu_n - k_1 > 0$, then $\mu_m < \mu_n$. Similarly we have $\Psi_m < 0$, $\Psi_n > 0$, thus we have $\tau_m = \frac{\arctan \Psi_m + \pi}{\omega_m} > \frac{\pi}{2\omega_m} = \frac{\pi}{2\sqrt{\mu_m}} > \frac{\pi}{2\sqrt{\mu_n}}$, $\tau_n = \frac{\arctan \Psi_n}{\omega_n} < \frac{\pi}{2\sqrt{\mu_n}}$, that is to say, $\tau_m > \tau_n$.

From (i) and (ii) we can see that for any two positive roots of (12), p and q, the corresponding time delay of p is bigger than q if only $k_3p - k_1 < 0$ and $k_3q - k_1 > 0$. So we only need to consider the case of $\mu_r > \frac{k_1}{k_3}$ to deduce the minimum tolerable time delay τ^*.

Remark 3. The proof of Theorem 2 presents us a method of computing the delay margin for consensus problem of third-order integrator system.

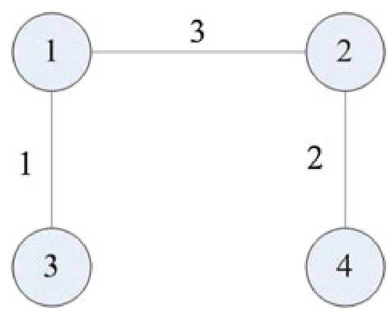

Fig. 1. The network topology. [Color figure can be viewed at wileyonlinelibrary.com]

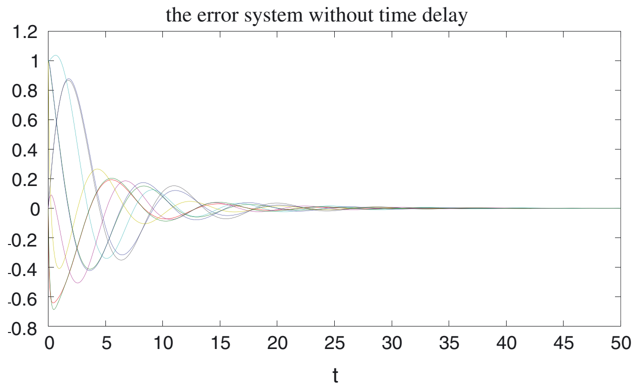

Fig. 2. The error system without time delay. [Color figure can be viewed at wileyonlinelibrary.com]

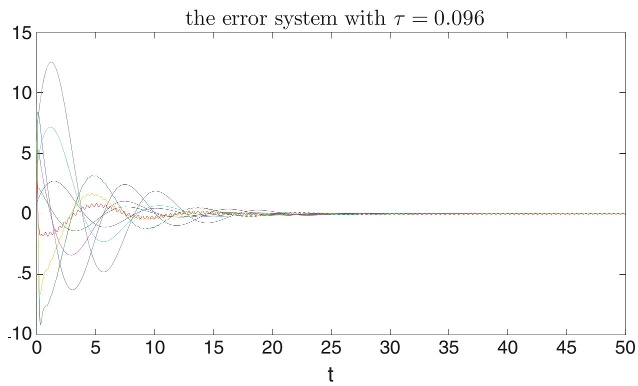

Fig. 3. The error system with $\tau = 0.096$. [Color figure can be viewed at wileyonlinelibrary.com]

V. SIMULATIONS

We consider an undirected connected network presented as Fig. 1 with four third-order integrator agents. By computing we can obtain the eigenvalues of L are $\lambda_2 = 1$, $\lambda_3 = 3$, $\lambda_4 = 8$. We take $k_1 = k_2 = 1$, $k_3 = 2$, and from Theorem 2 we know that this system can reach consensus under protocol (6). Fig. 2 presents the error system without time delay.

Also for the case of constant communication delay, from Theorem 2 we only need to consider the roots of $\mu^3 - 4\lambda_i^2\mu^2 + 3\lambda_i^2\mu - \lambda_i^2 = 0$ satisfying $\mu > \frac{k_1}{k_2} = 0.5$, and there are $\mu_2 = 3.1479$, $\mu_3 = 35.2411$, $\mu_4 = 255.25$, so we can get $\tau^* = \min\{\tau_2, \tau_3, \tau_4\} = \min\{0.7031, 0.2502, 0.0964\} = 0.0964$. So protocol (8) can makes this system reach consensus if and only if $\tau \in [0, \tau^*)$. Figs 3 and 4 display the error system with time delay $\tau = 0.096$, $\tau = 0.097$, respectively. Apparently, these simulations are consistent with our main results of Theorem 2.

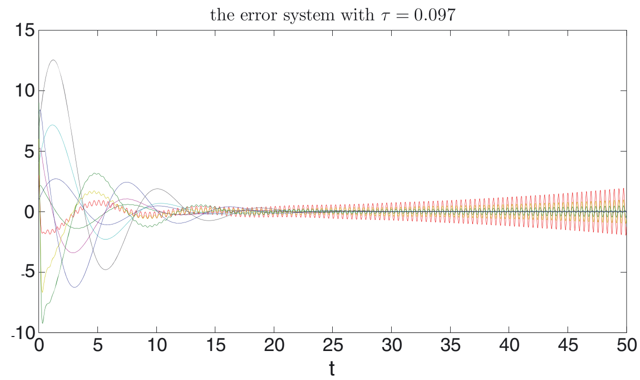

Fig. 4. The error system with $\tau = 0.097$. [Color figure can be viewed at wileyonlinelibrary.com]

VI. CONCLUSION

In this paper, we considered a consensus problem for a class of general third-order multi-agent systems on an undirected connected network. The consensus control problem can be turned into a synchronous stability problem by using variables transformation. Then we presented a necessary and sufficient condition for guaranteeing consensus. And this result can be applied to a special case of third-order integrator systems. Also we presented a method of computing tolerable communication time delay for third-order integrator systems under the assumption that MASs can reach consensus without communication delay.

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