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Brief paper

Three-dimensional formation merging control under directed and switching topologies[☆]

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ABSTRACT

This paper concentrates on the formation merging control problem for a leader–follower network. The objective is to control a team of agents called followers such that they are merged with another team of agents called leaders to form a single globally rigid formation. A method based on graph Laplacian is introduced to address this problem. Each follower selects its interaction neighbors and interaction weights according to the given target configuration. The graph modeling the interaction topology of all the agents is directed and time-varying. First, by assuming that the synchronized velocity of the leaders is known to all the followers, a necessary and sufficient condition is derived to ensure uniform asymptotic formation merging. Second, we relax this assumption and consider that the velocity of the leaders is known to only a subset of followers, for which the same necessary and sufficient condition is obtained with the help of an internal model for velocity synchronization.

1. Introduction

For multi-agent systems, maneuvering in a formation is often a basic requirement in many cooperative tasks such as source seeking, exploration, and map construction (Murray, 2007). The goal is to control the agents to achieve and maintain a desired formation (Bai, Arcak, & Wen, 2008; Krick, Broucke, & Francis, 2009; Wang, Han, & Lin, 2012).

In this paper, we consider a leader–follower network and the formation merging problem in the three-dimensional space. By formation merging we mean that two sub-formations are merged to form one single globally rigid formation. We assume that a group of agents called leaders move as a whole in a globally rigid formation while the other group of agents called followers are initially in an arbitrary configuration. The objective is to control the followers in a distributed way such that they merge into a single globally rigid formation with the leaders.

One approach addressing the formation merging problem is to figure out how many new distance constraints should be imposed for agent pairs in the two groups in order to form a single globally rigid formation and then work out a distributed control law to make the agents meet these new distance constraints. From this perspective, Eren, Anderson, Whiteley, Morse, and Belhumeur (2004) consider merging two globally rigid formations into a single globally rigid one; Yu, Fidan, and Anderson (2006) aim to control the merging efficiently and optimally in the sense of minimizing the number of newly added distance constraints. For directed graphs, the concept of persistent connectivity is introduced to address the feasibility problem of merging two sub-formations into a single one (Hendrickx, Yu, Fidan, & Anderson, 2008). However, in Eren et al. (2004), Hendrickx et al. (2008) and Yu et al. (2006), the formation merging control problem is studied by exploring how many distance constraints should be imposed in order to merge two sub-formations into one, but no distributed control scheme is proposed.

Another approach addressing the formation merging problem is to consider the displacement constraints between agent pairs and use relative positions as feedback information for the design

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of coordination laws (Lin, Broucke, & Francis, 2004; Lin, Ding, Yan, Yu, & Giua, 2013). In Lin et al. (2013), a complex Laplacian based control law is introduced to solve the leader-following formation control problem in the plane under a directed and fixed graph setting. Lin, Chen, and Fu (2013) extend this idea solving affine formation control problems in arbitrary higher dimensional space.

To our best knowledge, little work has been reported for formation merging control under directed and switching topologies. However, it is more practical but more challenging when the interaction graph is directed and time-varying (Cao, Anderson, Morse, & Yu, 2008; Guo, Lin, Cao, & Yan, 2010). As a first step towards the general formation merging control problem, we assume in this paper that the target formation of followers lies in the three-dimensional convex hull spanned by the leaders, which is important for convergence analysis. Under this assumption, we first study the formation merging control problem for the case that the synchronized velocity of the leaders is known to all the followers, for which a necessary and sufficient condition for uniform asymptotic formation merging is obtained. That is, every follower should frequently have a joint path from at least one leader. Second, to cope with the case that the synchronized velocity is known only to a subset of followers, an internal model (Wieland, Sepulchre, & Allgower, 2011) based velocity synchronization scheme is adopted, for which the same necessary and sufficient condition is obtained.

To some extent, the formation merging control problem is similar to the containment control, set tracking and shape control problems (Cao, Ren, & Egerstedt, 2012; Cheah, Hou, & Slotine, 2009; Shi, Hong, & Johansson, 2012; Yan, Chen, & Sun, 2012). However, the distinct feature of formation merging control is that the target formation needs to be globally rigid. On the other hand, compared to Eren et al. (2004), Hendrickx et al. (2008) and Yu et al. (2006), the contribution of our work is that we not only derive a necessary and sufficient condition on how many links are required for formation merging in terms of relative displacement constraints, but also provide a distributed control law for directed and time-varying networks to merge the followers with the leaders. We show in this paper that the key to solve the formation merging control problem is the selection of neighbors and interaction weights. This paper presents the rules for the selection of neighbors and interaction weights, and shows that under such construction, the followers can achieve uniform asymptotic formation merging with the leaders as desired.

Notation: $\mathbb{R}$ denotes the set of real numbers. Let $\overline{\mathbb{C}}_-$ represent the closed left-half complex plane. $\mathbf{1}_n$ represents the n-dimensional vector of ones and I_n represents the identity matrix of order n. The symbol $\otimes$ denotes the Kronecker product.

2. Preliminaries

A *directed graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a non-empty finite set $\mathcal{V}$ of elements called *nodes* and a finite set $\mathcal{E}$ of ordered pairs of nodes called *edges*. Let $\mathcal{U} \subset \mathcal{V}$ be a subset of nodes in $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. We say a node $v \in \mathcal{V} - \mathcal{U}$ is *reachable* from $\mathcal{U}$ if there exists a path from a node in $\mathcal{U}$ to v. A subset of nodes $\mathcal{U}$ is called *closed* if any node in $\mathcal{U}$ is not reachable from $\mathcal{V} - \mathcal{U}$. A *time-varying graph* $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ is a graph whose edge set changes over time. For a time-varying graph $\mathcal{G}(t)$, a node v is said to be *uniformly jointly reachable* from $\mathcal{U} \subset \mathcal{V}$ if there exists $T > 0$ such that for all t, v is reachable from $\mathcal{U}$ in the union graph $\mathcal{G}([t, t+T))$, whose edge set is the union of the edge set of $\mathcal{G}(t)$ over the time interval $[t, t+T)$. For a directed graph $\mathcal{G}$, the associated Laplacian L is a matrix such that its (i, j)th entry $(i \neq j)$ is the negative weight on edge (j, i) and 0 otherwise, and its (i, i)th entry is the negative sum of all off-diagonal entries in the same row.

A square matrix $E \in \mathbb{R}^{n\times n}$ is *nonnegative* if all its entries are nonnegative. Moreover, E is called *(row) stochastic* if it is nonnegative and every row sum equals 1. In addition, the *associated graph* $\mathcal{G}(E)$ is defined to be one consisting of n nodes $v_1, \ldots, v_n$ where an edge leads from v_j to v_i if and only if the (i, j)th entry of E is nonzero.

3. Formation merging control problem

Consider a leader–follower network of $N = m + n$ agents with m leaders labeled $1, \ldots, m$ and n followers labeled $m+1, \ldots, N$. Consider a target configuration $p_a = [{p_1}^\top, \ldots, {p_m}^\top]^\top$ for the leaders and $p_b = [{p_{m+1}}^\top, \ldots, {p_N}^\top]^\top$ for the followers. Moreover, we assume that agents do not overlap each other in the target configuration.

Denote by z_i the 3D position of agent i. We say the leaders move in a globally rigid formation p_a with $v_r(t)$ if

$$z_i(t) - \int_{t_0}^{t} v_r(\tau)d\tau = Ap_i + c, \quad \text{for } i = 1, \ldots, m,$$

where $A \in \mathbb{R}^{3\times 3}$ is a unitary matrix, and $c \in \mathbb{R}^3$ is a constant vector. This definition says that the configuration $\{z_i(t) - \int_{t_0}^{t} v_r(\tau)d\tau,\ i = 1, \ldots, m\}$ is obtained from p_a via a rigid-body transformation.

We assume $m \geq 4$ since at least four agents are needed to form a three-dimensional formation. Suppose the m leaders move in a globally rigid formation p_a and are governed by the following dynamics

$$\dot{z}_i(t) = v_r(t), \quad i = 1, \ldots, m, \tag{1}$$

where $v_r(t)$ is the synchronized velocity of the leaders.

This paper aims to merge a group of followers with another group of leaders and form one single large rigid formation. The precise definition of uniformly asymptotic formation merging is given as follows.

Definition 3.1. A globally rigid formation $p = [{p_a}^\top, {p_b}^\top]^\top$ is said to be *uniformly asymptotically merged* if for any $\delta > 0$ and for any $\varepsilon > 0$ there exists $T > 0$ such that for any t_0 and any $z_i(t_0)$ satisfying $\|z_i(t_0) - Ap_i - c\| \leq \delta$,

$$(\forall t \geq t_0 + T)(\forall i) \left\| z_i(t) - Ap_i - c - \int_{t_0}^{t} v_r(\tau)d\tau \right\| \leq \varepsilon,$$

where A and c are determined by the leaders.

The single-integrator model is assumed for the followers,

$$\dot{z}_i = u_i, \quad i = m+1, \ldots, N, \tag{2}$$

where $u_i \in \mathbb{R}^3$ represents the velocity control input.

Suppose that every agent is equipped with an onboard sensor to measure relative positions of its neighbors. We use a time-varying graph $\bar{\mathcal{G}}(t) = (\mathcal{V}, \bar{\mathcal{E}}(t))$ to model the information flow, where $\mathcal{V} = \mathcal{V}_a \cup \mathcal{V}_b$ with $\mathcal{V}_a = \{1, \ldots, m\}$ and $\mathcal{V}_b = \{m+1, \ldots, N\}$, and $(j, i) \in \bar{\mathcal{E}}(t)$ only if agent i can measure the relative position of agent j. Moreover, it is assumed that when $(j, i) \in \bar{\mathcal{E}}(t)$, agent j is able to communicate to agent i about its identity and p_j. Let $\bar{\mathcal{N}}_i(t)$ be the set of neighbors of agent i, namely, $j \in \bar{\mathcal{N}}_i(t)$ if and only if $(j, i) \in \bar{\mathcal{E}}(t)$.

In order to derive a solution for the formation merging control problem, we technically assume the following.

Assumption 3.1. The target formation of the followers entirely lies in the three-dimensional convex hull spanned by the leaders.

4. Formation merging control with the reference velocity available to all the agents

4.1. Distributed control law

We introduce a *neighbor-selecting rule* for the followers. That is to say, the followers may not interact with all the neighbors in the information flow graph $\bar{\mathcal{G}}(t)$. They pick their neighbors to interact with according to certain rules and then form the interaction graph

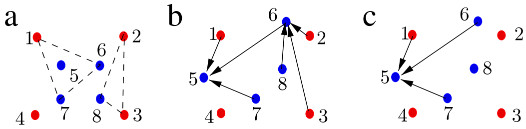

Fig. 1. (a) Target configuration. (b) Information flow graph $\bar{\mathcal{G}}(t)$. (Sensors such as cameras have a cone-like field of view, so in this example, the information flow graph is not determined based on vicinity.) (c) Interaction graph $\mathcal{G}(t)$.

$\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$. Let $\mathcal{N}_i(t)$ be the set of interaction neighbors of agents i, namely, $j \in \mathcal{N}_i(t)$ if and only if $(j, i) \in \mathcal{E}(t)$. Denote by $\mathrm{co}\{x_1, \ldots, x_n\}$ the convex hull of $x_1, \ldots, x_n \in \mathbb{R}^3$.

Neighbor-selecting rule: If $p_i \notin \mathrm{co}\{p_j : j \in \bar{\mathcal{N}}_i(t)\}$, then $\mathcal{N}_i(t) = \emptyset$. Otherwise, $\mathcal{N}_i(t) = \bar{\mathcal{N}}_i(t)$.

An example is given in Fig. 1 to demonstrate the neighbor-selecting rule. In Fig. 1(a), $p_5 \in \mathrm{co}\{p_1, p_6, p_7\}$ and $p_6 \notin \mathrm{co}\{p_2, p_3, p_8\}$, so it follows that $\mathcal{N}_5 = \bar{\mathcal{N}}_5$ and $\mathcal{N}_6 = \emptyset$. It is clear that by this neighbor-selecting rule, $p_i \in \mathrm{co}\{p_j : j \in \mathcal{N}_i(t)\}$. This fact is very important for the convergence analysis.

We consider the following control law for each follower i,

$$u_i(t) = v_r(t) + \sum_{j \in \mathcal{N}_i(t)} k_{ij}(t)(z_j(t) - z_i(t)), \tag{3}$$

where $k_{ij}(t)$'s are the control parameters to be designed.

To avoid infinite switching within a finite time interval, we assume the following.

Assumption 4.1. The interval between any two switching instants satisfies a dwell time condition. That is to say, there exists $\tau_D > 0$ such that

$$t_{i+1} - t_i \geq \tau_D \quad \text{for all } i = 0, 1, \ldots$$

if the interaction graph $\mathcal{G}(t)$ switches at $t_0, t_1, t_2, \ldots$.

4.2. Design of control parameters k_{ij}'s

We design k_{ij}'s in this subsection for (3). By Assumption 4.1, one knows that $\mathcal{N}_i(t)$ is piecewise constant. Since k_{ij}'s are different for different neighbor sets $\mathcal{N}_i$, $k_{ij}(t)$ is thus piecewise constant. In the sequel, we omit t for $k_{ij}(t)$ and $\mathcal{N}_i(t)$ for simplicity unless it is necessary.

By the neighbor-selecting rule, for each agent there are four possible cases:

(i) It has no neighbor;
(ii) $\mathrm{co}\{p_j : j \in \mathcal{N}_i\}$ is one-dimensional (a line segment);
(iii) $\mathrm{co}\{p_j : j \in \mathcal{N}_i\}$ is two-dimensional (a convex polygon);
(iv) $\mathrm{co}\{p_j : j \in \mathcal{N}_i\}$ is three-dimensional (a convex polyhedron).

We consider these four cases in the following for the design of k_{ij}'s. That is, for agent i, k_{ij}'s are chosen to be the barycentric coordinates about its neighbors in the target configuration. The barycentric coordinate was introduced by August Ferdinand Möbius in 1827 (Coxeter, 1969).

(i) If i has no neighbor, (3) degenerates to $u_i = v_r(t)$.

(ii) If $\mathrm{co}\{p_j : j \in \mathcal{N}_i\}$ is one-dimensional, then we take $k_{ij} = \alpha_j$ for $j \in \mathcal{N}_i$, where α_j is calculated as follows.

First, consider that i has only two neighbors, say i_1 and i_2. Then we obtain $p_i = \alpha_1 p_{i_1} + \alpha_2 p_{i_2}$, where $\alpha_1 = \frac{\|p_{i_2} - p_i\|}{\|p_{i_2} - p_{i_1}\|}$ and $\alpha_2 = \frac{\|p_{i_1} - p_i\|}{\|p_{i_2} - p_{i_1}\|}$. It is clear that $\alpha_1, \alpha_2 > 0$ and $\alpha_1 + \alpha_2 = 1$.

Second, if agent i has more than two neighbors, then we can take any two of them containing p_i inside and obtain the same formula, i.e., $p_i = \alpha_1^l p_{i_1^l} + \alpha_2^l p_{i_2^l}$, where l enumerates all possible combinations of two neighbors containing p_i inside. Then consider a convex combination of all these representations for p_i, i.e.,

$$p_i = \sum_l \gamma^l (\alpha_1^l p_{i_1^l} + \alpha_2^l p_{i_2^l}) := \sum_{j \in \mathcal{N}_i} \alpha_j p_j,$$

where $\gamma^l \in (0, 1)$ and $\sum_l \gamma^l = 1$. It is certain that $\alpha_j > 0$ for all $j \in \mathcal{N}_i$ and $\sum_{j \in \mathcal{N}_i} \alpha_j = 1$.

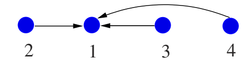

Fig. 2. An example of case (ii).

An example is given in Fig. 2. For agent 1, p_1 can be represented as $p_1 = \alpha_2^1 p_2 + \alpha_3^1 p_3$ and also can be represented as $p_1 = \alpha_2^2 p_2 + \alpha_4^2 p_4$. Then consider a convex combination of these two representations $p_1 = \gamma^1(\alpha_2^1 p_2 + \alpha_3^1 p_3) + \gamma^2(\alpha_2^2 p_2 + \alpha_4^2 p_4) := \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 p_4$.

(iii) If $\mathrm{co}\{p_j : j \in \mathcal{N}_i\}$ is two-dimensional, then we take $k_{ij} = \alpha_j$ for $j \in \mathcal{N}_i$, where α_j is calculated as follows.

First, consider that i has only three neighbors, say i_1, i_2 and i_3, and $\mathrm{co}\{p_{i_1}, p_{i_2}, p_{i_3}\}$ is a triangle. Let the coordinates of p_{i_1}, p_{i_2}, and p_{i_3} be $p_{i_1} = (x_{i_1}, y_{i_1}, z_{i_1})$, $p_{i_2} = (x_{i_2}, y_{i_2}, z_{i_2})$ and $p_{i_3} = (x_{i_3}, y_{i_3}, z_{i_3})$. Denote $x = [x_{i_1}, x_{i_2}, x_{i_3}]^\top$, $y = [y_{i_1}, y_{i_2}, y_{i_3}]^\top$ and $z = [z_{i_1}, z_{i_2}, z_{i_3}]^\top$. Let $S_{i_1 i_2 i_3}$ be the area of $\mathrm{co}\{p_{i_1}, p_{i_2}, p_{i_3}\}$, which can be calculated as $S_{i_1 i_2 i_3} = \frac{1}{2}\sqrt{S_1^2 + S_2^2 + S_3^2}$, where $S_1 = \det[x, y, \mathbf{1}_3]^\top$, $S_2 = \det[y, z, \mathbf{1}_3]^\top$ and $S_3 = \det[z, x, \mathbf{1}_3]^\top$. Then it holds that $p_i = \alpha_1 p_{i_1} + \alpha_2 p_{i_2} + \alpha_3 p_{i_3}$, where $\alpha_1 = \frac{S_{i i_2 i_3}}{S_{i_1 i_2 i_3}}$, $\alpha_2 = \frac{S_{i_1 i i_3}}{S_{i_1 i_2 i_3}}$ and $\alpha_3 = \frac{S_{i_1 i_2 i}}{S_{i_1 i_2 i_3}}$. It is known that $\alpha_1, \alpha_2, \alpha_3 > 0$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

Second, if agent i has more than three neighbors, similar to the procedure of case (ii) we can get the representation for p_i as $p_i = \sum_{j \in \mathcal{N}_i} \alpha_j p_j$, where $\alpha_j > 0$ for all $j \in \mathcal{N}_i$ and $\sum_{j \in \mathcal{N}_i} \alpha_j = 1$.

(iv) If $\mathrm{co}\{p_j : j \in \mathcal{N}_i\}$ is three-dimensional, then we take $k_{ij} = \alpha_j$ for $j \in \mathcal{N}_i$, where α_j is calculated as follows.

First, consider that i has only four neighbors, say i_1, i_2, i_3 and i_4, and $\mathrm{co}\{p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}\}$ is a tetrahedron. Denote by $V_{i_1 i_2 i_3 i_4}$ the signed volume of $\mathrm{co}\{p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}\}$. It can be calculated by $V_{i_1 i_2 i_3 i_4} = \frac{1}{6}\det([p_{i_2} - p_{i_1}, p_{i_3} - p_{i_1}, p_{i_4} - p_{i_1}]^\top)$. Then it is obtained that $p_i = \alpha_1 p_{i_1} + \alpha_2 p_{i_2} + \alpha_3 p_{i_3} + \alpha_4 p_{i_4}$, where $\alpha_1 = \frac{V_{i i_2 i_3 i_4}}{V_{i_1 i_2 i_3 i_4}}$, $\alpha_2 = \frac{V_{i_1 i i_3 i_4}}{V_{i_1 i_2 i_3 i_4}}$, $\alpha_3 = \frac{V_{i_1 i_2 i i_4}}{V_{i_1 i_2 i_3 i_4}}$, and $\alpha_4 = \frac{V_{i_1 i_2 i_3 i}}{V_{i_1 i_2 i_3 i_4}}$. It is certain that $\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$ and $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$.

Second, if agent i has more than four neighbors, similar to the procedure of case (ii) we can get the representation for p_i as $p_i = \sum_{j \in \mathcal{N}_i} \alpha_j p_j$, where $\alpha_j > 0$ for all $j \in \mathcal{N}_i$ and $\sum_{j \in \mathcal{N}_i} \alpha_j = 1$.

4.3. Stability analysis

In what follows, we show whether a globally rigid formation $p = [{p_a}^\top, {p_b}^\top]^\top$ can be uniformly asymptotically merged for a leader–follower network.

Let $L(t)$ be the Laplacian matrix for the graph with weights $k_{ij}(t)$'s associated to the edges (j, i)'s at time t. Define $z = [{z_a}^\top, {z_b}^\top]^\top$ as the aggregate state of all z_i's. Throughout the paper, we use subscript a to represent the corresponding aggregate state for the leaders and subscript b for the followers.

Under (1) and (3), the overall system can be written as

$$\dot{z} = -(L(t) \otimes I_3)z + \mathbf{1}_N \otimes v_r(t), \tag{4}$$

where $L(t)$ has the following form

$$L(t) = \left[\begin{array}{c|c} 0_{m \times m} & 0_{m \times n} \\ \hline L_l(t) & L_f(t) \end{array}\right]. \tag{5}$$

Also, by the way how we design $k_{ij}(t)$'s, we know that

$$(L(t)\otimes I_3)p = 0 \quad \text{and} \quad L(t)\mathbf{1}_N = 0. \tag{6}$$

The following theorem shows that a globally rigid formation $p = [{p_a}^\top, {p_b}^\top]^\top$ is a stable steady-state formation under the proposed control law.

Theorem 4.1. *Suppose the leaders move in the globally rigid formation p_a. Then*

$$z^*(t) = (I_N \otimes A)p + \mathbf{1}_N \otimes \left(c + \int_{t_0}^{t} v_r(\tau)d\tau\right),$$

where A and c are determined by the leaders, is a stable steady-state solution of system (4).

Proof. Let $y = z - \mathbf{1}_N \otimes \int_{t_0}^{t} v_r(\tau)d\tau$. Then system (4) is transformed to

$$\dot{y} = -(L(t)\otimes I_3)y. \tag{7}$$

To show $z^*(t)$ is a steady-state solution of system (4) is equivalent to show $y^* = (I_N \otimes A)p + \mathbf{1}_N \otimes c$ is an equilibrium point of system (7). Notice that $(L(t)\otimes I_3)p = 0$ in (6) yields

$$\begin{aligned}&(L(t)\otimes I_3)[(I_N\otimes A)p + \mathbf{1}_N \otimes c]\\ &\quad = (L(t)\otimes A)p = (I_N \otimes A)(L(t)\otimes I_3)p = 0,\end{aligned}$$

which means y^* is an equilibrium point of system (7).

Next, we show $z^*(t)$ is a stable solution of system (4), which is equivalent to show y^* is a stable equilibrium point of system (7). Suppose the switching time is $t_0, t_1, t_2, \ldots$. Consider any $t > 0$, without loss of generality, say $t \in [t_i, t_{i+1})$. Thus, the transition matrix is

$$\Phi(t, t_i) = \exp[-(L(t_i)\otimes I_3)(t - t_i)] \tag{8}$$

and the solution of system (7) can be described by $y(t) = \Phi(t, t_i)\Phi(t_i, t_{i-1})\cdots\Phi(t_1, t_0)y^0$ for the initial state y^0. Note that every transition matrix in the above formula is a stochastic matrix and the product of stochastic matrices is also a stochastic matrix (Lin, 2008, page 34, 51). It follows that every state $y_i(t)$ is a convex combination of $y_1^0, \ldots, y_N^0$. That is, $y_i(t) = \sum_{j=1}^{N}\alpha_j y_j^0$, where $\alpha_j \geq 0$ $(j = 1, \ldots, N)$ and $\sum_{j=1}^{N}\alpha_j = 1$.

Then let us consider any arbitrary $\varepsilon > 0$ and choose $\delta = \varepsilon$. Suppose initially $(\forall i)\|y_i^0 - y_i^*\| \leq \delta$. Since y^* is an equilibrium point, it is known $y_i^* = \sum_{j=1}^{N}\alpha_j y_j^*$. Thus, we have for every i,

$$\|y_i(t) - y_i^*\| = \left\|\sum_{j=1}^{N}\alpha_j(y_j^0 - y_j^*)\right\| \leq \sum_{j=1}^{N}\alpha_j\delta = \delta = \varepsilon.$$

The conclusion follows. ∎

The next result presents a necessary and sufficient graphical condition to ensure that a globally rigid formation $[{p_a}^\top, {p_b}^\top]^\top$ can be uniformly asymptotically merged.

Theorem 4.2. *Suppose the leaders move in the globally rigid formation p_a. The globally rigid formation $[{p_a}^\top, {p_b}^\top]^\top$ can be uniformly asymptotically merged under the distributed control law (3) if and only if every follower is uniformly jointly reachable from $\mathcal{V}_a$.*

The proof requires a lemma from graph theory.

Lemma 4.1 (*Beineke & Wilson, 1997, page 87*)**.** *Let E be a nonnegative matrix and denote $e_{ij}^{(k)}$ as the (i, j)th entry of E^k. Then $e_{ij}^{(k)} > 0$ if and only if the associated graph $\mathcal{G}(E)$ has a walk from node v_j to node v_i of length k.*

Proof of Theorem 4.2. ($\Leftarrow$) Suppose the interaction graph $\mathcal{G}(t)$ switches at $t_0, t_1, t_2, \ldots$. Recall that by our dwell time assumption, $t_{i+1} - t_i \geq \tau_D$ for all $i = 0, 1 \ldots$. Moreover, we are always able to find a $\tau_m > \tau_D$ large enough such that $t_{i+1} - t_i \leq \tau_m$ for all $i = 0, 1, \ldots$. If for some interval $[t_i, t_{i+1})$ there is no such a τ_m, we can partition $[t_i, t_{i+1})$ artificially.

Suppose now every follower is uniformly jointly reachable from $\mathcal{V}_a$. Then by definition there exists $T > 0$ such that for all t in the union graph $\mathcal{G}([t, t + T))$ every follower is reachable from $\mathcal{V}_a$. Now we generate a subsequence $\{t_{m_k}\}$ of the sequence $\{t_i\}$ as follows: (1) Set $m_0 = 0$; (2) If $t_{m_0} + T \in (t_{i-1}, t_i]$, set $m_1 = i$; (3) If $t_{m_1} + T \in (t_{i-1}, t_i]$, set $m_2 = i$; (4) And so on. Thus, for the transformed system (7), we have

$$y(t_{m_{k+1}}) = \Psi(t_{m_k})y(t_{m_k}) \tag{9}$$

where $\Psi(t_{m_k}) = \left[\exp\left(-\int_{t_{m_k}}^{t_{m_{k+1}}} L(t)dt\right)\right] \otimes I_3$. Denote by Ξ the set of all $\Psi(t_{m_k})$'s derived above. We regard (9) as a discrete-time switched system and rewrite it as

$$y(k+1) = \Psi(k)y(k) \quad \text{with } \Psi(k) \in \Xi. \tag{10}$$

Note that, due to the special structure of $L(t)$ described in (5), $\Psi(k)$ has the following form

$$\Psi(k) = \left[\begin{array}{c|c} I_{3m\times 3m} & 0_{3m\times 3n} \\ \hline \Psi_l(k) & \Psi_f(k)\end{array}\right].$$

Next we show that for all $\Psi(k) \in \Xi$, $\|\Psi_f(k)\|_\infty$ is uniformly upper-bounded by a constant less than one. For any $L(t)$, we can decompose it as $-L(t) = -D(t) + E(t)$, where $D(t)$ is a diagonal matrix and $E(t)$ is a nonnegative matrix with all diagonal entries zero. Thus,

$$\Psi(k) = \left[\exp\left(-\int_{t_{m_k}}^{t_{m_{k+1}}} D(t)dt\right)\exp\left(\int_{t_{m_k}}^{t_{m_{k+1}}} E(t)dt\right)\right]\otimes I_3.$$

We denote $\Sigma = \int_{t_{m_k}}^{t_{m_{k+1}}} E(t)dt$ and it is noted that $\Sigma = E(t_{m_k})(t_{m_k+1} - t_{m_k}) + \cdots + E(t_{m_{k+1}-1})(t_{m_{k+1}} - t_{m_{k+1}-1})$. By the condition that every follower is uniformly jointly reachable from $\mathcal{V}_a$, we can then know that every follower is reachable from $\mathcal{V}_a$ in the associated graph $\mathcal{G}(\Sigma)$. Then, considering the equality

$$\exp(\Sigma) = I + \Sigma + \frac{\Sigma^2}{2!} + \cdots$$

and the fact that $\exp\left(-\int_{t_{m_k}}^{t_{m_{k+1}}} D(t)dt\right)$ is a positive diagonal matrix, we can infer by Lemma 4.1 that each row of $\Psi_l(k)$ has a nonzero entry because each row in the corresponding block of $\exp(\Sigma)$ has a nonzero entry. On the other hand, as shown in Theorem 4.1, we know that $\Psi(k)$ is a stochastic matrix. The above two conclusions together imply that $\|\Psi_f(k)\|_\infty < 1$. Moreover, recall that $\tau_D \leq t_{i+1} - t_i \leq \tau_m$ and $L(t_i)$'s are taken in a finite set due to finite k_{ij}'s. Therefore, from the formula of $\Psi(k)$ above, there is a positive constant $\sigma < 1$ such that $\|\Psi_f(k)\|_\infty$ is uniformly upper-bounded by σ.

Since the m leaders move in the globally rigid formation p_a, from (10) we then have

$$y_b(k+1) = \Psi_f(k)y_b(k) + \Psi_l(k)y_a^*, \tag{11}$$

where $y_a^* = (I_m \otimes A)p_a + \mathbf{1}_m \otimes c$. Due to the fact that $\|\Psi_f(k)\|_\infty < 1$, we know that $I - \Psi_f(k)$ is invertible. Thus, the system (11) has a unique equilibrium point $y_b^* = (I_n \otimes A)p_b + \mathbf{1}_n \otimes c$. So by the coordinate transformation $q_b(k) = y_b(k) - y_b^*$ and applying the fact $y_b^* = \Psi_f(k)y_b^* + \Psi_l(k)y_a^*$, we get

$$q_b(k+1) = \Psi_f(k)q_b(k). \tag{12}$$

As we just showed that $\|\Psi_f(k)\|_\infty$ is uniformly upper-bounded by $\sigma < 1$, it follows straightforward that $q(k)$ asymptotically converges to 0. So we can reach the conclusion that $\lim_{j\to\infty} y_b(t_{m_j}) = y_b^*$.

Finally, let us look at the evolution of the continuous state $y_b(t)$ in the interval between any two consecutive switching instants. From the proof of Theorem 4.1, we know that for any $t \in [t_i, t_{i+1})$ and any arbitrary $\varepsilon > 0$

$$(\forall i)\|y_i(t_i) - y_i^*\| \le \varepsilon \Rightarrow \|y_i(t) - y_i^*\| \le \varepsilon.$$

Thus, we get $\lim_{t\to\infty} y_b(t) = y_b^*$.

From $\|\Psi_f(k)\|_\infty \le \sigma < 1$, it is certain that $\forall\delta > 0$ and $\forall\varepsilon > 0$ there exists $T' > 0$ (large enough) such that for any t_0 and any $z_i(t_0)$ satisfying $\|z_i(t_0) - Ap_i - c\| \le \delta$,

$$(\forall t \ge t_0 + T')(\forall i) \left\| z_i(t) - Ap_i - c - \int_{t_0}^{t} v_r(\tau)d\tau \right\| \le \varepsilon.$$

($\Rightarrow$) We prove it in a contrapositive way. Assume that there exists a follower, say b_i, that is not uniformly jointly reachable from $\mathcal{V}_a$. That is, for any $T > 0$ there exists $t^* \ge 0$ such that in the union graph $\mathcal{G}([t^*, t^* + T))$, b_i is not reachable from $\mathcal{V}_a$. Let Θ be the set including all such followers that are not reachable from $\mathcal{V}_a$ in $\mathcal{G}([t^*, t^* + T))$. Then it can be known that Θ is a closed set. We choose δ large enough and $\varepsilon > 0$. Then for all $T > 0$, there exists $t_0 = t^*$ and choose $z_i(t_0)$ $(i \in \Theta)$ such that the distance between $\mathrm{co}\{z_i(t_0) : i \in \Theta\}$ and $\mathrm{co}\{z_i(t_0) : i \in \mathcal{V}_a\}$ is bigger than ε. Since Θ is a closed set, the states of these followers at $t \in [t_0, t_0 + T)$ remain in the convex hull of their states at t_0. We have thus found $\delta > 0$ and $\varepsilon > 0$ such that, for all $T > 0$, there exists $t_0 = t^*$ and $z_i(t_0)$ satisfying $\|z_i(t_0) - Ap_i - c\| \le \delta$,

$$(\exists t = t_0 + T)(\exists i) \left\| z_i(t) - Ap_i - c - \int_{t_0}^{t} v_r(\tau)d\tau \right\| > \varepsilon.$$

Thus, the conclusion follows. ■

Remark 4.1. Theorem 4.2 shows that every follower only needs to have a joint path from at least one leader. The graph condition is mild such that the method can be extended to deal with multiple follower groups. As an illustration, we denote by V_a the group of leaders and denote by $\mathcal{V}_{b_1}, \mathcal{V}_{b_2}, \ldots, \mathcal{V}_{b_m}$ multiple groups of followers. The condition in Theorem 4.2 can then be generalized and stated as follows. Multiple follower groups can be uniformly asymptotically merged with the leader group to form a single globally rigid formation if every follower in each group $\mathcal{V}_{bi}$ is uniformly jointly reachable from $\mathcal{V}_a$ in the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \mathcal{V}_a \cup \mathcal{V}_{b_1} \cup \cdots \cup \mathcal{V}_{b_m}$.

5. Formation merging control with the reference velocity available to a subset of followers

In this section, we discuss how to cope with the situation that $v_r(t)$ is known only to a subset of followers.

Suppose the leaders are governed by the dynamics

$$\begin{cases} \dot z_i(t) = v_r(t), \quad i = 1, 2, \ldots, m, \\ \dot v_r(t) = \Gamma v_r(t), \end{cases} \tag{13}$$

where $\Gamma \in \mathbb{R}^{3\times 3}$ with its spectrum $\sigma(\Gamma) \subset \overline{\mathbb{C}}_-$.

Remark 5.1. Since Γ is a constant matrix, it is assumed that it can be known by all the followers via communications.

In this scenario, we propose the following alternative distributed formation merging control law for each follower:

$$\begin{cases} u_i(t) = \eta_i(t) + \sum_{j\in\mathcal{N}_i(t)} k_{ij}(t)(z_j(t) - z_i(t)) \\ \dot\eta_i(t) = \Gamma\eta_i(t) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\eta_j(t) - \eta_i(t)) \end{cases} \tag{14}$$

where $\underline{a} \le a_{ij}(t) \le \bar a$ for some $\underline{a} > 0$ and $\bar a > 0$, and η_i is the estimation of the reference velocity by follower i. For the leaders, we have $\eta_1 = \cdots = \eta_m = v_r(t)$.

The next result shows that a globally rigid formation $[{p_a}^\mathsf{T}, {p_b}^\mathsf{T}]^\mathsf{T}$ can be uniformly asymptotically merged under the same graphical condition.

Theorem 5.1. *Suppose the leaders move in the globally rigid formation p_a. The globally rigid formation $[{p_a}^\mathsf{T}, {p_b}^\mathsf{T}]^\mathsf{T}$ can be uniformly asymptotically merged under the distributed control law (14) if and only if every follower is uniformly jointly reachable from $\mathcal{V}_a$.*

The proof requires a lemma concerning a cascade system.

Lemma 5.1 (Loria, Panteley, Popovic, & Teel, 2005)**.** *For a cascade system*

$$\dot x = f(t, x, z), \tag{15}$$

$$\dot z = g(t, z). \tag{16}$$

If each initial condition (x_0, z_0) produces trajectories that are bounded uniformly in the initial time, the functions f and g are locally Lipschitz uniformly in t, and the origins of (16) and $\dot x = f(t, x, 0)$ are globally uniformly asymptotically stable, then the origin of the cascade system (15)–(16) is globally uniformly asymptotically stable.

Proof of Theorem 5.1. ($\Leftarrow$) Rewrite (13) and (14) as

$$\begin{bmatrix} \dot z \\ \dot\eta \end{bmatrix} = \begin{bmatrix} -L(t)\otimes I_3 & I_{3N} \\ 0 & I_N\otimes\Gamma - H(t)\otimes I_3 \end{bmatrix} \begin{bmatrix} z \\ \eta \end{bmatrix} \tag{17}$$

where $H(t)$ is also a Laplacian matrix with the same form as $L(t)$, i.e.,

$$H(t) = \left[\begin{array}{c|c} 0_{m\times m} & 0_{m\times n} \\ \hline H_l(t) & H_f(t) \end{array}\right]. \tag{18}$$

Define $y = z - \mathbf{1}_N \otimes \int_{t_0}^{t} v_r(\tau)d\tau$, $q = y - y^*$, and $\delta = \eta - \mathbf{1}_N \otimes v_r(t)$. Note that the leaders move in a globally rigid formation, i.e., $q_a^* = 0$ and $\delta_a^* = 0$. So (17) can be regarded as a cascade system

$$\dot q_b = -(L_f(t)\otimes I_3)q_b + \delta_b, \tag{19}$$

$$\dot\delta_b = (I_N\otimes\Gamma - H_f(t)\otimes I_3)\delta_b. \tag{20}$$

According to Lemma 5.1 to prove the uniform asymptotic stability of (17), it remains to prove the uniform asymptotic stability of the subsystem (20).

Let $\zeta_i = e^{-\Gamma(t-t_0)}\eta_i$. Recall that $\dot v_r(t) = \Gamma v_r(t)$ and $\eta_1 = \cdots = \eta_m = v_r(t)$. So $\zeta_i = v_r(t_0)$ $(i = 1, \ldots, m)$. For the followers, we have

$$\begin{aligned} \dot\zeta_i &= -\Gamma e^{-\Gamma(t-t_0)}\eta_i + e^{-\Gamma(t-t_0)}\Gamma\eta_i \\ &\quad + e^{-\Gamma(t-t_0)} \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\eta_j(t) - \eta_i(t)) \\ &= \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\zeta_j(t) - \zeta_i(t)), \end{aligned}$$

or equivalently in a compact form $\dot\zeta = -(H(t)\otimes I_3)\zeta$, which has the same form as (7). With the same proof of Theorem 4.2, we derive that $\zeta_i(t)$ uniformly asymptotically converges to $v_r(t_0)$. Moreover, for the switched linear system, global uniform asymptotic stability implies global uniform exponential stability (Liberzon, 2003, page 22). Thus there exist positive constants α_1 and α_2 such that for all $t > t_0$,

$$\|\zeta_i(t) - v_r(t_0)\| \le \alpha_1 e^{-\alpha_2(t-t_0)}\|\zeta_i(t_0) - v_r(t_0)\|,$$

which implies for all $t > t_0$,

$$\begin{aligned} \|\eta_i(t) - e^{\Gamma(t-t_0)}v_r(t_0)\| \le\ & \alpha_1 e^{-\alpha_2(t-t_0)}\|e^{\Gamma(t-t_0)}\| \\ & \times \|\eta_i(t_0) - v_r(t_0)\|. \end{aligned}$$

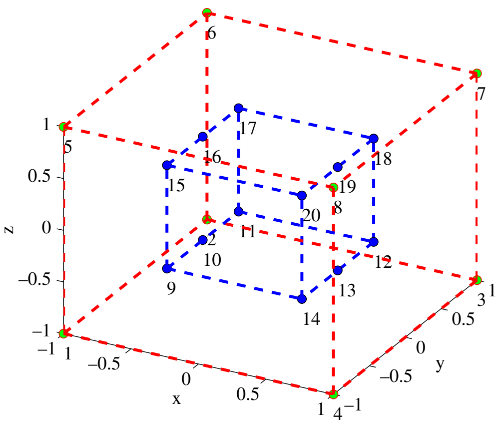

Fig. 3. The target formation with 8 leaders and 12 followers.

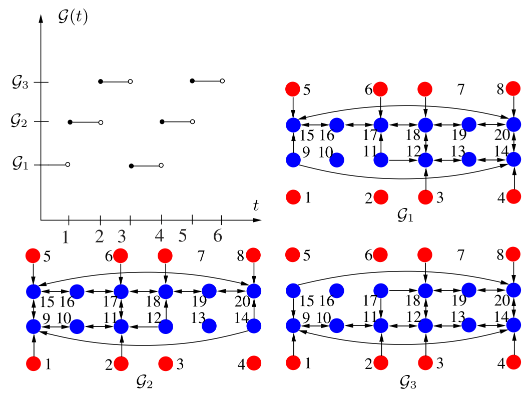

Fig. 4. A periodic switching graph $\mathcal{G}(t)$ that switches among three different topologies $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$.

Because $\sigma(\Gamma) \subset \overline{\mathbb{C}}_-$, there exist positive constants α_3 and α_4 such that

$$\|\eta_i(t) - v_r(t)\| \le \alpha_3 e^{-\alpha_4(t-t_0)} \|\eta_i(t_0) - v_r(t_0)\|,$$

or equivalently $\|\delta_i(t)\| \le \alpha_3 e^{-\alpha_4(t-t_0)} \|\delta_i(t_0)\|$, which proves the uniform asymptotic stability of (20).

($\Rightarrow$) The proof is the same as for Theorem 4.2. ■

6. Simulation

In this section, we present a simulation to validate our theoretic results. Consider 8 leaders moving in a globally rigid formation and consider 12 followers with any initial states. Denote the set of leaders by $\mathcal{V}_a = \{1, 2, \ldots, 8\}$ and the set of followers by $\mathcal{V}_b = \{9, 10, \ldots, 20\}$. Suppose the target formation $[{p_a}^{\mathsf{T}}, {p_b}^{\mathsf{T}}]^{\mathsf{T}}$ be the one shown in Fig. 3.

In the simulation, we consider a periodic switching interaction graph $\mathcal{G}(t)$ that switches among three different topologies as shown in Fig. 4. It can be checked that every follower is uniformly jointly reachable from $\mathcal{V}_a$. Moreover, suppose that $\Gamma = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Then under the control law (14), the simulation result is presented in Fig. 5, from which we see the followers are uniformly asymptotically merged with the leaders to form a single globally rigid formation.

7. Conclusion

This paper developed a distributed control law for the formation merging control problem. We showed that a group of followers can be asymptotically merged with another group of leaders if and only if every follower is jointly reachable from a leader over any time interval of certain length. However, there are still several interesting issues remaining open. For example, how can we remove the convex assumption about the neighbors, how to analyze the asymptotic convergence property under switching topologies, and how can we preserve the graph connectivity condition as the system evolves?

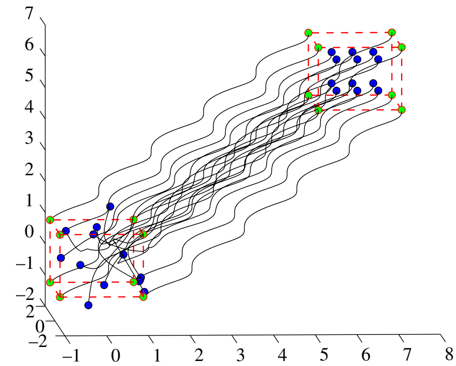

Fig. 5. Asymptotical formation merging.

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