State Estimation Subject to Random Network Delays without Time Stamping

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***Abstract*—The discrete-time state estimation problem is studied for networked control systems subject to random network delays without time stamping. A new time delay model is presented which allows the transmitted data to be received in bursts. Under the assumption that the data bursts are not out of order, we derive the optimal linear estimator which guarantees an unbiased estimate with minimum and uniformly bounded estimation error covariance. The estimator gains can be derived by solving a set of recursive discrete-time Riccati equations. A simulation example shows the effectiveness of the proposed algorithm.**

I. Introduction

The problem of state estimation for systems with random time delays has attracted great attention due to the wide applications in signal processing, control and communication systems [1-5]. The random measurement problem was studied as early as in [6] for state estimation. In the recent years, many results have been reported for networked control systems with random time delays [7-11].

In [8] and [15], the least mean square filtering problem was discussed for systems with a single random sampling delay. Estimation problems for systems with random delays and uncertain measurements are also investigated in [12-14]. Zhang and Xie [16] studied the optimal estimation problem for discrete-time systems with time-varying delays in the measurement channel, and the measurements are time stamped which can take only one value at each time instant. Schenato [18] proposed estimators subject to simultaneous random packet delay and packet dropout, and this allows packets to arrive in burst or even out of order at the receiver side, as long as the measurements are time stamped.

Without using time stamps, Sun [19] proposed the optimal filtering problem for discrete-time stochastic linear system with multiple random measurement delays. Sun [20] also investigated the estimation problem for systems with bounded random measurement delays and packet dropouts, which are described by some binary distributed random variables with known probabilities. But in [20], the network model can receive the same measurement multiple times, and at the same time, an excessively high packet loss rate can occur, which does not fit most communication protocols. In fact, for most network protocols, random time delays mean that more than one measurement may be received at each time instant. That is, measurements are received in bursts of various sizes.

In this paper, we propose the optimal estimation problem where observation packets are subject to bounded random delays. This allows packets to arrive in bursts at the receiver side. We assume that there are no packet dropouts and the packets can not be received repeatedly. Without using time stamping, a new network model is presented in which the measurement bursts satisfy the assumption that different bursts are not out of order. This is a realistic assumption for most network protocols when time delays are not serious. We derive an optimal estimator by minimizing the estimation error covariance subject to the constraints that the estimate is unbiased and estimation error covariance is uniformly bounded. The estimator gains are given in terms of Riccati equations.

This paper is organized as follows. Section II formulates the optimal estimation problem and describes the network model for random delays; Section III presents the solution for the optimal estimation using the new network model. Section IV gives a simulation example, and Section V draws some conclusions.

II. Problem formulation and network model

Consider the following discrete-time linear stochastic system:

$$x_{k+1} = Ax_k + \upsilon_k \quad (1)$$

$$y_k = Cx_k + \omega_k \quad (2)$$

where $x_k \in \mathbb{R}^n$ is the system state, $y_k \in \mathbb{R}^m$ is the measured output, $\upsilon_k \in \mathbb{R}^n$ and $\omega_k \in \mathbb{R}^m$ are system noise and measure noise, respectively. A, C are matrices of the appropriate dimensions. The initial state x_0 and υ_k, ω_k are Gaussian, uncorrelated, white, with mean $(\bar{x}_0, 0, 0)$ and covariance (P_0, Q_k, R_k), respectively. We also assume that A is unstable, the pair (A, C) is observable, and $R > 0$.

In the networked system, the output of the system is measured at every time instant and transmitted to the estimator through a communication channel with a random time delay. We consider that measurements are not time stamped through network transmission. Thus, at the receiver side, the received measurements can not be reordered as they can arrive out of order. The information which can be only confirmed is that the

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number of received data and the missing date at each time. We first make the following assumptions for the network system.

Assumption 1: Each measurement is received once and once only. The transmission delay for each measurement ranges from 0 to N, where N is the maximum delay which is finite and known. In particular, there is no packet loss and measurements are received in bursts with the size of each burst ranges from 0 to $N+1$.

Assumption 2: The measurements are not time stamped. But the received measurement bursts are such that they are in order. That is, the received bursts follow the first-in-first-out (FIFO) principle, but the order of the measurements within each burst is not known.

In order to understand the problem better, we first give the state transition diagram for the case when $N = 1$:

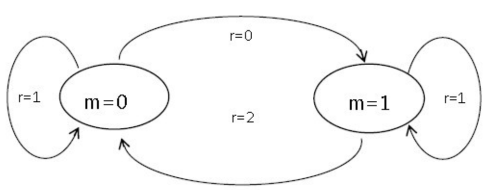

Fig. 1. the state transition for random time-delay.

let m_k be the number of missing measurements at time k, and r_k be the number of the received measurements at time k. There are two cases:

Case 1: $m_k = 0$, i.e., y_{k-1} was received at $k-1$. There two subcases according to r_k:

Case 1.1, when $r_k = 0$, time delay for y_k will happen, and $m_{k+1} = 1$;

Case 1.2, when $r_k = 1$ the measurement y_k is received on time, so $m_{k+1} = 0$.

Case 2: $m_k = 1$, i.e. y_{k-1} was not received at $k-1$, then we have:

Case 2.1, when $r_k = 1$, y_{k-1} must be received at k, and y_k must be missing, so $m_{k+1} = 1$;

Case 2.2, when $r_k = 2$, y_k, y_{k-1} are received simultaneously, then $m_{k+1} = 0$.

Form the above analysis, we know that the received measurement can be precisely deduced in the Case 1 and subcase 2.1. But for Case 2.2, because of absence of time stamps, we do not know the arrival order of $\{y_{k-1},\ y_k\}$.

The state transition diagram for random time delay when $N > 1$ can be also be given, as shown in Fig.2:

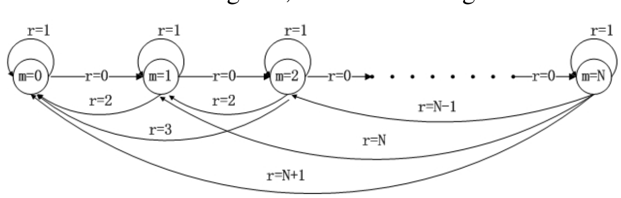

Fig. 2. the state transition with random delay bound $N > 1$.

The situation here is more complex. There is only one case where the received measurements can be precisely known: $r_k = 1$, where the received measurement must be the first missing one due to the FIFO assumption. In all other cases, the exact locations of the received data are not known due to the lack of time stamps. However, because the bursts are not out of order, the receive data are known to be the earliest missing data. The arrival sequences at time k is one of the permutations of $\{y_{k-m_k+1}, y_{k-m_k+2}, \cdots, y_{k-m_k+r_k}\}$ and will be denoted by $\tilde{y}_{ki}$, $(i = 1, 2, \cdots, r_k!)$. Thus, the measurements received by the estimator can be modeled as:

$$\tilde{y}_k = \gamma_k^{(1)} \tilde{y}_{k1} + \gamma_k^{(2)} \tilde{y}_{k2} + \cdots + \gamma_k^{(r_k!)} \tilde{y}_{kr_k!} \quad (3)$$

where $\gamma_k^{(i)}$ $(i = 1, 2, \cdots, r_k!)$ is a scalar quantity independent of k taking value 0 or 1, satisfying $\sum_{i=1}^{r_k!} \gamma_k^{(i)} = 1$ with prob$\{\gamma_k^{(i)} = 1\} = \rho_k^{(i)}$, $0 < \rho_k^{(i)} < 1$, and $\sum_{i=1}^{r_k!} \rho_k^{(i)} = 1$. We denote $\gamma_k = \{\gamma_k^{(1)}, \gamma_k^{(2)}, \ldots, \gamma_k^{(r_k!)}\}$.

At time k, based on the observations $\tilde{y}_k$ in (3) and the most recent estimate $\hat{x}_{k-m_k+1}$, we design the linear estimator as follows:

$$\hat{x}_{k-m_k+r_k+1} = F_k \hat{x}_{k-m_k+1} + [H_{k1} \ \ H_{k2} \ \ \cdots \ \ H_{kr_k}] \tilde{y}_k \quad (4)$$

The estimator error and error covariance are defined by

$$\begin{aligned} e_{k-m_k+r_k+1} &\triangleq x_{k-m_k+r_k+1} - \hat{x}_{k-m_k+r_k+1} \quad (5) \\ \bar{P}_{k-m_k+r_k+1} &\triangleq E_x E_\gamma [e_{k-m_k+r_k+1} e_{k-m_k+r_k+1}^T] \quad (6) \end{aligned}$$

where E_x is the expectation with respect to υ, ω and x_0; and E_γ is expectation with respect to $\gamma = \{\gamma_1, \gamma_2, \ldots\}$.

The estimate $\hat{x}_{k-m_k+r_k+1}$ needs to be optimal in the sense that it minimizes the error covariance, i.e. it is desired to find the estimator by minimizing (6). We demand that the estimator is unbiased, i.e. $E_x E_\gamma e_{k-m_k+r_k+1} = 0$, and we also want the estimation error covariance to be uniformly bounded, as defined below.

Definition 1: the estimation error covariance is called uniformly bounded if there exists a constant $M > 0$ independent of P_0, such that

$$\bar{P}_k \leq M \quad (7)$$

for all $k = 0, 1, 2, \ldots$.

III. Estimator design with random delays

In this section, we will present the solution to the optimal estimation with random delays.

The estimator error $e_{k-m_k+r_k+1}$ is defined in (5), substituting (1), (3) and (4) into it, we have

$$\begin{aligned} e_{k-m_k+r_k+1} =& x_{k-m_k+r_k+1} - \hat{x}_{k-m_k+r_k+1} \\ =& A^{r_k} x_{k-m+1} + \sum_{i=0}^{r_k-1} A^i \upsilon_{k-m_k+r_k-i} \\ & - F_k \hat{x}_{k-m_k+1} - [H_{k1} \ \ H_{k2} \ \ \cdots \ \ H_{kr_k}] \tilde{y}_k \end{aligned} \quad (8)$$

Assume Π_i is the i-th permutation matrix of the sequence $\tilde{\tilde{y}}_k = \begin{bmatrix} y_{k-m_k+1} \\ \cdots \\ y_{k-m_k+r_k} \end{bmatrix}$. Then in (8),

$$\begin{aligned}
&[H_{k1}\ H_{k2}\ \cdots\ H_{kr_k}]\tilde{y}_k \\
=&[H_{k1}\ \cdots\ H_{kr_k}][\gamma_k^{(1)}\tilde{y}_{k1}+\gamma_k^{(2)}\tilde{y}_{k2}+\cdots+\gamma_k^{(r_k!)}\tilde{y}_{kr_k!}] \\
=&[H_{k1}\ \cdots\ H_{kr_k}][\gamma_k^{(1)}\Pi_1\tilde{\tilde{y}}_k+\gamma_k^{(2)}\Pi_2\tilde{\tilde{y}}_k\cdots+\gamma_k^{(r_k!)}\Pi_{r_k!}\tilde{\tilde{y}}_k] \\
=&[H_{k1}\ \cdots\ H_{kr_k}][\gamma_k^{(1)}\Pi_1+\gamma_k^{(2)}\Pi_2+\cdots+\gamma_k^{(r_k!)}\Pi_{r_k!}]\tilde{\tilde{y}}_k \\
=&[\gamma_k^{(1)}\tilde{H}_{k1}+\gamma_k^{(2)}\tilde{H}_{k2}+\cdots\quad+\gamma_k^{(r_k!)}\tilde{H}_{kr_k!}]\tilde{\tilde{y}}_k \qquad (9)
\end{aligned}$$

where $\tilde{H}_{ki}$ is the ith permutation of $[H_{k1}\ \cdots\ H_{kr_k}]$, $i = 1,2,\cdots,r_k!$. Then substituting (9) into (8), we get

$$\begin{aligned}
e_{k-m_k+r_k+1} =& A^{r_k}x_{k-m_k+1}+\sum_{i=0}^{r_k-1}A^i v_{k-m_k+r_k-i}- \\
&F_k\hat{x}_{k-m_k+1}-[\gamma_k^{(1)}\tilde{H}_{k1}+\gamma_k^{(2)}\tilde{H}_{k2}\cdots+\gamma_k^{(r_k!)}\tilde{H}_{kr_k!}]\tilde{\tilde{y}}_k \\
=&A^{r_k}x_{k-m_k+1}-F_k\hat{x}_{k-m_k+1}+\text{noise} \\
&-[\gamma_k^{(1)}\tilde{H}_{k1}+\gamma_k^{(2)}\tilde{H}_{k2}+\cdots \\
&\cdots+\gamma_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}x_{k-m_k+1} \qquad (10)
\end{aligned}$$

where the "noise" in (10) is the system noise and the measurement noise with zero mean, and is not important for the next deduction.

By the unbiased estimation property $E_xE_\gamma e_{k-m_k+r_k+1} = 0$, with the probability of $\gamma_k^{(i)}$, we get

$$F_k = A^{r_k}-[\rho_k^{(1)}\tilde{H}_{k1}+\cdots+\rho_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix} \qquad (11)$$

Substituting F_k back into (10), it is rewritten as

$$\begin{aligned}
e_{k-m_k+r_k+1} =& A^{r_k}x_{k-m_k+1}+\text{noise}-[A^{r_k}- \\
&[\rho_k^{(1)}\tilde{H}_{k1}+\cdots+\rho_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}]\hat{x}_{k-m_k+1} \\
&-[\gamma_k^{(1)}\tilde{H}_{k1}+\cdots+\gamma_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}x_{k-m_k+1} \\
=&\text{noise}+\Big[A^{r_k}-[\rho_k^{(1)}\tilde{H}_{k1}+\cdots+ \\
&\rho_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}\Big]e_{k-m_k+1}-[(\gamma_k^{(1)}-\rho_k^{(1)})\tilde{H}_{k1} \\
&+\cdots+(\gamma_k^{(r_k!)}-\rho_k^{(r_k!)})\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}x_{k-m_k+1} \qquad (12)
\end{aligned}$$

Theorem 1 Consider the system (1)-(2), and the network model as described earlier. A necessary condition for the estimation error to be unbiased and estimation error covariance to be uniformly bounded is that $H_{k1} = H_{k2} = \cdots = H_{kr_k}$ for all k. Consequently, the optimal estimator has the form:

$$\hat{x}_{k-m_k+r_k+1} = F_k\hat{x}_{k-m_k+1}+H_k\frac{1}{r_k}\sum_{i=1}^{r_k}y_{k-m_k+i} \qquad (13)$$

Proof: Since A is unstable, $E\|x_k\| \to \infty$ as $k \to \infty$,. From Definition 1, in order for the error $e_{k-m_k+r_k+1}$ to be uniformly bounded, in (12) we must have

$$\begin{aligned}
&[(\gamma_k^{(1)}-\rho_k^{(1)})\tilde{H}_{k1}+(\gamma_k^{(2)}-\rho_k^{(2)})\quad\cdots \\
&\qquad+(\gamma_k^{(r_k!)}-\rho_k^{(r_k!)})\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}=0 \qquad (14)
\end{aligned}$$

Then

$$\begin{aligned}
&[\gamma_k^{(1)}\tilde{H}_{k1}+\gamma_k^{(2)}\tilde{H}_{k2}\quad\cdots\quad+\gamma_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix} \\
=&[\rho_k^{(1)}\tilde{H}_{k1}+\rho_k^{(2)}\tilde{H}_{k2}\quad\cdots\quad+\rho_k^{(r_k!)}\tilde{H}_{kr_k!}]\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix} \qquad (15)
\end{aligned}$$

Denote the right-hand side of (15) by W. Note that W is a constant.
Since $\sum_{i=1}^{r_k!}\gamma_k^{(i)} = 1$, we have:

$$\tilde{H}_{ki}\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix} = W \qquad (16)$$

for all i. Recalling that (C, A) is observable, then:

1) When $r_k \leq n$, $\begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1}\end{bmatrix}$ is full row rank, then from (16), we get $\tilde{H}_{ki}$ is constant for $i = 1,2,\cdots,r_k!$. So it follows that

$$H_{k1} = H_{k2} = \cdots = H_{kr_k} \qquad (17)$$

2) When $r_k > n$, then choose all the permutations such that

$$\Pi_i = \begin{bmatrix} \tilde{\Pi}_i & 0 \\ 0 & I \end{bmatrix}$$

with $\tilde{\Pi}_i \in \mathbb{R}^{n\times n}$. From 1), we know that $H_{k1} = H_{k2} = \cdots = H_{kn}$.
Similarly consider the following permutation as

$$\Pi_i = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \tilde{\Pi}_i & 0 \\ 0 & 0 & I_2 \end{bmatrix}$$

with $I_1 \in \mathbb{R}^{n_1\times n_1}$, $\tilde{\Pi}_i \in \mathbb{R}^{n\times n}$ and $n_1 < n$, then it has $H_{k(n_1+1)} = H_{k(n_1+2)} = \cdots = H_{k(n_1+n)}$.
. Following the same idea, we get $H_{ki_1} = H_{ki_2} = \cdots = H_{ki_n}$ for $\forall\ (ki_1, ki_2, \cdots.) \in (k1, k2, \cdots, kr_k)$. Thus, (17) also holds.
Let $\frac{1}{r_k}H_k = H_{k1} = H_{k2} = \cdots = H_{kr_k}$, substituting it into (4), the estimator (4) is now equivalent to (13).■

Remark 1 : Theorem 1 basically means that, under the requirements of unbiased estimation and uniformly bounded estimation error covariance, we just need use the average of all the received measurements in a burst to get the optimal estimation. Equivalently, the measurement model received by the estimator is given by:

$$\mathbf{y}_k = \begin{cases} \frac{1}{r_k}\sum_{i=1}^{r_k} y_{k-m_k+i} & r_k \neq 0 \\ 0 & r_k = 0 \end{cases} \tag{18}$$

Next, we will give the optimal estimation gain H_k. It is clear that there is no packet received when $r_k = 0$, there is no need to update the state estimate in (13), i.e., the most recent state estimate remains $\hat{x}_{k-m_k+1}$, and the error covariance is $\bar{P}_{k-m_k+1}$. So we only consider $r_k > 0$ in the sequel.

Theorem 2 Considering the system (1)-(2), and suppose the estimation error covariance $\bar{P}_{k-m_k+1}$ is given. The estimation gain H_k that minimizes $\bar{P}_{k-m_k+r_k+1}$ is given by

$$\begin{aligned} H_k =& r_k(A^{r_k}\bar{P}_{k-m_k+1}(\sum_{i=0}^{r_k-1} CA^i)^T \\ &+ \sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} A^i Q_{k-m_k+r_k-i-1} A^{jT} C^T) M_k^{-1} \end{aligned} \tag{19}$$

where

$$\begin{aligned} M_k =& \sum_{i=0}^{r_k-1} CA^i \bar{P}_{k-m_k+1}(\sum_{i=0}^{r_k-1} CA^i)^T + \\ & \sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} CA^j Q_{k-m_k+i-j-1} A^{jT} C^T + \sum_{i=0}^{r_k-1} R_{k-m_k+i} \end{aligned} \tag{20}$$

The corresponding estimation the error covariance is given by

$$\begin{aligned} \bar{P}_{k-m_k+r_k+1} =& A^{r_k}\bar{P}_{k-m_k+1}A^{r_k T} - H_k M_k H_k^T \\ &+ \sum_{i=0}^{r_k-1} A^i Q_{k-m_k+i} A^{iT} \end{aligned} \tag{21}$$

$$\bar{P}_0 = E x_0 x_0^T \tag{22}$$

Proof: substituting H_k into (11), and $\sum_{i=1}^{r_k!}\rho_k^{(i)} = 1$, we have

$$F_k = A^{r_k} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1} CA^i \tag{23}$$

From (1) (2) (13), the error is :

$$\begin{aligned} & e_{k-m_k+r_k+1} = x_{k-m_k+r_k+1} - \hat{x}_{k-m_k+r_k+1} \\ &= A^{r_k}x_{k-m_k+1} + \sum_{i=0}^{r_k-1} A^i \upsilon_{k-m_k+r_k-i} - F_k\hat{x}_{k-m_k+1} \\ &- \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1} CA^i x_{k-m_k+1} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1}\omega_{k-m_k+i+1} \\ &- \frac{1}{r_k}H_k\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} CA^j\upsilon_{k-m_k+i-j} \end{aligned} \tag{24}$$

It is obvious that the noises are correlated, and the estimator error covariance is

$$\begin{aligned} & \bar{P}_{k-m_k+r_k+1} = E_x[e_{k-m_k+r_k+1}e_{k-m_k+r_k+1}^T] = \\ & (A^{r_k} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1} CA^i)\bar{P}_{k-m_k+1}(A^{r_k} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1} CA^i)^T \\ & + \sum_{i=0}^{r_k-1} A^i Q_{k-m_k+r_k-i}A^{iT} + \frac{1}{{r_k}^2}H_k\sum_{i=0}^{r_k-1} R_{k-m_k+i+1}H_k^T \\ & + \frac{1}{{r_k}^2}H_k\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} CA^j Q_{k-m_k+i-j}A^{jT}C^T H_k^T \\ & - \frac{1}{r_k}\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} A^i Q_{k-m_k+r_k-i}A^{jT}C^T H_k^T \\ & - \frac{1}{r_k}H_k\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} CA^j Q_{k-m_k+r_k-i}A^{iT} \\ & = (H_k + H_k^*)M_k(H_k + H_k^*)^T - H_k M_k H_k^{*T} \\ & - H_k^* M_k H_k^T - H_k^* M_k H_k^{*T} + A^{r_k}\bar{P}_{k-m_k+1}A^{r_k T} \\ & + \sum_{i=0}^{r_k-1} A^i Q_{k-m_k+r_k-i}A^{iT} \\ & - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1} CA^i\bar{P}_{k-m_k+1}A^{r_k}T \\ & - \frac{1}{r_k}A^{r_k}\bar{P}_{k-m_k+1}(H_k\sum_{i=0}^{r_k-1} CA^i)^T \\ & - \frac{1}{r_k}\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} A^i Q_{k-m_k+r_k-i}A^{jT}C^T H_k^T \\ & - \frac{1}{r_k}H_k\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} CA^j Q_{k-m_k+r_k-i}A^{iT} \end{aligned} \tag{25}$$

where M_k is (20), and
$H_k^* = -r_k(A^{r_k}\bar{P}_{k-m_k+1}(\sum_{i=0}^{r_k-1} CA^i)^T + \sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1} A^i Q_{k-m_k+r_k-i-1}A^{jT}C^T)M_k^{-1}$.
Then $H_k = -H_k^*$ makes $\bar{P}_{k-m_k+r_k+1}$ minimum, and the estimator gain (19) is obtained.
Substituting H_k back to (25), we get (21), with the initial condition is $P_0 = Ex_0x_0^T$.■

Remark 2: It is well-known that for a standard Kalman filter, the estimation error covariance P_{k+1} is a monotonic function of P_k. Thus we want to know whether this property extends to the proposed estimator. The answer turns out to be affirmative, as shown below.

From (25), we have

$$\begin{aligned}
&\bar{P}_{k-m_k+r_k+1} = E_x[e_{k-m_k+r_k+1}e^T_{k-m_k+r_k+1}] = \\
&(A^{r_k} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1}CA^i)\bar{P}_{k-m_k+1}(A^{r_k} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1}CA^i)^T \\
&+ \sum_{i=0}^{r_k-1}A^iQ_{k-m_k+r_k-i}A^{iT} + \frac{1}{{r_k}^2}H_k\sum_{i=0}^{r_k-1}R_{k-m_k+i+1}H_k^T \\
&+ \frac{1}{{r_k}^2}H_k\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1}CA^jQ_{k-m_k+i-j}A^{jT}C^TH_k^T \\
&- \frac{1}{r_k}\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1}A^iQ_{k-m_k+r_k-i}A^{jT}C^TH_k^T \\
&- \frac{1}{r_k}H_k\sum_{i=1}^{r_k-1}\sum_{j=0}^{i-1}CA^jQ_{k-m_k+r_k-i}A^{iT}
\end{aligned} \tag{26}$$

We denote the mapping (26) from $\bar{P}_{k-m_k+1}$ to $\bar{P}_{k-m_k+r_k+1}$ by $\mathcal{F}(\cdot) : S^n_+ \to S^n_+$, i.e.,

$$\bar{P}_{k-m_k+r_k+1} = \mathcal{F}(\bar{P}_{k-m_k+1}) \tag{27}$$

Lemma 1 $\mathcal{F}(\cdot)$ is a monotonic function, i.e., if $\bar{P}^{(1)}_{k-m_k+1} \geq \bar{P}^{(2)}_{k-m_k+1} > 0$, then

$$\mathcal{F}(\bar{P}^{(1)}_{k-m_k+1}) \geq \mathcal{F}(\bar{P}^{(2)}_{k-m_k+1}) \tag{28}$$

Proof: denote the mapping (26) from $\bar{P}_{k-m_k+1}$ and H_k to $\bar{P}_{k-m_k+r_k+1}$ by $G(\cdot,\cdot) : S^n_+ \times R^n \to S^n_+$, then since the solution H_k in (19) is obtained by minimizing (26), that it is

$$H_k = \arg\min_{\tilde{H}_k} G(\bar{P}_{k-m_k+1}, \tilde{H}_k) \tag{29}$$

with the suppose $\bar{P}^{(1)}_{k-m_k+1} \geq \bar{P}^{(2)}_{k-m_k+1}$, let $H^{(1)}_k$ and $H^{(2)}_k$ be the corresponding H_k as obtained in (19) by (29), then

$$\begin{aligned}
\bar{P}^{(2)}_{k-m_k+r_k+1} &= G(\bar{P}^{(2)}_{k-m_k+1}, H^{(2)}_k) \\
&\leq G(\bar{P}^{(2)}_{k-m_k+1}, H^{(1)}_k) \\
&\leq G(\bar{P}^{(1)}_{k-m_k+1}, H^{(1)}_k) \\
&= \bar{P}^{(1)}_{k-m_k+r_k+1}
\end{aligned} \tag{30}$$

Hence, the lemma holds.■

In the above, the two equalities follow from (29). The first inequality follows from (29) as well. The second inequality follows from (26), i.e., $G(\bar{P}_{k-m_k+1}, \tilde{H}_k)$ is linear in $\bar{P}_{k-m_k+1}$ when H_k is fixed.

Remark 3: The implication of Lemma 1 is that the structure for state estimator (4) is indeed an optimal choice for a linear estimator. That is, it is sufficient to consider a linear combination of $\hat{x}_{k-m_k+1}$ and $\tilde{y}_k$ instead of considering a linear combination of all the received measurement bursts.

Theorem 2 gives the optimal estimation of $x_{k-m_k+r_k+1}$ at time k. The next theorem extends this by providing an optimal estimate of x_{k+1} at time k. We will denote this estimate by $\hat{x}_{k+1|k}$.

Theorem 3 Considering the system (1)(2) with random time delay bounded with $N > 1$ and the initial m_1, the number of received packets r_k is given for all k. At time k, the observation measurements is modeled in (18), then the optimal estimator is given by:

• When $r_k = 0$, there is no need to update the state estimate and the error covariance. Therefore,

$$\hat{x}_{k+1|k} = A^{m_k}\hat{x}_{k-m_k+1} \tag{31}$$

• When $r_k > 0$, the update state estimation is obtained in Theorem 2, based on these, the estimator $\hat{x}_{k+1|k}$ is

$$\hat{x}_{k+1|k} = A^{m_k-r_k}\hat{x}_{k-m_k+r_k+1} \tag{32}$$

with

$$m_{k+1} = m_k - r_k + 1 \tag{33}$$

Proof: The (31) is obvious when there is no update measurement, and we get the estimation just by prediction. When $r_k > 0$, the most recent update estimation is $\hat{x}_{k+l_k-m_k+1}$ which has been given in Theorem 2. Since the measurements $\{y_{k-m_k+l_k+1}, y_{k-m_k+l_k+1}, \cdots, y_k\}$ are missing or without known order at time k, there is no measurement can be used to estimate x_{k+1}. Thus it can only use prediction for getting the $\hat{x}_{k+1|k}$ based on the last update estimation $\hat{x}_{k+r_k-m_k+1}$. The number of missing packets is $m_k - r_k$, therefore, the prediction state estimate is obtained in (32).■

IV. Simulation Example

In this section, we present a numerical example for the case of the maximum time delay $N = 1$.

Consider a system described in (1) and (2) with the following specifications:

$$A = \begin{bmatrix} 1.1 & -0.1 \\ 0.5 & 0.9 \end{bmatrix}, \ C = [1 \ \ 2]$$

and $R = 0.1$, $Q = 0.25I_2$, $P_0 = 0.25I_2$, where I_2 is the identity matrix.

We know that r_k is obtained according to the transition diagram in Fig. 1, and suppose the transition probabilities are as follows:

$$\begin{aligned}
p_{00} &= P(m(k+1) = 0|m(k) = 0) = 0.85; \\
p_{01} &= P(m(k+1) = 1|m(k) = 0) = 0.15; \\
p_{10} &= P(m(k+1) = 0|m(k) = 1) = 0.75; \\
p_{11} &= P(m(k+1) = 1|m(k) = 1) = 0.25
\end{aligned}$$

Then Fig. 3 shows the comparison of the trace of the error covariance for three scenarios:

method 1: standard Kalman filtering (without delays);

method 2: the proposed method in this paper;

method 3: when there receive two measurements, the estimator just use the newest measurement.

It can be seen from the simulation results that the proposed estimator in the paper has a better performance.

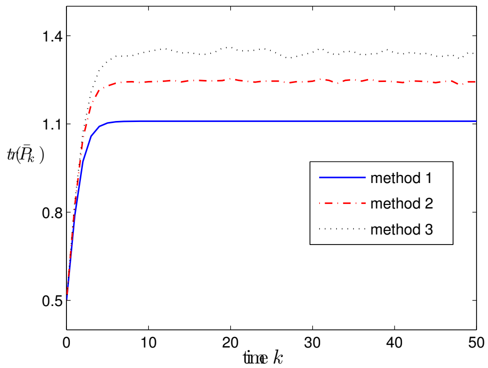

Fig. 3. Comparison of the trace of error covariance

V. Conclusion

In this paper, we have studied an optimal state estimation problem for the case when the measurements are subject to random network delays without time stamps. By assuming that the received measurements are in bursts and the bursts are not out of order, a new network model is deduced. This network model mimics many real-world network protocols. An optimal linear state estimator is derived with the properties that the state estimate is unbiased and that the state estimation error covariance is uniformly bounded and minimized. It turns out that this estimator essentially employs the averaged received measurements in each burst. The assumption that the received measurement bursts are not out of order is void in the case when the maximum time delay equals one because in this case the received measurement bursts are always in order.

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