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Brief paper

Stability of MMSE state estimators over lossy networks using linear coding[☆]

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ABSTRACT

This paper studies the state estimation problem for a stochastic discrete-time system over a lossy channel where the packet loss is modeled as an independent and identically distributed binary process. To counter the effect of random packet loss, we propose a linear coding method to preprocess the measured output, and prove that the coded output is information preserving when packet loss is void and is information enhancing when packet loss is present. An optimal state estimator under the minimum mean square error (MMSE) criterion is derived for the coded output when subject to packet loss. The maximum packet loss rate for ensuring a stable estimator is then derived and shown to be very close to a well-known lower bound. Also considered is a compressed linear coding method where the measured output is first compressed onto a lower dimensional space before encoding, and it is shown that the similar packet rate condition for stability holds.

1. Introduction

This work is concerned with a networked state estimation problem where the sensor and the estimator are connected through a digital network with packet loss. The packet loss process of the network is modeled by an independent and identically distributed (i.i.d.) binary process. If the packet loss rate is above a certain threshold, it is known that the optimal intermittent Kalman filter (IKF) can be unstable, i.e., its estimation error can be unbounded (Sinopoli et al., 2004). In this paper, we ask whether it is possible to obtain a weaker condition on the packet loss rate for a stable state estimator by preprocessing the raw output measurement. We provide an affirmative answer by applying a simple linear coding method and developing the corresponding MMSE state estimator.

With the rapid development of sensing, signal processing, and communication technologies, networked systems have been widely used in many important areas such as monitoring, detection and tracking. One of the key issues is to estimate the state of the networked system over an unreliable communication network, which has received considerable interest in the recent years (Hespanha, Naghshtabrizi, & Xu, 2007; Sinopoli et al., 2004). By modeling the packet loss process as an i.i.d. binary process, Sinopoli et al. (2004) proved the optimality of the IKF and the existence of a critical packet loss rate under which the IKF is guaranteed to be stable. This seminal result raises a fundamental problem of quantifying the critical loss rate. A lot of efforts have been devoted towards finding this critical rate (Huang & Dey, 2006; Mo & Sinopoli, 2012; Plarre & Bullo, 2009; Sinopoli et al., 2004; You, Fu, & Xie, 2011).

In Sinopoli et al. (2004), both upper and lower bounds for the critical packet loss rate are provided. The lower bound is simply characterized by the largest magnitude of the unstable eigenvalues of the system, and is shown to be tight for several cases: (1) The observation matrix is invertible in the observable subspace (Plarre & Bullo, 2009); (2) All the system eigenvalues have

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distinct magnitudes (Mo & Sinopoli, 2008); (3) The system is non-degenerate (Mo & Sinopoli, 2010). However, it was shown in You et al. (2011) that the lower bound is not always tight, and even for a class of second-order systems, the critical rate strictly lies between the lower and upper bounds. Exact characterization of the critical rate for the general case is complicated and can be found in Rohr, Marelli, and Fu (2014).

In Schenato (2008), it was shown that by transmitting the output of the Kalman filter instead of transmitting the measured output directly, the critical packet loss rate equals to the aforementioned lower bound. However, the dimension of the estimated state is usually much higher than that of the measured output, which intuitively requires more communication resources on the sensor side. Other disadvantages include the need for state estimation at the transmitter side and the difficulty to generalize the method when the measurements are not collated at one transmitter. An alternative is to send a finite linear combination of outputs. This idea was explored by Robinson and Kumar (2007) for scalar systems. He, Han, Wang, and Shi (2013) studied linear minimum mean square error (MMSE) estimation and smoothing using the statistics of the packet loss process. Note that they do not study the stability of the derived estimators. Differently, there are other complex coding methods for quantized stabilization problems where the sensor transmits the quantized state (or state estimate) in Yüksel (2009), You and Xie (2011), Minero, Coviello, and Franceschetti (2013).

In this paper, we propose a new coding method which constructs a finite-length linear combination of the measured output, which is then transmitted to the remote estimator via the lossy network. Since the dimension of the coded output is not greater than the measured output, no additional communication load will be induced. The coded output has two important features. Firstly, it is *information preserving*, when the transmission is perfect, in the sense that the raw output can be perfectly reconstructed from the coded output. Secondly, it is *information enhancing*, when packet loss exists, in the sense that a higher packet loss rate can be tolerated to ensure a stable state estimator. More specifically, we show that the maximum packet loss rate for ensuring a stable state estimator can be made very close to the aforementioned lower bound.

To study the information preserving and enhancing properties of the coded output, we introduce the notion of *strong observability*. Roughly speaking, the standard observability condition requires an uninterrupted sequence of measurements to achieve stable estimation, whereas the strong observability condition allows the measurements to be intermittent. We will show that the role of our coding method is to enhance an observability condition by turning it into a strong observability condition, allowing us to effectively combat packet loss.

Using the coded output, we develop an MMSE state estimator by augmenting the state dimension. The maximum packet loss rate for ensuring a stable MMSE state estimator is shown to be very close to the aforementioned lower bound. To further reduce the dimension of the coded output, we use a *compressed encoding* method which first compresses the measured output onto a lower dimensional subspace before applying the linear coding method. This allows us to transmit a lower-dimensional (or even scalar) coded output with a similar packet loss condition for a stable state estimator.

The rest of the paper is organized as follows. The estimation problem is formulated in Section 2. Section 3 introduces the linear coding method and gives the MMSE estimator using the intermittent coded output. Section 4 analyzes the stability condition for the estimator. Section 5 generalizes the coding method by reducing the dimension of the coded output. Simulation and concluding remarks are drawn in Sections 6 and 7, respectively. Many proofs are in the Appendix.

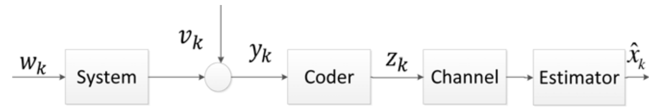

Fig. 1. Networked estimation with output coded.

2. Problem formulation

Consider the following discrete-time stochastic system

$$\begin{aligned} x_{k+1} &= Ax_k + w_k, \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{q\times n}$, $x_k \in \mathbb{R}^n$ denotes the state and $y_k \in \mathbb{R}^q$ denotes the measured output, at time k. The initial state x_0 is a Gaussian random vector with mean $\bar{x}_0$ and covariance matrix P_0. Both w_k and v_k are white Gaussian noises with zero mean and positive definite covariance matrices Q and R, respectively. In addition, x_0, w_k and v_k are mutually independent.

We are concerned with a networked state estimation problem where the sensor (which measures y_k) and the estimator are linked via a lossy network. Due to the channel unreliability, packets from the sensor to the estimator may be lost and the loss information is assumed to be known by the estimator. We use a binary random process γ_k to model this process. More precisely, $\gamma_k = 1$ indicates that the packet transmitted from the sensor is successfully delivered to the estimator at time k, whereas $\gamma_k = 0$ means that the packet is lost. We assume that the packet loss process is an i.i.d. process with packet receiving rate $p = \text{Prob}(\gamma_k = 1) = \mathbb{E}[\gamma_k]$. As opposite to Mo and Sinopoli (2012); Sinopoli et al. (2004); You et al. (2011), we study the networked estimation configuration depicted in Fig. 1, where the signal z_k transmitted to the estimator is a coded version of y_k.

Since we focus on the state estimation problem and its stability property, there is no loss of generality to make the following assumption.

Assumption 1. The matrix A is unstable and invertible, and (A, C) is observable.

Remark 1. We assume the instability of the system because for stable systems, the estimation error covariance would be bounded even all the packets from sensor get lost, which would make the stability analysis for the state estimator meaningless. In practice, however, even if the system is stable, we may want to impose some stability margin for the state estimator. One way to achieve this is to simply consider the related system obtained by replacing matrix A with the scaled matrix αA for some $\alpha > 1$. This scaled system may no longer be stable, and would therefore satisfy Assumption 1. The parameter α represents the stability margin, in the sense that if the state estimator for the scaled system is stable, then the state estimator for the original system is stable with a positive exponential decay rate.

For any matrices X and Y with compatible dimensions, denote $\text{col}\{X, Y\} = [X^T, Y^T]^T$. Let

$$m = n - \text{rank}(C) + 1.$$

Then, the matrix $\text{col}\{C, CA, \ldots, CA^{m-1}\}$ has full column rank.

We focus on a class of encoders with finite memory size, whose output can be computed recursively. More precisely, $z_k = \mathcal{E}_k(y_k, y_{k-1}, \ldots, y_{k-m+1})$, where the map $\mathcal{E}_k(\cdot)$ denotes the encoder at time k, and the memory size is m. Since z_k is to be transmitted to the estimator through an unreliable channel, the maximum information available to the estimator at time k is given by (Anderson & Moore, 1979)

$$\mathcal{F}_k = \{(\gamma_i, z_i\gamma_i) : i = 0, 1, \ldots, k\}.$$

The MMSE predictor and estimator of x_k are given by

$$\hat{x}_{k|k-1} = \mathbb{E}[x_k|\mathcal{F}_{k-1}], \quad \text{and} \quad \hat{x}_{k|k} = \mathbb{E}[x_k|\mathcal{F}_k], \tag{2}$$

respectively. Their error covariance matrices are

$$P_{k|k-1} = \mathbb{E}[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^T|\mathcal{F}_{k-1}];$$

$$P_{k|k} = \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T|\mathcal{F}_k], \tag{3}$$

respectively.

Our objective is to recursively compute the above quantities, and derive the network condition for the stability of the filter, i.e.,

$$\sup_{k\in\mathbb{N}} \mathbb{E}[\|x_k - \hat{x}_{k|k}\|^2] < \infty, \tag{4}$$

where the expectation is taken with respect to the packet loss processes $\{\gamma_k\}$.

In general, the higher the dimension of the coded output z_k, the larger the required communication resources. For this reason, our goal is to design a coder to reduce the effect of packet loss with the dimension of its output no larger than that of the measured output. In the next section, we give a coding method which keeps z_k of the same dimension as that of y_k. A compressed coding method, where the dimension of z_k is reduced, will be given in Section 5.

3. Linear coding and state estimation

3.1. Linear coding

If the measured output is directly transmitted, i.e., $z_k = y_k$, the MMSE estimator is computed by the IKF (Sinopoli et al., 2004). This is given as

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + \gamma_k A K_k (y_k - C\hat{x}_{k|k-1}); \tag{5}$$

$$P_{k+1|k} = AP_{k|k-1}A^T - \gamma_k A K_k C P_{k|k-1} A^T + Q, \tag{6}$$

with the initial values $P_{0|-1} = P_0$ and $\hat{x}_{0|-1} = x_0$, where $K_k = P_{k|k-1}C^T(CP_{k|k-1}C^T + R)^{-1}$ is the Kalman gain.

As discussed in Section 1, the critical packet loss rate for ensuring a stable IKF depends on the system structure in a complicated way and is in general greater than the known lower bound (Rohr et al., 2014; You et al., 2011). This motivates the idea of constructing the output of the Kalman filter at the sensor end and transmitting it to the estimator (Schenato, 2008), i.e. transmitting $z_k = \mathbb{E}[x_k|y_1, \ldots, y_k]$. Then, the MMSE estimator is given by

$$\hat{x}_{k|k} = \begin{cases} z_k, & \text{if } \gamma_k = 1; \\ A\hat{x}_{k-1|k-1}, & \text{if } \gamma_k = 0. \end{cases} \tag{7}$$

The necessary and sufficient condition for the stability of the above estimator is simply given by

$$|\lambda_{\max}|^2(1-p) < 1, \tag{8}$$

where $\lambda_{\max}$ is a maximum eigenvalue of A in magnitude. That is, the critical packet loss rate $1-p$ equals $|\lambda_{\max}|^{-2}$, which is the well-known lower bound (Sinopoli et al., 2004). However, the dimension of the state estimate is generally much higher than that of y_k.

Our proposed linear coding method is as follows. Take $\alpha_k^T = [\alpha_{k1}, \ldots, \alpha_{k(m-1)}, 1] \in \mathbb{R}^{1\times m}$ (Recall $m = n - \text{rank}(C) + 1$). The coded output is given by

$$\begin{aligned} z_k &= y_k + \alpha_{k(m-1)}y_{k-1} + \cdots + \alpha_{k1}y_{k-m+1} \\ &= (\alpha_k^T \otimes I_q)\text{col}\{y_{k-m+1}, \ldots, y_k\} \in \mathbb{R}^q \end{aligned} \tag{9}$$

with the convention that $y_k = 0$ for $k < 0$, where $I_q \in \mathbb{R}^{q\times q}$ is the identity matrix and $\otimes$ is the Kronecker product (Horn & Johnson, 1985). The design of $\{\alpha_k : k \in \mathbb{N}\}$ will be detailed later.

It is clear from (9) that the sequence $\{y_0, y_1, \ldots, y_k\}$ can be uniquely recovered from the sequence $\{z_0, z_1, \ldots, z_k\}$ for any $k \geq 0$. For this reason, the coded output is information preserving when there is no packet loss.

3.2. The MMSE estimator

From (9), the noise in z_k is correlated with those in $z_{k-1}, z_{k-2}, \ldots, z_0$. Hence, we cannot obtain an MMSE estimator by simply running a Kalman filter with the system's output z_k. To get around this, we define $\mu_k = \text{col}\{y_{k-m+1}, y_{k-m+2}, \ldots, y_{k-1}\}$ and obtain

$$\mu_{k+1} = F\mu_k + Gy_k, \tag{10}$$

where $G = \text{col}\{0, 0, \ldots, 0, I_q\}$ and $F = \begin{bmatrix} 0 & I_{(m-1)q} \\ 0 & 0 \end{bmatrix}$. Define the augmented state $u_k = [x_k^T \ \ \mu_k^T]^T$. With the notation $H_k = [C \ \alpha_{k1}I_q \ \ldots \ \alpha_{k(m-1)}I_q]$, (1) and (9) are rewritten as the following augmented system

$$\begin{aligned} u_{k+1} &= \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix} u_k + \begin{bmatrix} w_k \\ Gv_k \end{bmatrix}, \\ z_k &= H_k u_k + v_k. \end{aligned} \tag{11}$$

Clearly, the noise components in (11) are temporally independent. Hence, we can obtain an MMSE estimator of u_{k+1} via a Kalman filter (Anderson & Moore, 1979) by

$$\begin{aligned} \hat{u}_{k+1|k} &= \Phi\hat{u}_{k|k-1} + \gamma_k(\Phi\Sigma_{k|k-1}H_k^T + S_k) \\ &\quad \times (H_k\Sigma_{k|k-1}H_k^T + R)^{-1}(z_k - H_k\hat{u}_{k|k-1}), \end{aligned} \tag{12}$$

$$\begin{aligned} \Sigma_{k+1|k} &= \Phi\Sigma_{k|k-1}\Phi^T + \bar{Q} - \gamma_k(\Phi\Sigma_{k|k-1}H_k^T + S_k) \\ &\quad \times (H_k\Sigma_{k|k-1}H_k^T + R)^{-1}(\Phi\Sigma_{k|k-1}H_k^T + S_k)^T \end{aligned} \tag{13}$$

with $\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & GRG^T \end{bmatrix}$, $\Phi = \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix}$ and $S_k = \begin{bmatrix} 0 \\ GR \end{bmatrix}$.

Remark 2. With the coding scheme (9), the dimension of the augmented state in the MMSE estimator is $n + (m-1)q$. On the other hand, as detailed in Remark 12, the compressed coding schemes in Section 5 permit reducing the dimension of augmented state to $2n - 1$.

If there is no packet loss, our next result shows that the MMSE estimator (12) using $\{z_k\}$, and the IKF using $\{y_k\}$, are equivalent.

Theorem 3. *Suppose packet loss is not present. Then, for any coding vectors $\{\alpha_k : k \in \mathbb{N}\}$, we have $\hat{x}_{k+1|k} = [I_n \ \ 0]\hat{u}_{k+1|k}$, where $\hat{x}_{k+1|k}$ is given by (2) and $\hat{u}_{k+1|k}$ is given by (12).*

Proof. Note from (12) and Theorem 3.2 of Anderson and Moore (1979) that $\hat{u}_{k+1|k} = \mathbb{E}[u_{k+1}|z_0, z_1, \ldots, z_k]$. Similarly, (2) implies $\hat{x}_{k+1|k} = \mathbb{E}[x_{k+1}|y_0, y_1, \ldots, y_k]$. Following the fact that the sequences $\{y_0, y_1, \ldots, y_k\}$ and $\{z_0, z_1, \ldots, z_k\}$ are equivalent (information preserving), we have

$$\mathbb{E}[u_{k+1}|z_0, z_1, \ldots, z_k] = \mathbb{E}[u_{k+1}|y_0, y_1, \ldots, y_k].$$

Multiplying its both sides by $[I_n \ \ 0]$, the right-hand side becomes

$$[I_n \ \ 0]\mathbb{E}[u_{k+1}|y_0, y_1, \ldots, y_k] = \mathbb{E}[x_{k+1}|y_0, y_1, \ldots, y_k].$$

It follows that $[I_n \ \ 0]\hat{u}_{k+1|k} = \hat{x}_{k+1|k}$. ■

Although both estimators (12) and (2) yield the same estimate in the absence of packet losses, we will show in Section 4 that, in the presence of packet losses, the coded output is information enhancing, in the sense that (12) permits a larger critical packet loss rate for stability.

4. Stability analysis

In this section we study the stability of the MMSE estimator in (12) when the coded output is transmitted to the estimator over a lossy channel. To this end, we introduce the notion of *strong observability*, and show that the coded output is strongly observable. Using this we then derive the stability condition.

Consider the following discrete-time system

$$x_{k+1} = Ax_k + w_k, \quad y_k = C_k x_k + v_k, \tag{14}$$

which is similar to (1), with the difference in that C_k is allowed to be time-varying. Nevertheless, $m = n - \text{rank}(C_k) + 1$ is constant.

Definition 1. For any $\tau \geq m$, the system (14), or the pair $(A, \{C_k : k \in \mathbb{N}\})$, is said to be strongly observable with period τ, if for any $1 \leq i_1 < i_2 < \cdots < i_{m-1} < \tau$ and $k \geq \tau - 1$, the following regression matrix

$$O(k, k - i_1, \ldots, k - i_{m-1}) = \text{col}\{C_k, C_{k-i_1}A^{-i_1}, \ldots, C_{k-i_{m-1}}A^{-i_{m-1}}\} \tag{15}$$

has full column rank.

Remark 4. If a pair (A, C) is observable, then (A, C) is strongly observable with period $\tau = m$. However, (A, C) may not be strongly observable with period $\tau > m$. An example of this case is given by

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \qquad C = [1 \ \ 1], \tag{16}$$

for which the observability index is $m = 2$ and (A, C) is observable. However, (A, C) is not strongly observable with period $\tau > 2$, because $\text{col}\{C, CA^{-2}\} = col\{C, 0.25C\}$ does not have full column rank.

Denote

$$\mathcal{C} = \text{col}\{CA^{m-1}, CA^{m-2}, \ldots, C\} \quad \text{and} \quad C_k = (\alpha_k^T \otimes I_q)\mathcal{C}.$$

The following lemma states that the periodically coded output turns an observable system into a strongly observable one. Its proof is given in Appendix A.1.

Lemma 5. *Consider the system (1), together with Assumption 1 and the coding scheme (9). If the coding vectors $\{\alpha_k : k \in \mathbb{N}\}$ are periodic with period $\tau \geq m$ (i.e., $\alpha_k = \alpha_{k+\tau}$), and $\alpha_0, \alpha_1, \ldots, \alpha_{\tau-1}$ are randomly drawn from an absolutely continuous probability distribution,[2] then, with probability one,[3] $(A, \{C_k : k \in \mathbb{N}\})$ is strongly observable with period τ.*

We are now ready to state the main result of this paper.

Theorem 6. *Consider the system (1), together with Assumption 1 and the coding scheme (9). For any $\tau \geq m$, suppose that the coding vectors $\{\alpha_k : k \in \mathbb{N}\}$ are periodic with period τ, and that $\alpha_0, \alpha_1, \ldots, \alpha_{\tau-1}$ are randomly drawn from an absolutely continuous probability distribution. Then, the MMSE estimator (12) is stable with probability one if*

$$|\lambda_{\max}|^2(1-p)(P(\tau, m))^{1/\tau} < 1, \tag{17}$$

where

$$P(\tau, m) = \sum_{i=0}^{m-1} \binom{\tau}{i} \left(\frac{p}{1-p}\right)^i \geq 1,$$

and $\binom{\tau}{i}$ is the binomial coefficient for choosing i from τ.

Proof. See Appendix A.2. ■

Remark 7. Notice that $(P(\tau, m))^{1/\tau} \to 1$ as $\tau \to \infty$. This implies that the gap between the sufficient condition (17) and the necessary condition (8) vanishes as $\tau \to \infty$. Fig. 2 shows this convergence for system (16).

[2] Loosely speaking, this means that the probability density function does not contain impulses.

[3] The probability of having a system which is not strongly observable is zero.

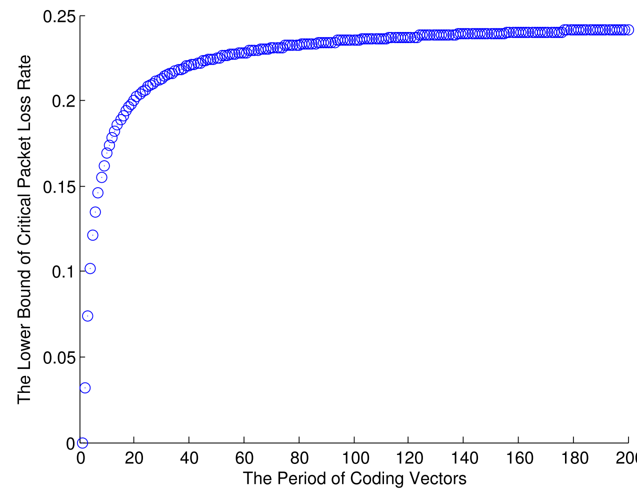

Fig. 2. The lower bound of critical packet loss rate.

Example 1. We now illustrate the advantage of coding in terms of stability using the system (16). From You et al. (2011), the IKF (i.e., without coding) for (16) is stable if and only if

$$|\lambda_{\max}|^4(1-p) < 1. \tag{18}$$

Clearly, it follows from Theorem 6 that we can always choose a period τ such that the stability condition resulting from coding is strictly weaker than (18).

5. Compressed coding

The coding method in the previous section requires the coded output to be of the same dimension as that of the raw measurement. In this section we show how to reduce the dimension of the coded output. We also develop a similar packet loss condition for the existence of a stable state estimator. We study two approaches.

5.1. First approach

Let $\mathcal{M}_A$ be the maximum geometric multiplicity of every eigenvalue of A. Since the number of Jordan blocks associated with a given eigenvalue equals its geometric multiplicity (Horn & Johnson, 1985), it follows from the Jordan form of an observable system (Chen, 1984) that $\mathcal{M}_A \leq \text{rank}(C)$. Select any integer ℓ with $\mathcal{M}_A \leq \ell \leq \text{rank}(C)$, and consider the coded output

$$z_k = (\alpha_k^T \otimes I_\ell)\text{col}\{\Lambda y_{k-n+\ell}, \ldots, \Lambda y_k\} \in \mathbb{R}^\ell, \tag{19}$$

where $\Lambda \in \mathbb{R}^{\ell \times q}$ is a constant matrix and $\alpha_k^T \in \mathbb{R}^{1\times(n-\ell+1)}$ is the coding vector at time k. We have the following result whose proof is given in Appendix A.3.

Lemma 8. *Under Assumption 1 and $\mathcal{M}_A \leq \ell \leq rank(C)$. Let $\Lambda \in \mathbb{R}^{\ell\times q}$ randomly drawn from an absolutely continuous probability distribution. Then, $(A, \Lambda C)$ is observable and ΛC has full row rank with probability one.*

Moreover, the MMSE estimator for the coding scheme (19) is straightforwardly obtained by replacing μ_k in (10) with $\mu_k = \text{col}\{\Lambda y_{k-m+1}, \Lambda y_{k-m+2}, \ldots, \Lambda y_{k-1}\}$. Then, we obtain the following corollary of Theorem 6.

Corollary 9. *Consider the system (1) satisfying Assumption 1, and the coding scheme (19), with $\mathcal{M}_A \leq \ell \leq rank(C)$, where $\Lambda \in \mathbb{R}^{\ell\times q}$ are randomly drawn from an absolutely continuous probability distribution. For any $\tau \geq n - \ell + 1$, suppose that $\{\alpha_k^T : k \in \mathbb{N}\}$*

are periodic with period τ, and that $\alpha_0, \alpha_1, \ldots, \alpha_{\tau-1}$ are randomly drawn from an absolutely continuous probability distribution. Then, the corresponding MMSE estimator is stable with probability one if

$$|\lambda_{\max}|^2(1-p)(P(\tau, n-\ell+1))^{1/\tau} < 1. \tag{20}$$

Proof. In view of Lemma 8, $(A, \Lambda C)$ is observable and ΛC has full row rank with probability one. Note that $\text{rank}(\Lambda C) = \ell$. Then, the stability condition follows from Theorem 6.

Remark 10. Notice that $P(\tau, n-\ell+1)$ increases as ℓ decreases (see its definition in Theorem 6). Corollary 9 states that the larger the dimension of the coder output, the larger the critical packet loss rate will be. This reveals a tradeoff between the state estimator stability and the communication load.

5.2. Second approach

Using the above compressed coding method, the dimension of the coded output can be reduced to $\mathcal{M}_A$. This is done by first compressing the measured output, while maintaining observability, and then applying the proposed linear coding. In this subsection we give an alternative method, which permits reducing the dimension of the coded output to a scalar by directly coding each dimension of measured output.

Taking a coding vector $\Lambda_k \in \mathbb{R}^{nq}$, we define a scalar coded output

$$z_k = \Lambda_k^T \text{col}\{y_{k-n+1}, y_{k-n+2}, \ldots, y_k\} \in \mathbb{R}. \tag{21}$$

Let $\widetilde{\mathcal{C}} := \text{col}\{CA^{-n+1}, CA^{-n+2}, \ldots, C\}$. We have the following lemma on the strong observability of the system resulting from the coding scheme (21).

Lemma 11. *Consider the system (1) with the coding scheme (21) and let $\tau \geq m$ be any given integer. Under Assumption 1, suppose that $\{\Lambda_k^T : k \in \mathbb{N}\}$ are periodic with period τ and $\Lambda_0, \Lambda_1, \ldots, \Lambda_{\tau-1}$ are randomly drawn from an absolutely continuous probability distribution. With probability one, $(A, \{\Lambda_k^T\widetilde{\mathcal{C}} : k \in \mathbb{N}\})$ is strongly observable with period τ.*

Proof. See Appendix A.4.

Note that the MMSE estimator for the coding scheme (21) is straightforwardly obtained by replacing μ_k in (10) with $\mu_k = \text{col}\{\Lambda_{k1}y_{k-n+1}, \ldots, \Lambda_{k(n-1)}y_{k-1}\}$, where $\Lambda_{ki} \in \mathbb{R}^q$, $i = 1, 2, \ldots, n$ and $\Lambda_k = \begin{bmatrix}\Lambda_{k1} & \Lambda_{k2} & \cdots & \Lambda_{kn}\end{bmatrix}$.

Remark 12. The dimension of augmented state u_k in MMSE estimator (12) under coding schemes (9), (19) and (21) are $n+(m-1)q$, $n+(m-1)\ell$ and $2n-1$, respectively. This means that the compressed coding can reduce the dimension of augmented state in MMSE estimator, which also reduce the computation burden.

Then, the stability condition of the corresponding MMSE state estimator follows directly from Theorem 6.

Corollary 13. *For the same setting as the one in Lemma 11, the corresponding MMSE state estimator is stable with probability one if*

$$|\lambda_{\max}|^2(1-p)(P(\tau, n))^{1/\tau} < 1.$$

Remark 14. The dimension of the output of coder (21) is lower than that of coder (19). However, its stability requirement is stronger. Also, both coding schemes require storing τ coding vectors. This means that coder (21) needs to store $nq\tau$ scalars, while coder (19) only needs $[(n-\ell+1)\tau + q\ell]$, which is far less. Hence, the choice between these coding schemes permits accommodating a tradeoff between communication load on one side, and memory capacity and stability requirement on the other.

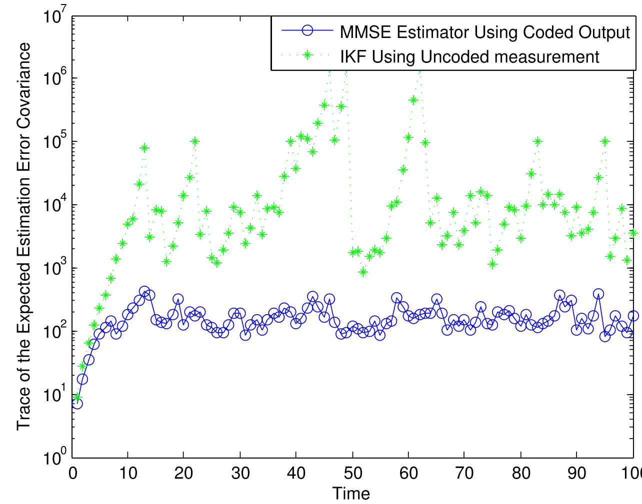

Fig. 3. Comparison between estimators under packet receival rate 0.85.

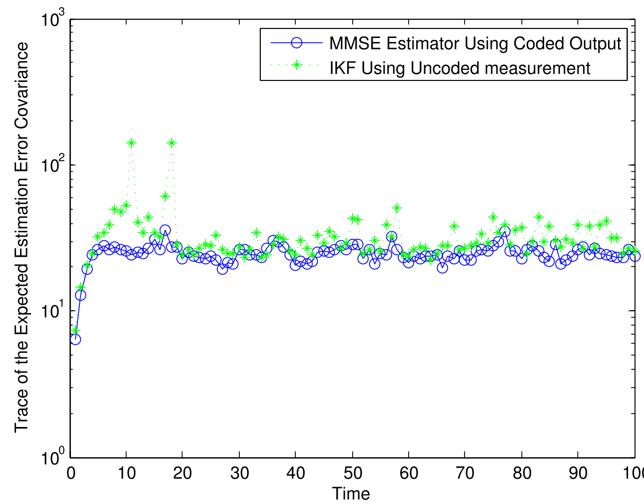

Fig. 4. Comparison between estimators under packet receival rate 0.95.

6. Simulation

To illustrate the advantages of the proposed linear coding method, we use two examples to compare the MMSE estimators obtained using uncoded and coded outputs. The first estimator is the IKF in Sinopoli et al. (2004) using the measured output directly. The second one is the proposed estimator (12) using the coded output (9). In both examples we use the second-order system in (16). In order to approximate the expected estimation error covariance we use 1500 Monte Carlo runs.

In the first example we use a packet arrival rate of $p = 0.85$ and a period of $\tau = 20$. From Theorem 7 in You et al. (2011), the stability condition (18) for the IKF is not satisfied, i.e., $|\lambda_{\max}|^4(1-p) = 2.4 > 1$. However, from Theorem 6 and (18), the stability condition of the MMSE estimator using the coded output is satisfied (with probability one over the choice of the coding vectors), i.e., $|\lambda_{\max}|^2(1-p)(P(20, 2))^{1/20} = 0.74742 < 1$. This is confirmed by Fig. 3, which shows the trace of the expected estimation error covariance on x_k in a logarithmic scale.

In the second example we use $p = 0.95$ and $\tau = 20$. In this case, $|\lambda_{\max}|^4(1-p) = 0.8$ and $|\lambda_{\max}|^2(1-p)(P(20, 2))^{1/20} = 0.24914$, thus both estimators are stable. We see from Fig. 4 that the performance of the proposed estimator using coded output is not worse than that of the IKF. In conclusion, the MMSE estimator using the coded output is more robust to packet loss and has a similar performance than that of the IKF, when they are both stable.

7. Conclusion

Motivated by the necessity of using unreliable channels for data communication, several coding methods on the system outputs have been proposed to reduce the effect of random packet losses on stability of the MMSE state estimator. A new notation called strong observability is defined and the key idea designing the coding vectors is to make a observable system strongly observable. We show that the proposed linear coding approach can enhance the stability of the state estimator under packet losses. In addition, our results reveal a tradeoff between the dimension of transmitted data and the required network reliability.

Appendix

A.1. Proof of Lemma 5

For each $k \in \mathbb{N}$, define $\alpha_{km} = 1$, so that we can write $\alpha_k^T = [\alpha_{k1}, \dots, \alpha_{km}]$. Let $1 \le i_1 < \cdots < i_{m-1} < \tau$ and $O \triangleq O(k, k - i_1, \dots, k - i_{m-1})$. Let also $A = MJM^{-1}$ be the Jordan decomposition of A, with $J = J_1 \oplus \cdots \oplus J_B, J_b, b = 1, \dots, B$ being the Jordan blocks in J, and j_b being the eigenvalue associated to J_b. Then,

$$C_k = \sum_{l=1}^{m} \alpha_{kl} CA^{m-l} = CM \left(\sum_{l=1}^{m} \alpha_{kl} J^{m-l} \right) M^{-1}$$
$$= CM \left(\bigoplus_{b=1}^{B} U_{k-i_1,b} \right) M^{-1}, \tag{A.1}$$

with $U_{k,b} = \sum_{l=1}^{m} \alpha_{k-i_1,l} J_b^{m-l}$. Now, for each $b \in \{1, \dots, B\}$, all the entries on the main diagonal of $U_{k,b}$ have the same value, which we denote by $u_{k,b}$. Hence, $U_{k,b}$ is invertible if and only if $u_{k,b} \neq 0$. We have

$$u_{k,b} = \sum_{l=1}^{m} \alpha_{kl} j_b^{m-l}. \tag{A.2}$$

Since $\alpha_{kl}, k \in \{1, \dots, \tau\}, l \in \{1, \dots, m\}$ are randomly drawn from an absolutely continuous probability distribution, it follows that the measure of the event on which $u_{k,b} = 0$ is zero. This means that, with probability one, for each $k \in \mathbb{N}$ and $b \in \{1, \dots, B\}$, $u_{k,b} \neq 0$ and therefore $U_{k,b}$ is invertible. Hence, so is $\bigoplus_{b=1}^{B} U_{k-i_1,b}$, and in view of (A.1), rank $(C_k) = \text{rank}(C)$.

The rest of the argument then follows by induction. For $0 \le j < m - 1$, with $i_0 = 0$, let $O_j = \text{col}\left\{C_k, C_{k-i_1}A^{-i_1}, \dots, C_{k-i_j}A^{-i_j}\right\}$. Since (A, C) is observable, and A is invertible, it follows that $\text{rank}(\mathcal{C}A^{-i_j}) = n$, for all $j = 0, \dots, m - 1$. Hence, if $\text{rank}(O_j) < n$, there exists at least one row of $\mathcal{C}A^{-i_{j+1}}$ which is not included in the linear span rowspan $\left\{O_j\right\}$ of the rows of O_j. Again, since $\alpha_{k-i_{j+1}}$ is randomly chosen, with probability one,

$$\text{rowspan}\left\{C_{k-i_{j+1}}A^{-i_{j+1}}\right\}$$
$$= \text{rowspan}\left\{\left(\alpha_{k-i_{j+1}}^T \otimes I_q\right) \mathcal{C}A^{-i_{j+1}}\right\}$$
$$\subsetneq \text{rowspan}\left\{O_j\right\}.$$

Hence, it follows that $\text{rank}(O_{j+1}) > \text{rank}(O_j)$ with probability one. This in turn implies $n \ge \text{rank}(O) \ge \text{rank}(C) + m - 1 = n$, and completes the proof. ■

A.2. Proof of Theorem 6

We first introduce the following lemma.

Lemma 15. *Suppose that $\{\alpha_k^T : k \in \mathbb{N}\}$ is periodic and $(A, \{\alpha_k^T \mathcal{C} : k \in \mathbb{N}\})$ is strongly observable with period τ. Under Assumption 1, if there are m packets received in time period $[(j-1)\tau, j\tau), j \ge 1$, there exists a positive value $\beta > 0$ (independent of P_0) such that $P_{j\tau|j\tau} < \beta I$.*

Proof. Suppose that $z_{t_k}, z_{t_{k-1}}, \dots, z_{t_{k-m+1}}$ are received in time period $[(j-1)\tau, j\tau), j \ge 1$, i.e., $j\tau > t_k > t_{k-1} > \cdots > t_{k-m+1} \ge (j-1)\tau$. For ease of writing, here we call $O(t_k, t_{k-1}, \dots, t_{k-m+1})$ as $O(k)$. Since $(A, \{\alpha_k^T \mathcal{C} : k \in \mathbb{N}\})$ is strongly observable with period τ, $O(k)$ is of full column rank. One can obtain a direct estimator of x_{t_k} by using $z_{t_k}, z_{t_{k-1}}, \dots, z_{t_{k-m+1}}$, i.e.,

$$\breve{x}_{t_k|t_k} = O^{\dagger}(k)\text{col}\{z_{t_k}, z_{t_{k-1}}, \dots, z_{t_{k-m+1}}\}, \tag{A.3}$$

where the superscript † denotes the Moore–Penrose pseudo-inverse (Horn & Johnson, 1985). Let $z_k = C_k x_k + n_k$, since $x_{t-i} = A^{-i}x_t - \sum_{j=1}^{i} A^{-j} w_{t+j-i-1}$, $C_k = (\alpha_k^T \otimes I_q)\mathcal{C}$ and $n_k = \sum_{i=1}^{m} \alpha_{ki}(v_{k-i+1} - \sum_{j=i}^{m-1} CA^{-m+j} w_{k-j+i-1})$, the estimator in (A.3) is rewritten as

$$\begin{aligned}\breve{x}_{t_k|t_k} &= O^{\dagger}(k)\text{col}\Bigg\{C_{t_k}x_{t_k} + n_{t_k}, C_{t_{k-1}}A^{-t_k+t_{k-1}} \\ &\quad \times x_{t_k} - C_{t_{k-1}} \sum_{j=1}^{t_k - t_{k-1}} A^{-t_k+t_{k-1}+j-1} w_{t_k-j} \\ &\quad + n_{t_{k-1}}, \dots, C_{t_{k-m+1}}A^{-t_k+t_{k-m+1}}x_{t_k} + C_{t_{k-m+1}} \\ &\quad \times \sum_{j=1}^{t_k - t_{k-m+1}} A^{-t_k+t_{k-m+1}+j-1} w_{t_k-j} + n_{t_{k-m+1}}\Bigg\} \\ &= O^{\dagger}(k)O(k)x_{t_k} + O^{\dagger}(k)\tilde{n}_{t_k},\end{aligned} \tag{A.4}$$

where $\tilde{n}_{t_k}$ is a linear combination of the noises from time t_k to t_{k-m+1}. Denote the estimation error covariance of $\breve{x}_{t_k|t_k}$ by $\breve{P}_{t_k|t_k}$, it follows that

$$\breve{P}_{t_k|t_k} = O^{\dagger}(k)\mathbb{E}[\tilde{n}_{t_k}\tilde{n}_{t_k}^T](O^{\dagger}(k))^T. \tag{A.5}$$

Since $t_k - t_{k-m+1}$ is finite, there exists a positive value $c > 0$ such that $\mathbb{E}[\tilde{n}_{t_k}\tilde{n}_{t_k}^T] < cI$, which results in that

$$\breve{P}_{t_k|t_k} < c(O^T(k)O(k))^{\dagger}, \tag{A.6}$$

Since $O(k)$ is full column rank, then $O^T(k)O(k) > 0$. Combining with that $\{\alpha_k^T : k \in \mathbb{N}\}$ is periodic and $t_k - t_{k-m+1} < \tau$, there exist a positive value $\kappa > 0$ such that $O^T(k)O(k) > \kappa I$. Substituting the above into (A.6), $\breve{P}_{t_k|t_k} < c\kappa^{-1}I$. Since the estimation error covariance of the MMSE estimator is lower than that of $\breve{P}_{t_k|t_k}$, then $P_{t_k|t_k} < c\kappa^{-1}I$. Based on the upper bounded divergence speed of estimation error covariance (i.e., $|\lambda_{max}|^2$), there exists $\varepsilon > 1$ such that $P_{j\tau|j\tau} \le \varepsilon|\lambda_{max}|^{2(j\tau - t_k)}P_{t_k|t_k}$. Since $j\tau - t_k < \tau$, the proof is completed by letting $\beta = \varepsilon|\lambda_{max}|^{2\tau}c\kappa^{-1}$.

Proof of Theorem 6. Firstly, we prove that $\sup_{k\in\mathbb{N}} \mathbb{E}[P_{k\tau|k\tau}] < \infty$. For any $j \in \{0, 1, \dots, k\}$, denote the event that there are less than m packets received in each of $[(k-1)\tau, k\tau), [(k-2)\tau, (k-1)\tau), \dots, [j\tau, (j+1)\tau)$ but no less than m packets received in $[(j-1)\tau, j\tau)$ by $\Omega_{j,k}^m$. Let its probability be $p_{j,k}^m$. Specially, $\Omega_{0,k}^m$ means that there are less than m packets received in each of $[(k-1)\tau, k\tau), [(k-2)\tau, (k-1)\tau), \dots, [0, \tau)$. Based on Lemma 5, with probability one, $(A, \alpha_k^T \mathcal{C} : k \in \mathbb{N})$ is strong observable with period τ, which satisfies the conditions in Lemma 15, and leads to that

$\mathbb{E}[P_{j\tau|j\tau}|\Omega^m_{j,k}] < \beta I$. Then there exists a positive value $\varepsilon > 1$ such that

$$\begin{aligned}\mathbb{E}[P_{k\tau|k\tau}] &= \sum_{j=0}^{k} \mathbb{E}[P_{k\tau|k\tau}|\Omega^m_{j,k}]p^m_{j,k} \\ &< \varepsilon \sum_{j=0}^{k} |\lambda_{\max}|^{2(k-j-1)\tau}\mathbb{E}[P_{j\tau|j\tau}|\Omega^m_{j,k}]p^m_{j,k} \\ &< \varepsilon\beta|\lambda_{\max}|^{-2\tau}\sum_{j=0}^{k}|\lambda_{\max}|^{2(k-j)\tau}p^m_{j,k}I. \qquad \text{(A.7)}\end{aligned}$$

Note that the probability of that $\Omega^m_{j,k}$ is

$$\begin{aligned}p^m_{j,k} &= \left(\sum_{i=0}^{m-1}\binom{\tau}{i}p^i(1-p)^{\tau-i}\right)^{k-j} \\ &\quad\times\left(1-\sum_{i=0}^{m-1}\binom{\tau}{i}p^i(1-p)^{\tau-i}\right) \\ &= [(1-p)^\tau P(\tau,m)]^{k-j}(1-(1-p)^\tau P(\tau,m)). \qquad \text{(A.8)}\end{aligned}$$

Substituting the above into (A.7) yields that

$$\begin{aligned}\mathbb{E}[P_{k\tau|k\tau}] &< \varepsilon\beta|\lambda_{\max}|^{-2\tau}(1-(1-p)^\tau P(\tau,m)) \\ &\quad\times\sum_{j=0}^{k}(|\lambda_{\max}|^2(1-p)P(\tau,m)^{1/\tau})^{(k-j)\tau}. \qquad \text{(A.9)}\end{aligned}$$

From (A.9), it is clear that $|\lambda_{\max}|^2(1-p)P(\tau,m)^{1/\tau} < 1$ is a sufficient condition for $\sup_{k\in\mathbb{N}}\mathbb{E}[P_{k\tau|k\tau}] < \infty$. Since τ is finite, the proof is completed ■

A.3. Proof of Lemma 8

Denote the n_A distinct eigenvalues of A by $\lambda_1,\ldots,\lambda_{n_A}$ and let $\mathcal{M}_A(\lambda_i)$ be the geometric multiplicity of λ_i, $i = 1,2,\ldots,n_A$. Let $\mathcal{M}_A = \max_{i\in\{1,\ldots,n_A\}}\mathcal{M}_A(\lambda_i)$. For ease of notation, there is no loss of generality to assume that A is already in a Jordan form, i.e., $A = \operatorname{diag}\{A_1,A_2,\ldots,A_{n_A}\}$, where A_i is the Jordan blocks of λ_i. By abusing the use of notation, let $C = [C_1,\ldots,C_{n_A}]$ in conformity of A. By Horn and Johnson (1985), we know that the number of elementary Jordan blocks of λ_i is equal to its geometry multiplicity. This, together with the observability of (A,C), implies that $\operatorname{rank}(C_i) \geq \mathcal{M}_A(\lambda_i)$ for all $i \in \{1,\ldots,n_A\}$. Similar to the proof of Lemma 5, since each row of $\Lambda \in \mathbb{R}^{\ell\times q}$ is randomly generated from an absolutely continuous probability distribution, then $\operatorname{rank}(\Lambda C) = \ell$ and $\operatorname{rank}(\Lambda C_i) = \min\{\ell,\operatorname{rank}(C_i)\} \geq \min\{\ell,\mathcal{M}_A(\lambda_i)\}$ with probability one. Note that $\ell \geq \mathcal{M}_A \geq \mathcal{M}_A(\lambda_i)$, we have $\operatorname{rank}(\Lambda C_i) \geq \mathcal{M}_A(\lambda_i)$ for all $i = 1,2,\ldots,n_A$. Thus $(A,\Lambda C)$ is observable and $\operatorname{rank}(\Lambda C) = \ell$ with probability one, which completes the proof. ■

A.4. Proof of Lemma 11

Taking any $t_{k-n+1} < t_{k-n+2} < \cdots < t_k$ with $t_k - t_{k-n+1} < \tau$. Similar to the proof of Lemma 5, it is sufficient to prove the fact that if $\operatorname{rank}(\operatorname{col}\{\Lambda^T_{t_k}\widetilde{\mathcal{C}},\ldots,\Lambda^T_{t_{k-j}}\widetilde{\mathcal{C}}A^{-t_k+t_{k-j}}\}) < n$ for any $j = 0,1,\ldots,n-2$, then we have, with probability one,

$$\begin{aligned}&\operatorname{rank}\left(\operatorname{col}\left\{\Lambda^T_{t_k}\widetilde{\mathcal{C}},\ldots,\Lambda^T_{t_{k-j-1}}\widetilde{\mathcal{C}}A^{-t_k+t_{k-j-1}}\right\}\right) \\ &= \operatorname{rank}\left(\operatorname{col}\left\{\Lambda^T_{t_k}\widetilde{\mathcal{C}},\ldots,\Lambda^T_{t_{k-j}}\widetilde{\mathcal{C}}A^{-t_k+t_{k-j}}\right\}\right)+1. \qquad \text{(A.10)}\end{aligned}$$

Since A is invertible and (A,C) is observable, we have $\operatorname{rank}(\widetilde{\mathcal{C}}A^{-t_k+t_{k-j-1}}) = n$ for any $0 \leq j \leq n-2$. This implies that there must exist a row of $\widetilde{\mathcal{C}}A^{-t_k+t_{k-j-1}}$ that cannot be linearly represented by the rows in $\operatorname{col}\{\Lambda^T_{t_k}\widetilde{\mathcal{C}},\ldots,\Lambda^T_{t_{k-j}}\widetilde{\mathcal{C}}A^{-t_k+t_{k-j}}\}$. Since $t_k - t_{k-n+1} < \tau$, then $\Lambda^T_{t_k},\ldots,\Lambda^T_{t_{k-n+1}}$ are randomly generated from an absolutely continuous probability distribution. It means that, with probability one, $\Lambda_{t_{k-j-1}}\widetilde{\mathcal{C}}A^{-t_k+t_{k-j-1}}$ is independent with $\operatorname{col}\{\Lambda^T_{t_k}\widetilde{\mathcal{C}},\ldots,\Lambda^T_{t_{k-j}}\widetilde{\mathcal{C}}A^{-t_k+t_{k-j}}\}$, which completes the proof. ■

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