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Robust Nonlinear Forwarding with Smooth State Feedback Control

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Abstract. In this paper, a robust forwarding control technique is introduced for a class of uncertain nonlinear systems admitting the so-called *up-augmented structure*. This technique gives a recursive design procedure which yields a smooth controller. Further, it can be combined with the well-known back-stepping technique to solve the robust stabilization problem for a large class of uncertain nonlinear systems. In contrast to previous methods on nonlinear forwarding, our technique is able to handle systems with large parameter uncertainties.

Keywords: Nonlinear systems; Forwarding; Robust stabilization; State feedback.

1 Introduction

In this paper, we consider the robust stabilization problem for a class of uncertain nonlinear systems admitting the so-called *up-augmented structure*. This structure starts with a *base system* of the following form:

$$\dot{x}(t) = f(x(t), q) + b(q)u(t) \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}$ is the control, $q \in \mathbf{R}^l$ is an uncertain parameter vector contained in a compact set Ω, $b(q)$ is continuous in q, $f(x, q)$ is continuous in q and smooth in x with $f(0, q) = 0$.

An *up-augmented system* is given by

$$\begin{aligned} \dot{x}_0 &= f_0(x, q) \\ \dot{x} &= f(x, q) + b(q)[u + d(x, x_0, \eta, q)] \end{aligned} \tag{2}$$

where $x_0 \in \mathbf{R}$ is a new state variable, t represents time, $f_0(x, q)$ is continuous in q and smooth in x with $f_0(0, q) = 0$, $d(x, x_0, \eta, q)$ is continuous in q and smooth in (x, x_0, η) with $d(0, 0, 0, q) = 0$. The parameter η represents state variables other than x_0 and x if (2) is a subsystem of a larger one, or void otherwise. We will denote $x^+ = [x_0, x^T]^T$ and $d(x^+, \eta, q) = d(x, x_0, \eta, q)$.

The stabilization problem for the up-augmented structure has been heavily studied recently for the case where the nonlinear system does not involve uncertain parameters; see e.g., [6], [1], [3] and [4]. However, these methods, called forwarding or feed-forwarding, all involve some kind of cancellation of nonlinearities. Subsequently, they do not apply to systems with uncertain parameters, especially of large sizes. This is in great contrast to robust stabilization of linear systems. In fact, a seminal paper of Wei [7] in 1990 proposed a method capable to deal with the up-augmented structure involving large uncertain parameters.

In a recent paper by the authors [5], Wei's robust stabilization technique was generalized to the nonlinear case. Our solution to the robust stabilization problem involves a two-step controller. In the first step, a controller is designed to drive the base system to a local region. Subsequently, a second controller is used to maintain the state of the base system small while driving the augmented state to zero. This robust forwarding technique is conceptually different from the standard forwarding ones as no nonlinear cancellation is involved. When specialized to the linear case, this technique recovers the result of Wei [7]. Further, a recursive application of this technique leads to a solution to robust stabilization of systems with the so-called *upper-triangular structure*; see [5].

However, our robust forwarding technique in [5] has a weakness which serves as the motivation for this paper. That is, the two-step controller is non-smooth. This implies that it is difficult to combine it with the well-known back-stepping technique for handling a larger class of uncertain nonlinear systems.

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The back-stepping technique deals with the so-called *down-augmented structure.* For the same base system (1), a down-augmented system is of the form:

$$\begin{array}{rcl}\dot{x} & = & f(x,q)+b(q)x_{n+1} \\ \dot{x}_{n+1} & = & \theta_{n+1}(q)[u+d(x,x_0,\eta,q)]\end{array} \tag{3}$$

where $x_{n+1} \in \mathbf{R}$ is a new state variable, $\theta_{n+1}(q)$ is a continuous function bounded away from zero, and $d(\cdot)$ is the same as before. It is well-known now that this structure can be easily handled using the standard back-stepping technique; see, e.g., [2]. A striking feature of the back-stepping technique is that it is capable to treat uncertain parameters of large sizes. A key assumption in the back-stepping technique is that the base system has a smooth stabilizer, say $u_n(x)$. This is because a state transformation

$$z = x; \quad z_{n+1} = x_{n+1} - u_n(x) \tag{4}$$

is used and the differentiability of the new state is required. It is worth to note that the back-stepping technique yields a smooth stabilizer, provided that the down-augmented system is smooth.

The need for a smooth stabilizer for the up-augmented structure is now clear: Such a stabilizer would allow the base system to be an up-augmented structure. That is, robust stabilization can be achieved for a much larger class of uncertain nonlinear systems generated using recursive up-augmentations and down-augmentations. This structure, when specialized to the linear case, corresponds to the so-called *anti-symmetric stepwise configuration*, coined by Wei [7].

With the above introduction, the objective of this paper is simply stated: We aim to propose a robust forwarding technique that would yield a smooth stabilizing controller for the up-augmented system in (2).

We point out that all the results allow the system to be time-varying. But for notational convenience, the time-dependence is suppressed.

2 Robust Forwarding with Non-smooth Control

In this section we revisit the robust forwarding technique in [5]. This technique does not yield a smooth controller, but will serves as the basis for a smooth controller later. As mentioned in the previous section, the key mechanism involved is a two-step control law. In the first step, a nonlinear controller is applied to the base system so that its state x converges to a "small" bounded set Ω while x_0 is not regulated. In the second step, a nonlinear controller is designed to maintain x within Ω while driving the augmented state x^+ to zero. Overall, this two-step control law achieves robust global asymptotic stabilization (RGAS) and robust local exponential stabilization (RLES).

Assumptions

Assumption 2.1 (*Local Exponential Stabilizability*): There exists a smooth controller $u_n(x)$ for the base system (1) such that, with

$$u(t) = u_n(x(t)), \tag{5}$$

the state of the system (1) is RLES.

Assumption 2.2 (*Local Smoothness Properties*): For the same local region Ω and local controller $u_0(x)$ as above, any $x \in \Omega$ and $q \in Q$, there holds

$$\begin{array}{l} f_0(x,q) = a(x,q)x; \\ f(x,q)+b(q)u_0(x) = A(x,q)x \\ b(q) = \theta_n(q)\begin{pmatrix}0\\1\end{pmatrix} = \theta_n(q)b \end{array} \tag{6}$$

where

$$\begin{array}{l} A(x,q) = \begin{bmatrix} 0 & A^-(x,q) \\ d_{n1}(x,q) & \star \end{bmatrix} \\ a(x,q) = [\theta_0(x,q) \quad \star] \end{array} \tag{7}$$

with $*$ representing an arbitrary term, $1 \geq \theta_0(x,q) \geq \underline{\theta}_0 > 0$; $1 \geq \theta_n(x,q) > \underline{\theta}_n > 0$, where $\underline{\theta}_0$ and $\underline{\theta}_n$ are constants. □

Remark 2.1 With the local smoothness assumption, the RLES property in Assumption 2.1 can be modified to the following: there exists $\mu > 0, \varepsilon > 0$ and matrix $P = P^T > 0$ such that

$$PA(x,q) + A^T(x,q)P \leq -\varepsilon I, \ \ \forall x \in \Omega, q \in Q. \tag{8}$$

where

$$\Omega = \{x: \ x^T P x < \mu\}. \tag{9}$$

Assumption 2.3 (*Global Properties*): Consider the following system derived from (2):

$$\dot{x} = f(x,q) + b(q)[u + d(x^+,\eta,q)] \tag{10}$$

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Given any smooth function $\eta(t)$ and $0 < \rho < 1$, there exists a smooth controller $u_d(x^+, \eta)$ such that, with

$$u(t) = u_d(x^+(t), \eta(t)), \qquad (11)$$

the state of the system (2) will be driven into $\rho\Omega$ in a finite time T, where Ω is given in (9), and $\rho\Omega = \{\rho x : x \in \Omega\}$.

Remark 2.2 Assumption 2.3 appears to be strong. However, we note that it is automatically satisfied for first order systems because a "high-gain" $u(t)$ can be designed to "overcome" both $f(x,q)$ and $d(x^+, \eta, q)$, forcing the state to converge to $\rho\Omega$. In Section 4, we will show that this property can be preserved under robust forwarding design. □

LYAPUNOV FUNCTION AND CONTROLLER DESIGN

Now we pay attention to controller design for (2). First, we utilize Assumption 2.3 and apply (11) to drive $x(t)$ into $\rho\Omega$ in a finite time T. In this step, $x_0(t)$ is not regulated. Once $x(t) \in \rho\Omega$, we switch to a local mode where a different controller $u^+(x^+, \eta)$ will be applied. This controller will maintain $x(t)$ in Ω while driving $x^+(t)$ to zero. The design of $u^+(x^+, \eta)$ relies on a local Lyapunov function for (2)

$$V^+(x^+) = (x_0 - (\gamma \ 0)Px)^2 + \int_0^{V(x)} s(w)dw > 0, \ \forall x \in \Omega \qquad (12)$$

where $\gamma < 0$ is a constant to be specified and $s(\cdot)$ is a locally smooth function satisfying: $s(w) > 0$ $\forall w \in [0, \mu)$;

$$\int_0^v s(w)dw < \infty, \ \forall v \in [0, \mu); \qquad (13)$$

and

$$\lim_{v \to \mu} \int_0^v s(w)dw \to \infty. \qquad (14)$$

Remark 2.3 Note that (12) includes a quite large set of the Lyapunov functions. A particular choice of $s(\cdot)$ is given by [5]:

$$s(w) = \frac{\mu}{\mu - w}.$$

For linear systems we can take $\mu = \infty$ and $s(w)$ a constant, which yields a quadratic Lyapunov function. It can be verified that this is the same Lyapunov function used in Wei [7]. In generally, these Lyapunov functions are non-quadratic. However, as $x \to 0$, $V^+(x^+)$ becomes quadratic in x^+ because $s(0) > 0$.

We also note that the function $\int_0^{V(x)} s(w)dw$ resembles a "potential barrier" and the Lyapunov function (12) is valid only for $x \in \Omega$, i.e.,

$$V^+(x^+) \to \infty \quad as \quad x^T P x \to \mu \qquad (15)$$

This implies that future $x \in \Omega$ as long as that $V^+(x^+)$ remains bounded.

For notational simplicity, we will denote $s(V(x))$ by $s(x)$. Defining

$$P^+ = \begin{bmatrix} 1 & -(\gamma \ 0)P \\ -P\begin{pmatrix} \gamma \\ 0 \end{pmatrix} & s(x)P + P\begin{pmatrix} \gamma \\ 0 \end{pmatrix}(\gamma \ 0)P \end{bmatrix} \qquad (16)$$

which is positive definite for all $x \in \Omega$. The inverse of P^+ is given by

$$S^+ = \begin{bmatrix} s(x) + (\gamma \ 0)P\begin{pmatrix} \gamma \\ 0 \end{pmatrix} & (\gamma \ 0) \\ \begin{pmatrix} \gamma \\ 0 \end{pmatrix} & P^{-1} \end{bmatrix}. \qquad (17)$$

Also define a (nonlinear) state transformation

$$z^+ = (z_0, \ z^T)^T = P^+ x^+. \qquad (18)$$

To simplify the analysis, we also assume in this section that η is void, i.e., $d(x^+, \eta, q) = d_1(x^+, q)$. Since $d_1(x^+, q)$ is smooth in x^+ and $d_1(0, q) = 0$, we can rewrite

$$d_1(x^+, q) = D^+(x^+, q)x^+ = D^+(x^+, q)S^+ z^+ \qquad (19)$$

for some $D^+(x^+, q)$ smooth in x^+ and continuous in q.

Differentiating $V^+(x^+(t))$ along the trajectory of (2), we have

$$\begin{aligned} \dot{V}^+ &= 2[x_0 - (\gamma \ 0)Px][\dot{x}_0 - (\gamma \ 0)P\dot{x}] \\ &\quad + 2s(V(x))x^T P\dot{x} \\ &= 2(x^+)^T P^+ \dot{x}^+ \end{aligned} \qquad (20)$$

Theorem 2.1 For the up-augmented system (2) satisfying Assumptions 2.1-2.3 and $d(x^+, \eta, q) = d_1(x^+, q)$, there exist $\gamma < 0$ and $\alpha(x^+) > 0$ such that the nonlinear controller

$$u(t) = u^+(x^+) = u_n(x) - \alpha(x^+)z^T b \qquad (21)$$

will render

$$\dot{V}^+(x^+) \le -\varepsilon^+(x)V^+(x^+), \quad \forall x \in \Omega, x_0 \in \mathbf{R} \qquad (22)$$

for some continuous $\varepsilon^+(x) > 0$, $x \in \Omega$.

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Moreover, the following choice of γ, $\alpha^+(x^+)$ and $\varepsilon^+(x)$ will suffice :

$$0 < \bar{\varepsilon} < \bar{\varepsilon}_{max} = \min_{q\in Q; x\in\Omega} \lambda_{min}\left[-P^{-1}\left(A^T(x,q)P + PA(x,q)\right)P^{-1}\right] \quad (23)$$

$$\gamma < \gamma_{max} = \min_{q\in Q;\ x\in\Omega} \frac{1}{2\theta_0(q)}\left[a(x,q)\left(A^T(x,q)P + PA(x,q) + \bar{\varepsilon}P^2\right)^{-1} a^T(x,q) - \bar{\varepsilon}\right] \quad (24)$$

$$\varepsilon^+(x) = \frac{\bar{\varepsilon}}{2}\lambda_{min}\left(s^{-1}(x)P^+(x)\right) > 0 \quad (25)$$

$$\alpha(x^+) = s^{-1}(x)\frac{\delta^2(x^+)}{\bar{\varepsilon}} \quad (26)$$

where $\delta(x^+)$ is any smooth function satisfying

$$\delta(x^+) \geq \max_{q\in Q}\left\| s(x)D(x^+,q)S^+(s) + \gamma\frac{d_{n1}(x,q)}{\theta_n(q)}\right\| \quad (27)$$

Proof : Using Assumptions 2.1-2.2, we obtain

$$\dot{V}^+ = (x^+)^T\left[P^+A^+(x,q) + A^+(x,q)^T P^+\right]x^+ + 2(x^+)^T P^+ b^+(q)[u + d_1(x^+,t,q)] \quad (28)$$

where $A^+(x,q)$ is defined as

$$A^+(x,q) = \begin{bmatrix} 0 & a(x,q) \\ 0 & A(x,q)\end{bmatrix}; \quad b^+(q) = \begin{bmatrix} 0 \\ b(q)\end{bmatrix}.$$

It is easy to verify that

$$A^+(x,q)S^+ + S^+A^+(x,q)^T = s^{-1}(x)\begin{bmatrix} 2a(x,q)\begin{pmatrix}\gamma\\0\end{pmatrix} & a(x,q)P^{-1} + (\gamma\ \ 0)A^T(x,q) \\ P^{-1}a^T(x,q) + A(x,q)\begin{pmatrix}\gamma\\0\end{pmatrix} & P^{-1}A^T(x,q) + A(x,q)P^{-1}\end{bmatrix}. \quad (29)$$

It follows from (28) and (29) that

$$\dot{V}^+ = s^{-1}(x)\,(z^+)^T\begin{bmatrix} 2\theta_0(x,q)\gamma & a(x,q)P^{-1} \\ P^{-1}a^T(x,q) & P^{-1}A^T(x,q) + A(x,q)P^{-1}\end{bmatrix}z^+ + 2s^{-1}(x)z^T A(x,q)\begin{pmatrix}\gamma\\0\end{pmatrix}z_0 + 2\,(z^+)^T b^+(q)[u + d_1(x^+,q)]. \quad (30)$$

The choice of $\bar{\varepsilon}$ and γ in (23) and (24) assures that, for $\forall q \in Q,\ \ x\in\Omega$,

$$\begin{bmatrix} 2\theta_0(x,q)\gamma & a(x,q)P^{-1} \\ P^{-1}a^T(x,q) & P^{-1}A^T(x,q) + A(x,q)P^{-1}\end{bmatrix} \leq -\bar{\varepsilon}I.$$

Also, with Assumption 2.2 and $(z^+)^T b^+ = z^T b$, we have

$$z^T A(x,q)\begin{pmatrix}\gamma\\0\end{pmatrix} = z^T\gamma d_{n1}(x,q)b = \gamma\frac{d_{n1}(x,q)}{\theta_n(q)}(z^+)^T b^+(q).$$

Therefore, from the above discussion and (30), we have

$$\dot{V}^+ \leq -s^{-1}(x)\bar{\varepsilon}\left(z^+\right)^T z^+ + 2s^{-1}(x)(z^+)^T b^+(q) \cdot \left[s(x)u + \gamma\frac{d_{n1}(x,q)}{\theta_n(q)}z_0 + s(x)d_1\right]. \quad (31)$$

Using (19) and (27), we have

$$\left\|\gamma\frac{d_{n1}(q)}{\theta_n(q)}z_0 + s(x)d_1(x^+,q)\right\| \leq \delta(x^+)\|z^+\|. \quad (32)$$

Then substituting (32), (26) and the controller (21) into (31) results in

$$\begin{aligned}\dot{V}^+ &\leq -s^{-1}(x)\frac{\bar{\varepsilon}}{2}(z^+)^T z^+ \\ &\leq -\frac{\bar{\varepsilon}}{2}\left(P^{+1/2}x^+\right)^T\left(s^{-1}(x)P^+\right)\left(P^{+1/2}x^+\right) \\ &\leq -\varepsilon^+(x^+)V(x^+) \end{aligned} \quad (33)$$

In particular, $\varepsilon^+(0) > 0$. Hence, we have RGAS and RLES for the closed-loop system (2). ∇∇∇

3 Robust Forwarding with Smooth Control

In this section, we show how to modify the non-smooth controller in the previous section so that it becomes a smooth one.

Let $\xi(\cdot) : \mathbf{R} \to \mathbf{R}$ be any monotonic smooth function satisfying the following properties:

$$\begin{aligned}\xi(v) &= 0,\ v \leq 0 \\ \xi(v) &= 1,\ v \geq 1\end{aligned} \quad (34)$$

Denote

$$F(v) = \int_0^v s(w)dw \quad (35)$$

Note that $F(v)$ is monotonic.

Then, define the new smooth stabilizing controller:

$$u(t) = \begin{cases} u_d(x^+,t) & t < T; \\ (1-\xi(\tau))u_d + \xi(\tau)u^+ & t \geq T\end{cases} \quad (36)$$

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where $T \geq 0$ is the first instant when $x \in \rho\Omega$ (see Assumption 2.3), and

$$\tau = \kappa_1 \left(F(V(x(t))) - F(V(x(T))) \right) + \kappa_2 (t - T) \tag{37}$$

with any $\kappa_i > 0$. The smoothness of this controller is due to the definition of $\xi(\cdot)$ and $\tau = 0$ at switching time $t = T$. The key feature of this new controller is that it becomes u^+ when either t or $V(x)$ becomes sufficiently large. Combining with the properties of u^+ in Theorem 2.1, this will assure that x remains Ω for $t \geq T$ and that $x^+ \to 0$ as $t \to \infty$. This result is summarized in the following theorem:

Theorem 3.1 For the up-augmented system (2) satisfying Assumptions 2.1-2.3 and $d(x^+, \eta, q) = d_1(x^+, q)$, the smooth controller in (36) will render the closed-loop system RGAS and RLES.

Proof: Follows from Theorem 2.1 and the argument above. ▽▽▽

4 Recursive Application of Robust Forwarding

The purpose of this section is to show that the robust nonlinear forwarding technique studied in the previous sections can be applied recursively. A simple motivation for this is the need to deal with the so-called *upper-triangular structure*:

$$\begin{array}{rcl} \dot{x}_1 & = & f_1(x_2, \cdots, x_n, q) \\ & \vdots & \\ \dot{x}_{n-1} & = & f_{n-1}(x_n, q) \\ \dot{x}_n & = & f_n(x_1, \cdots, x_n, q) + u \end{array} \tag{38}$$

where $x_1, \cdots, x_n$ are state variables, q is an uncertain parameter vector as before and f_i, $i = 1, \cdots, n$ are smooth in x and continuous in q.

But the significance of the recursive applicability goes beyond the upper-triangular structure. As we will point out in the next section, a much richer class of nonlinear systems can be robustly stabilized by combining robust nonlinear forwarding and backstepping.

Technically speaking, we need to show that the robust nonlinear forwarding technique used in the previous sections can preserve Assumptions 2.1-2.3 for the up-augmented structure. The results below apply to both smooth and non-smooth controllers.

First, we return to the general case where η is not void. Since $d(x^+, \eta, q) = 0$ when $(x^+, \eta) = 0$, the function $d(x^+, \eta, q)$ can be decomposed into two smooth terms as follows:

$$d(x^+, \eta, q) = d_1(x^+, q) + d^+(x^+, \eta, q)\eta \tag{39}$$

with $d_1(0, q) = 0$.

Consider the following control law

$$u(x^+, \eta) = u_1(x^+) + u_2(x^+, \eta) \tag{40}$$

where $u_1(x^+)$ is as in (36). Then, the system

$$\begin{array}{rcl} \dot{x}_0 & = & f_0(x, q) \\ \dot{x} & = & f(x, q) + b(q)[u_1(x^+) + d_1(x^+, q)] \end{array} \tag{41}$$

is RGAS and RLES.

We first justify Assumption 2.2. Note that $f_0(x, q)$, $f(x, q)$ and $b(q)[u_1(x^+) + d(x^+, q)]$ are locally smooth. It follows that

$$\begin{array}{rl} & \left[\begin{array}{c} f_0(x, q) \\ f(x, q) + b(q)[u_1(x^+) + d_1(x^+, q)] \end{array} \right] \\ = & A_c^+(x^+, q) x^+ \end{array} \tag{42}$$

where

$$A_c^+(x^+, q) = \left[\begin{array}{cc} 0 & a^+(x, q) \\ \left(\begin{array}{c} 0 \\ \star \end{array} \right) & A_c(x^+, q) \end{array} \right].$$

Hence , the system (41) satisfies Assumption 2.2.

For Assumptions 2.1 and 2.3, we have the following two theorems, respectively:

Theorem 4.1 For the system (41), take $s_0 > s(0) > 0$ and let

$$P_0^+ = \left[\begin{array}{cc} 1 & -\left(\gamma \;\; 0\right) P \\ -P \left(\begin{array}{c} \gamma \\ 0 \end{array} \right) & s_0 P + P \left(\begin{array}{c} \gamma \\ 0 \end{array} \right) \left(\gamma \;\; 0\right) P \end{array} \right] \tag{43}$$

Then, there exists $\mu^+ > 0$ such that

$$P_0^+ A_c^+(x^+, q) + \left(A_c^+(x^+, q) \right)^T P_0^+ \leq -\varepsilon_0^+ I \tag{44}$$

for all $x^+ \in \Omega^+ = \{x^+ : (x^+)^T P_0^+ x^+ < \mu^+\}$.

Proof: This follows from the RLES property of the system. The details are omitted.

Theorem 4.2 Suppose the up-augmented system (2) satisfies Assumptions 2.1-2.3. For any given $\beta > 0$, the controller (40) with

$$u_2(x^+, \eta) = -\frac{1}{\beta s(x)} (1 + \eta^2) z^T b \delta_2^2 (x^+, \eta) \tag{45}$$

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will locally render

$$\dot{V}^+(x^+) \leq -\varepsilon^+ V^+(x^+) + \beta \quad (46)$$

where $\delta_2(x^+, \eta)$ is a smooth function satisfying

$$\delta_2(x^+, \eta) \geq \max_{q \in Q} |d_2(x^+, \eta, q)|. \quad (47)$$

and $\varepsilon^+ = \varepsilon_0^+/2$ with ε_0^+ given in Theorem 4.1.

Proof: Modified from Theorem 2.1. Details are omitted.

5 Concluding Remarks

In this paper, we have proposed a new design technique, robust nonlinear forwarding, for robust stabilization of nonlinear systems with an up-augmented structure. This technique allows us to deal with nonlinear systems with large parameter uncertainties, can provide either smooth or non-smooth (but simpler) stabilizing controllers.

The importance of smooth controllers is that the robust nonlinear forwarding technique can be combined with the back-stepping technique to produce a rich class of uncertain nonlinear systems which can be robustly stabilized. Roughly speaking, the structure of such a system is mainly characterized by its locally linearized version and admits the so-called *anti-symmetric stepwise configuration* (ASSC) which has been studied by Wei in [7] for uncertain linear systems.

To explain the ASSC, we consider the following system

$$\dot{x} = f(x, q) + b(q)u \quad (48)$$

where $f(x, q)$ is smooth in x and continuous in q, $b(q)$ is continuous in q, and $q \in Q$ is an uncertain parameter vector as before. Define

$$M(x, q) = \left[\frac{\partial f(x, q)}{\partial x} \quad b(q) \right] \quad (49)$$

and adopt the following convention:

- $*$ = any scalar function of x and q with a known bound over $\Omega \times Q$;
- θ = any scalar function of x and q with $1 \geq |\theta| \geq \underline{\theta} > 0$ over $\Omega \times Q$.

Then, examples of ASSC are given as follows:

$$\begin{bmatrix} 0 & \theta & * & 0 \\ 0 & 0 & \theta & 0 \\ * & * & * & \theta \end{bmatrix}$$

$$\begin{bmatrix} 0 & \theta & * & 0 & 0 \\ 0 & 0 & \theta & 0 & 0 \\ * & * & * & \theta & 0 \\ * & * & * & * & \theta \end{bmatrix}$$

$$\begin{bmatrix} 0 & \theta & * & * & 0 \\ 0 & 0 & \theta & 0 & 0 \\ 0 & 0 & * & \theta & 0 \\ * & * & * & * & \theta \end{bmatrix}$$

These examples are all generated via a sequence of up- and down-augmentations. For example, the first example is generated via an up-augmentation from the lower-right 2×3 structure. A precise definition of the ASSC can be found in Wei [7] with the exception that the matrix $M(x, q)$ in [7] is independent of x.

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