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Brief paper

Distributed event-driven control for finite-time consensus☆

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ABSTRACT

This paper is concerned with how multi-agent networks achieve finite-time consensus using distributed event-driven control. Due to the hybrid nonlinearities arising from the nonsmooth control and the triggering condition, finite-time consensus analyses are more challenging with event-driven control than with continuous-time control. We study agents with single integrator dynamics and scalar states and present a distributed event-driven control protocol for the finite-time consensus, with comparison to continuous-time control. It is shown that using the proposed event-driven control scheme, agents can reach consensus within a limited time and without Zeno behavior. We also obtain an estimate for the settling time and demonstrate that it is not only related to the initial condition and network connectivity, but is also linked with the event-triggering condition. Simulations are given to demonstrate the theoretical results.

1. Introduction

Multi-agent networks (MANs) find applications in various engineering fields such as unmanned aerial vehicles (UAVs), cooperative robots, wireless sensor networks, and power systems. Distributed control for multi-agent cooperation has gained much attention in the past decade (Cai, Lewis, Hu, & Huang, 2017; Hou, Fu, Zhang, & Wu, 2017; Lu, Han, Zhang, Liu, & Liu, 2017; Olfati-Saber & Murray, 2004). Consensus control theory plays a fundamental role in studying multi-agent cooperation since other collective behaviors such as flocking, formation, and distributed optimization are mostly consensus-based, see, e.g., Chen, Wen, Liu, and Liu (2016), Guan, Hu, Chi, He, and Cheng (2014) and Li, Liao, and Huang (2013).

In real-world applications such as UAVs cruising, vehicles tracking, and robots environmental monitoring, it is important for agents to do all planed tasks within a limited time (Cao & Ren, 2014; Franceschelli, Pisano, Giua, & Usai, 2015; Lu et al., 2017). From this practical consideration, the convergence performance should be taken into the design of control protocols. Previous studies suggest that with possible control protocols, MANs can achieve consensus asymptotically with an exponential convergence rate, and the rate of convergence is related to the network connectivity, i.e., the second-smallest eigenvalue of the Laplacian matrix. One further characterization of the convergence performance could be finite-time dynamics (Guan, Sun, Wang, & Li, 2012; Meng, Jia, & Du, 2016). Differing from asymptotic convergence, finite-time convergence requires a prescribed settling time that may depend on the initial conditions. Comparing with asymptotic consensus control, finite-time control ensures consensus in a limited time setup meanwhile with calculation/communication cost reduced, thus enabling better application in UAVs and cooperative robots.

The finite-time consensus analysis is based largely on the finite-time stability theory of continuous-time autonomous systems. Early in Tang (1998), a Lyapunov-based approach was presented to ensure the finite-time stability using terminal sliding mode control. Finite-time stability of continuous but non-Lipschitzian autonomous systems was investigated in Bhat and Bernstein (2000). There are many existing results on distributed finite-time control of MANs, including finite-time consensus and finite-time tracking. The finite-time consensus problems under bidirectional and unidirectional interaction cases were studied in Wang and Xiao (2010). A distributed binary consensus protocol and pinning control were used to ensure finite-time consensus in Chen, Lewis,

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and Xie (2011). Finite-time consensus for leader-following second-order MANs was studied with a continuous-time nonsmooth control in Guan et al. (2012). Finite-time convergence of a nonlinear, continuous consensus algorithm for MANs with unknown inherent nonlinear dynamics was analyzed in Cao and Ren (2014). Finite-time consensus with disturbance rejection by discontinuous local interactions was considered in Franceschelli et al. (2015). Recently in Liu, Lam, Yu, and Chen (2016), finite-time consensus of MANs was studied with a switching protocol, while in Meng et al. (2016) finite-time consensus of agents over antagonistic signed networks was investigated.

In addition to convergence performance, the control cost is another key factor for designing control protocols. It is unnecessary to update controller in a time-clocked manner, continuously or periodically, especially for sensor agents with limited resources. The event-driven method has proved an alternative to the time-clocked ones (Dimarogonas, Frazzoli, & Johansson, 2012; Guo, Ding, & Han, 2014; Li, Yu, Yu, Huang, & Liu, 2016; Tabuada, 2007; Zhang, Feng, Yan, & Chen, 2014). In an event-driven setup, the control updating occurs only at event-based time instants, thus reducing the number of actuator updates (Fan, Feng, Wang, & Song, 2013; Hu, Liu, & Feng, 2016; Zhu, Jiang, & Feng, 2014). Early in Tabuada (2007), an event-triggered strategy was adopted for stabilizing nonlinear systems. Both centralized and distributed event-triggered controls were designed for consensus in Dimarogonas et al. (2012). A novel sampling-event-based control scheme was presented for designing consensus protocols in Meng and Chen (2013). A distributed event-triggering sampling scheme was designed to ensure leader-following consensus in Li, Liao, Huang, and Zhu (2015). The probabilistic consensus of MANs was studied with output feedback event-triggered control in Ding, Wang, Shen, and Wei (2015). A practical layered event control scheme was developed for multi-agent consensus in Xu, Chen, and Ho (2017).

A recent survey of event-triggered control for asymptotic consensus has been done in Ding, Han, Ge, and Zhang (2018). However, there has been very little work on designing event-driven control for finite-time stability or consensus. Most of the existing works on finite-time consensus use nonsmooth but continuous-time control methods (Cao & Ren, 2014; Franceschelli et al., 2015; Fu & Wang, 2016). An event-triggered control for finite-time consensus was reported in Lu et al. (2017), considering MANs with position and velocity dynamics and switching topologies. Beyond control updates, the triggering condition should have an influence on the convergence performance (time cost), as shown in Tabuada (2007). From this point of view, it is still in demand to bridge the settling time and the event-triggering condition in seeking multi-agent cooperation. The event-driven finite-time control algorithm cannot be dealt with either similarly to asymptotic consensus analyses in Dimarogonas et al. (2012), Hu et al. (2016), Xie, Xu, Chu, and Zou (2015) and Zhang et al. (2014), or using the existing finite-time convergence analyses in Bhat and Bernstein (2000), Lu et al. (2017), Meng et al. (2016) and Wang and Xiao (2010). Difficulties thus arise from synthesizing the hybrid nonlinearity caused by the nonsmooth control and the event-triggering condition in the Lyapunov analysis, as well as showing a link between the convergence time and the event condition.

The above observations motivate to develop an event-driven finite-time control protocol for MANs with single-integrator model and fixed topology in this paper. The objective here is to design a distributed event-driven control protocol for ensuring finite-time consensus, meanwhile showing a link between the settling time and the triggering condition. To this end, an event-driven finite-time control protocol is first presented and analyzed with comparison to continuous-time control. Then, new criteria including an effective triggering condition are established to guarantee consensus in a finite-time manner. We also obtain an estimate for the settling time that is determined by the MAN model, the control gain and the event-triggering threshold. The developed results thus offer an insight into how networked agents cooperate with event-driven control and data in a finite-time manner.

The rest of the paper is organized as follows. Section 2 presents some preliminaries and formulates the research problem. Section 3 proposes an event-driven control protocol for the finite-time consensus and derives an estimate for the settling time involving the triggering condition. Section 4 gives simulation results while Section 5 concludes the paper.

Notations: Let $\Re$ be the set of real numbers, $\Re_+$ the set of nonnegative real numbers, and $\Re^n$ the set of $n \times 1$ real vectors. Let $y = \text{col}(y_1, y_2, \ldots, y_n)$ be the $n \times 1$ real vector, and $y^\top$ its transposition. Denote $\|y\| = \|y\|_2 = \left(\sum_{i=1}^n y_i^2\right)^{1/2}$ and $\text{dist}\big(y, Q\big) = \inf_{q \in Q} \|y - q\|^2$ the distance of vector y on set $Q \subset \Re^n$. Let $[y_1]^{[\mu]} = \text{sign}(y_1)|y_1|^\mu$ and $\text{sign}(\cdot)$ denote the sign function. For a continuous function $f(t) : \Re_+ \to \Re^n$, the upper Dini derivative is denoted by $\text{D}^+ f(t) = \limsup_{\tau \to 0^+} (f(t+\tau) - f(t))/\tau$.

2. Preliminaries and problem formulation

In this section, preliminaries for finite-time convergence and the research problem of the paper are formulated.

2.1. Preliminaries

Consider the autonomous system

$$\dot{z}(t) = g(z(t)), \quad t \in \Re_+, \quad z(0) = z_0\,, \tag{1}$$

where $z = z(t) \in \Re^n$ is the state at time t, $g : \Re^n \to \Re^n$ is continuous, and $g(0) = 0$.

Definition 1. The set $\Theta \subset \Re^n$ is said to be finite-time attractive for system (1) if there exists a settling time $T_s > 0$, such that for any initial state z_0,

$$\begin{cases} \lim\limits_{t \to T_s} \text{dist}\big(z(t), \Theta\big) = 0, \\ \text{dist}\big(z(t), \Theta\big) = 0, \quad t > T_s. \end{cases}$$

Lemma 2. *Let $V(z) : \Re^n \to \Re_+$ be a continuous and positive definite function, satisfying*

(i) $V(z) = 0 \Leftrightarrow z \in \Theta$,
(ii) *the Dini derivative of $V(z)$ along the solutions of system (1) is bounded as*

$$\text{D}^+ V(z(t))\big|_{(1)} \le -aV(z(t))^\mu, \tag{2}$$

where $0 < \mu < 1$ and $a > 0$ are given constants.

Then, the set Θ is finite-time attractive for system (1) with the settling time

$$T_s = \frac{1}{1-\mu}\frac{V(z_0)^{1-\mu}}{a} \ll \infty. \tag{3}$$

Proof. The proof is similar to that were reported in Bhat and Bernstein (2000) and Tang (1998), thus is omitted. □

Lemma 3. *For any $\tilde{y}, \tilde{z} \in \Re$ and $0 < \mu \le 1$,*

$$\text{(i)} \quad \big|\tilde{y} + \tilde{z}\big|^\mu \le |\tilde{y}|^\mu + |\tilde{z}|^\mu, \tag{4}$$

(ii) *if $|\tilde{z}| \le |\tilde{y}|$, then*

$$-\tilde{y}[\tilde{y} + \tilde{z}]^{[\mu]} \le -\tilde{y}[\tilde{y}]^{[\mu]} + |\tilde{y}|\,|\tilde{z}|^\mu. \tag{5}$$

Proof. See the Appendix. □

Lemma 4. (i) *For any* $\xi_1, \xi_2, \ldots, \xi_N \in \Re_+$, $0 < \mu < 1$, *and* $\nu > 1$,

$$\Big(\sum_{r=1}^{N} \xi_r\Big)^{\mu} \le \sum_{r=1}^{N} \xi_r^{\mu} \le N^{1-\mu}\Big(\sum_{r=1}^{N} \xi_r\Big)^{\mu},$$

$$N^{1-\nu}\Big(\sum_{r=1}^{N} \xi_r\Big)^{\nu} \le \sum_{r=1}^{N} \xi_r^{\nu}.$$

(ii) *For continuous function* $\xi(t) = \mathrm{col}\big(\xi_1(t), \xi_2(t), \ldots, \xi_N(t)\big)$, *with* $\xi_i(t) \ne 0$, $i = 1, 2, \ldots, N$, *and* $t \in [b_1, b_2]$, *there exists a constant* $\theta_i \ge 1$ *such that*

$$\theta_i|\xi_i(t)| \ge (1/\sqrt{N})\|\xi(t)\|, \quad t \in [b_1, b_2],$$

where $\theta_i = \frac{\max_i\{M_i\}}{m_i}$, $M_i = \max_{t\in[b_1,b_2]}\big\{|\xi_i(t)|\big\}$, *and* $m_i = \min_{t\in[b_1,b_2]}\big\{|\xi_i(t)|\big\}$, $0 \le b_1 \le b_2 < \infty$.

Proof. See the Appendix. □

2.2. Problem formulation

Consider an MAN consisting of N agents, the communication topology is a weighted undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, with a node set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and an edge set $\mathcal{E} = \{(i,j)|i, j \in \mathcal{V}\}$. Each agent has the dynamics

$$\dot{x}_i(t) = u_i(t), \quad t \in \Re_+, \quad i = 1, 2, \ldots, N, \tag{6}$$

where $x_i(t) \in \Re$ is the state variable, and $u_i(t) \in \Re$ is the control input. Denote $x = \mathrm{col}(x_1, x_2, \ldots, x_N)$.

Let $\mathcal{A} = \big(a_{ij}\big)_{N\times N}$ be the adjacency matrix, where $a_{ij} > 0$ if $(i,j) \in \mathcal{E}$, otherwise $a_{ij} = 0$, and $a_{ii} = 0$ for all $i \in \mathcal{V}$. $N_i = \{j : j \in \mathcal{V},\ a_{ij} > 0\}$ denotes the neighboring set of agent i. The Laplacian matrix $L = \big(l_{ij}\big)_{N\times N}$ of graph $\mathcal{G}$ is defined as: $l_{ij} = -a_{ij}$ $(i \ne j)$ and $l_{ii} = \sum_{r=1, r\ne i}^{N} a_{ir}$, $i, j = 1, 2, \ldots, N$.

According to Guan et al. (2014) and Olfati-Saber and Murray (2004), if the graph $\mathcal{G}$ is undirected and connected, then the Laplacian matrix L is positive semi-definite, and its eigenvalues satisfy $0 = \lambda_1(L) < \lambda_2(L) \le \cdots \le \lambda_N(L)$.

Definition 5. MAN (6) is said to reach finite-time consensus if for any initial condition $x(0)$, the set $\mathcal{M} = \{x : Lx = 0\}$ is finite-time attractive for (6).

As mentioned above, an event-driven control protocol uses only certain sampling data for the updates of controller. While being used for seeking the finite-time consensus, the advantages of event-driven control can be a reduction in the updating number as well as a tradeoff between the settling time and the event-triggering condition. *The main objective* of the paper then is to design a distributed event-driven control with a specific event-based settling time $T_s > 0$ such that the set $\mathcal{M} = \{x : Lx = 0\}$ is finite-time attractive for (6), or equivalently $\lim_{t\to T_s} \|x_i(t) - x_j(t)\| = 0$ and $x_i(t) = x_j(t)$, for all $t > T_s$ and $i, j = 1, 2, \ldots, N$.

3. Control design with event-driven data

In this section, a distributed event-driven control scheme is designed and analyzed with the goal of solving the finite-time consensus problem for MAN (6).

For agent i, denote by t_k^i the sampling instant for $x_i(t)$, $i = 1, 2, \ldots, N$, $k = 0, 1, 2, \ldots$. The state error for agent i is defined as

$$e_i(t) = x_i(t_k^i) - x_i(t), \quad t \in \big[t_k^i, t_{k+1}^i\big).$$

Similarly to related work (Dimarogonas et al., 2012; Xu et al., 2017), the event condition for agent i is then defined as

$$|e_i(t)| \le \sigma_i|\hat{x}_i(t)|, \tag{7}$$

where $\hat{x}_i = L_i x$ is the ith element of $\hat{x} = Lx$, and $\sigma_i > 0$ denotes the event threshold.

More specifically, the sampling instant t_{k+1}^i is determined by

$$t_{k+1}^i = \inf\Big\{t > t_k^i\Big||e_i(t)| > \sigma_i|\hat{x}_i(t)|\Big\}.$$

For the purpose of finite-time consensus, an event-driven control protocol is given as

$$u_i(t) = -\alpha\Big[\sum_{j=1}^{N} a_{ij}\big(x_i(t_k^i) - x_j(t_{k_j'(t)}^j)\big)\Big]^{[\mu]}, \tag{8}$$

where $t \in \big[t_k^i, t_{k+1}^i\big)$, $k_j'(t) = \arg\min_{b\in\mathbb{N}}\{t - t_b^j | t_b^j \le t_k^i\}$, $t_{k_j'(t)}^j$ denotes the last event instant of agent j from time t, $0 < \mu < 1$, and $\alpha > 0$ is the control gain.

Remark 6. The control protocol (8) suggests that each agent updates its controller only at event instants t_k^i. Due to the ZOH (zero-order-hold) rule, for $t \in [t_k^i, t_{k+1}^i)$, the controller (8) takes a constant input that is a combination of its own data at the current event instant and the most recent neighboring data from the last event instant. Comparing with the continuous-time counterpart:

$$u_i(t) = -\alpha\Big[\sum_{j=1}^{N} a_{ij}\big(x_i(t) - x_j(t)\big)\Big]^{[\mu]}, \quad t \in \Re_+, \tag{9}$$

the event-driven controller (8) allows a reduction in the number of control updates. Under the triggering condition (7), the protocol (8) can improve control implementation, while it may require a larger settling time to reach the consensus, as will be discussed later.

For comparison, the finite-time consensus of MAN (6) is first studied using the continuous-time control protocol (9).

Consider the following Lyapunov function candidate

$$\begin{aligned} W(x) &= \frac{1}{2}x^{\top}Lx \\ &= \frac{1}{2}\Big[\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\big(x_i - x_j\big)^2\Big]. \end{aligned} \tag{10}$$

Lemma 7. *Suppose that the graph* $\mathcal{G}$ *is connected. Then under continuous-time control protocol* (9), *MAN* (6) *reaches finite-time consensus with the estimated settling time*

$$T_S = \frac{1}{1-\mu}\frac{W(x(0))^{\frac{1-\mu}{2}}}{\alpha 2^{\frac{\mu-1}{2}}\lambda_2^{\frac{\mu+1}{2}}}, \tag{11}$$

where $\alpha > 0$, *and* λ_2 *is the second-smallest eigenvalue of the Laplacian matrix L.*

Proof. The proof is similar to that from Parsegov, Polyakov, and Shcherbakov (2013), thus is omitted. □

In the following, the finite-time consensus of MAN (6) with the event-driven control protocol (8) is analyzed.

According to the event-driven scheme, when an event occurs, i.e., the condition $|e_i(t)| \le \sigma_i|\hat{x}_i(t)|$ breaks, $e_i(t) = 0$ is enforced. By the definitions of $e_i(t)$ and $k_j'(t)$, one has $x_i(t_k^i) = e_i(t) + x_i(t)$ and $x_j(t_{k_j'(t)}^j) = x_j(t) + e_j(t)$, $j \in N_i$, $t \in \big[t_k^i, t_{k+1}^i\big)$. Then

$$u_i(t) = -\alpha\Big[\sum_{j=1}^{N} a_{ij}\big(x_i(t) - x_j(t) + e_i(t) - e_j(t)\big)\Big]^{[\mu]}. \tag{12}$$

Substituting (12) into (6) gives the closed loop

$$\dot{x}_i(t) = -\alpha\Big[\sum_{j=1}^{N} a_{ij}\big(x_i(t) - x_j(t) + e_i(t) - e_j(t)\big)\Big]^{[\mu]}. \tag{13}$$

The following theorem shows that MAN (6) can also reach finite-time consensus under the event-driven control protocol (8).

Theorem 8. *Suppose that the graph $\mathcal{G}$ is connected and the controller (8) is triggered by the event condition (7), with the thresholds σ_i, $i = 1, 2, \ldots, N$ satisfying*

$$0 < \sigma_{\max} = \max_i\{\sigma_i\} < \sqrt{\frac{\lambda_2}{\lambda_N^3 N^{\frac{1-\mu}{1+\mu}}}}. \tag{14}$$

Then under the event-driven control protocol (8)

(i) *MAN (6) reaches finite-time consensus within the settling time estimate*

$$\hat{T}_S = \frac{1}{1-\mu}\frac{W(x(0))^{\frac{1-\mu}{2}}}{\hat{\alpha}2^{\frac{\mu-1}{2}}\lambda_2^{\frac{\mu+1}{2}}}, \tag{15}$$

(ii) *MAN (6) does not exhibit Zeno behavior on time interval $[0, \hat{T}_S)$,*

where $\hat{\alpha} = \alpha\frac{\mu}{1+\mu}\left[1-N^{\frac{1-\mu}{2}}\left(\frac{\lambda_N^3\sigma_{\max}^2}{\lambda_2}\right)^{\frac{\mu+1}{2}}\right] > 0$, $0 < \mu < 1$, $\alpha > 0$, λ_2 and λ_N are respectively the second-smallest and largest eigenvalues of the Laplacian matrix L, and N is the number of agents.

Proof. (i) The first part verifies the finite-time convergence for consensus. The Dini derivative of $W(x)$ along the state trajectories of MAN (13) satisfies

$$\begin{aligned}
&\mathrm{D}^+W(x(t))\big|_{(13)}\\
&= -\alpha\sum_{i=1}^N\Bigg\{\Big[\sum_{j=1}^N a_{ij}\big(x_i(t)-x_j(t)\big)\Big]\\
&\cdot\Big[\sum_{j=1}^N a_{ij}\big(x_i(t)-x_j(t)+e_i(t)-e_j(t)\big)\Big]^{[\mu]}\Bigg\}\\
&= -\alpha\sum_{i=1}^N\Big\{\hat{x}_i(t)\big[\hat{x}_i(t)+\hat{e}_i(t)\big]^{[\mu]}\Big\},
\end{aligned}$$

where $\hat{x}_i = L_i x = \sum_{j=1}^N a_{ij}(x_i - x_j)$, $\hat{e}_i = L_i e = \sum_{j=1}^N a_{ij}(e_i - e_j)$, and L_i is the ith row of matrix L.

It is now to show that $|\hat{e}_i| < |\hat{x}_i|$, $i = 1, 2, \ldots, N$, under the event condition (7) with the threshold satisfying (14). Since $\sum_{i=1}^N |\hat{e}_i|^2 = \sum_{i=1}^N e^\top L_i^\top L_i e = e^\top L^\top Le$, it follows that

$$\begin{aligned}
\sum_{i=1}^N|\hat{e}_i|^2 &\le \lambda_N^2\sum_{i=1}^N|e_i|^2 \le \lambda_N^2\sigma_{\max}^2\sum_{i=1}^N|\hat{x}_i|^2\\
&< \frac{\lambda_2}{\lambda_N N^{\frac{1-\mu}{1+\mu}}}\sum_{i=1}^N|\hat{x}_i|^2 < \sum_{i=1}^N|\hat{x}_i|^2.
\end{aligned} \tag{16}$$

By the definitions: $e_i = x_i(t_k^i) - x_i$, $\hat{e}_i = L_i e$ and $\hat{x}_i = L_i x$, without loss of generality, let $\hat{e}_i = q_i\hat{x}_i$. The inequality (16) is then equivalent to $\sum_{i=1}^N q_i^2|\hat{x}_i|^2 < \sum_{i=1}^N|\hat{x}_i|^2$. The matrix form is $x^\top L^\top Q^2 Lx < x^\top L^\top Lx$, where $Q = \mathrm{diag}(q_1, q_2, \ldots, q_N)$ is the diagonal matrix. Note that the initial state of MAN (6) is arbitrary. That is, the inequality $x^\top L^\top Q^2 Lx < x^\top L^\top Lx$ holds for all $x \in \Re^N$. Thus, one has $0 < |q_i| < 1$, implying $|\hat{e}_i| < |\hat{x}_i|$.

Using Lemma 3(ii), one has

$$\begin{aligned}
&\mathrm{D}^+W(x(t))\big|_{(13)}\\
&\le -\alpha\sum_{i=1}^N\hat{x}_i(t)\big[\hat{x}_i(t)\big]^{[\mu]} + \alpha\sum_{i=1}^N\big|\hat{x}_i(t)\big|\cdot\big|\hat{e}_i(t)\big|^{\mu}\\
&\le -\alpha\sum_{i=1}^N\Big|\sum_{j=1}^N a_{ij}\big(x_i(t)-x_j(t)\big)\Big|^{\mu+1}\\
&\quad+\alpha\sum_{i=1}^N\Bigg[\frac{1}{1+\mu}\Big|\sum_{j=1}^N a_{ij}\big(x_i(t)-x_j(t)\big)\Big|^{\mu+1}\\
&\quad+\frac{\mu}{1+\mu}\Big|\sum_{j=1}^N a_{ij}\big(e_i(t)-e_j(t)\big)\Big|^{\mu+1}\Bigg],
\end{aligned} \tag{17}$$

where the last inequality is based on the Young's inequality.

Recalling the event condition (7), one obtains

$$\begin{aligned}
&\sum_{i=1}^N\Bigg[\Big|\sum_{j=1}^N a_{ij}\big(e_i(t)-e_j(t)\big)\Big|^2\Bigg]^{\frac{\mu+1}{2}}\\
&\le N^{\frac{1-\mu}{2}}\Bigg\{\sum_{i=1}^N\Big[\sum_{j=1}^N a_{ij}\big(e_i(t)-e_j(t)\big)\Big]^2\Bigg\}^{\frac{\mu+1}{2}}\\
&\le N^{\frac{1-\mu}{2}}(2\lambda_N^3\sigma_{\max}^2)^{\frac{\mu+1}{2}}(W(x))^{\frac{\mu+1}{2}}.
\end{aligned}$$

Substituting the above inequality into (17) gives

$$\begin{aligned}
&\mathrm{D}^+W(x(t))\big|_{(13)}\\
&\le -\alpha\left(1-\frac{1}{1+\mu}\right)\sum_{i=1}^N\Big|\sum_{j=1}^N a_{ij}\big(x_i(t)-x_j(t)\big)\Big|^{\mu+1}\\
&\quad+\alpha\frac{\mu}{1+\mu}\Bigg\{\sum_{i=1}^N\Big[\sum_{j=1}^N a_{ij}\big(e_i(t)-e_j(t)\big)\Big]^2\Bigg\}^{\frac{\mu+1}{2}}\\
&\le -\alpha\frac{\mu}{1+\mu}(2\lambda_2)^{\frac{\mu+1}{2}}W(x)^{\frac{\mu+1}{2}}\\
&\quad+\alpha\frac{\mu}{1+\mu}N^{\frac{1-\mu}{2}}(2\sigma_{\max})^{\frac{\mu+1}{2}}W(x)^{\frac{\mu+1}{2}}\\
&= -\frac{\alpha\mu}{1+\mu}(2\lambda_2)^{\frac{\mu+1}{2}}\left[1-N^{\frac{1-\mu}{2}}\left(\frac{\sigma_{\max}^2\lambda_N^3}{\lambda_2}\right)^{\frac{\mu+1}{2}}\right]W(x)^{\frac{\mu+1}{2}}\\
&= -\hat{\alpha}(2\lambda_2)^{\frac{\mu+1}{2}}W(x)^{\frac{\mu+1}{2}}.
\end{aligned} \tag{18}$$

Since $0 < \frac{\mu+1}{2} < 1$ and $\mathcal{M} = \{x : W(x) = 0\} = \{x : Lx = 0\}$, it follows from Lemma 2 that the set $\mathcal{M}$ is finite-time attractive for MAN (13) under the event condition (7). Therefore, MAN (6) reaches the finite-time consensus within the estimated settling time (15).

(ii) This part shows the exclusion of Zeno behavior. It will be verified that each inter-event time $t_{k+1}^i - t_k^i$ that implicitly defined by (7) is positively lower bounded.

Consider that an event of agent i occurs at time t_k^i. According to the event-driven scheme, one has $|e_i(t_k^i)| = 0$, and only when the error $|e_i(t)|$ is about to exceed the twisted threshold $\sigma_i|\hat{x}_i(t)|$, for $\hat{x}_i(t) \ne 0$, agent i will be reactivated. Thus, before the next event time, one has $|e_i(t)|\big/|\hat{x}_i(t)| \le \sigma_i$.

Similarly to Dimarogonas et al. (2012) and Zhang et al. (2014), the comparison principle of differential equations is used to obtain a positive lower bound for $t_{k+1}^i - t_k^i$. Clearly, $|e_i(t)| \le \|e(t)\|$, $i = 1, 2, \ldots, N$. By Lemma 4(ii), for $\hat{x}_i(t) \ne 0$ and $|\hat{x}_i(t)| < \infty$, there exists a finite constant $\vartheta_i \ge 1$ such that $\vartheta_i|\hat{x}_i(t)| \ge (1/\sqrt{N})\|\hat{x}(t)\|$. One obtains $\frac{|e_i(t)|}{|\hat{x}_i(t)|} \le \vartheta_i\sqrt{N}\frac{\|e(t)\|}{\|\hat{x}(t)\|}$. Then, the time interval for which $|e_i(t)|\big/|\hat{x}_i(t)|$ ranges from 0 to σ_i is greater than that $\vartheta_i\sqrt{N}\|e(t)\|\big/\|\hat{x}(t)\|$ needs.

The time derivative of $\|e(t)\|\big/\|\hat{x}(t)\|$ satisfies

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\frac{\|e\|}{\|\hat{x}\|} &\le \frac{\|\dot{x}\|\cdot\|\hat{x}\| + \|e\|\cdot\|\dot{\hat{x}}\|}{\|\hat{x}\|^2}\\
&\le \frac{\|\dot{x}\|\cdot\|\hat{x}\| + \|L\|\cdot\|e\|\cdot\|\dot{x}\|}{\|\hat{x}\|^2}.
\end{aligned} \tag{19}$$

Based on Lemma 3(i) and Lemma 4(i), one gets

$$\begin{aligned}\|\dot{x}\| &\le \sum_{i=1}^{N} |\dot{x}_i| \\ &= \alpha \sum_{i=1}^{N} \Big|\hat{x}_i(t) + \sum_{j=1}^{N} a_{ij}\big(e_i(t) - e_j(t)\big)\Big|^{\mu} \\ &\le \alpha N^{1-\frac{\mu}{2}} \Big(\sum_{i=1}^{N} \hat{x}_i(t)^2\Big)^{\frac{\mu}{2}} \\ &\quad + \alpha N^{1-\frac{\mu}{2}} \Big[\sum_{i=1}^{N}\Big(\sum_{j=1}^{N} a_{ij}\big(e_i(t) - e_j(t)\big)\Big)^2\Big]^{\frac{\mu}{2}} \\ &= \alpha N^{1-\frac{\mu}{2}} \Big(\|\hat{x}\|^{\mu} + \|Le\|^{\mu}\Big). \end{aligned} \tag{20}$$

According to the event-driven scheme, one has $\|e\| \le \sigma_{\max}\|\hat{x}\|$ and e_i can be detected at any time t. Let $E_{ik} = \max\{|e_i(t)| : t > t_k^i, e_i \neq 0\}$. Substituting (20) into (19) then gives

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\frac{\|e\|}{\|\hat{x}\|} \\ &\le \alpha N^{1-\frac{\mu}{2}} \Big(1 + \|L\|\frac{\|e\|}{\|\hat{x}\|}\Big)\Big[1 + \|L\|^{\mu}\Big(\frac{\|e\|}{\|\hat{x}\|}\Big)^{\mu}\Big]\frac{1}{\|\hat{x}\|^{1-\mu}} \\ &\le \alpha N^{1-\frac{\mu}{2}} \frac{\sigma_{\max}^{1-\mu}}{E_{ik}^{1-\mu}}\Big(1 + \|L\|\frac{\|e\|}{\|\hat{x}\|}\Big)\Big[1 + \|L\|^{\mu}\Big(\frac{\|e\|}{\|\hat{x}\|}\Big)^{\mu}\Big]. \end{aligned} \tag{21}$$

By the comparison principle, the inequality (21) yields $\|e(t)\|/\|\hat{x}(t)\| \le \psi(t)$, where $\psi(t)$ is the solution of differential equation

$$\dot{\psi}(t) = b_{ik}\big(1 + \|L\|\psi(t)\big), \quad \psi(t_k^i) = 0, \quad t \in [t_k^i, t_{k+1}^i),$$

with $b_{ik} = \alpha N^{1-\frac{\mu}{2}}\big(\sigma_{\max}^{1-\mu} + \|L\|^{\mu}\sigma_{\max}\big)/E_{ik}^{1-\mu} > 0$.

Solving the above differential equation gives $\psi(t) = \frac{\exp\left(\|L\| b_{ik}(t-t_k^i)\right)-1}{\|L\|}$. Denote $y_i(t) = |e_i(t)|/|\hat{x}_i(t)|$. Before the next event instant, one has $y_i(t) \le \vartheta_i\sqrt{N}\int_{t_k^i}^{t}\dot{\psi}(s)\mathrm{d}s$. Under the event condition (7), the next event instant is no less than $t_k^i + \tau_{ik}$, where τ_{ik} satisfies $\vartheta_i\sqrt{N}\int_{t_k^i}^{t_k^i+\tau_{ik}}\dot{\psi}(s)\mathrm{d}s = \sigma_i$. Thus

$$t_{k+1}^i - t_k^i \ge \tau_{ik} = \frac{1}{\lambda_N b_{ik}}\ln\Big(1 + \frac{\lambda_N\sigma_i}{\vartheta_i\sqrt{N}}\Big) > 0\,. \tag{22}$$

This completes the proof. □

Remark 9. In (14), the triggering threshold σ_i is supposed to be no greater than $\sqrt{\lambda_2/\big(\lambda_N^3 N^{\frac{1-\mu}{1+\mu}}\big)}$ which, with $0 < \mu < 1$, is inversely proportional to the network scale N. Meanwhile the lower bound of the inter-event time $t_{k+1}^i - t_k^i$ is positively determined by the event threshold σ_i, as shown in (22). These observations are consistent with the phenomenon that as the network scale N goes bigger, MANs require more amount of control actuation/load. According to Zhang et al. (2014), the scalar ϑ_i also influences the inter-event time. One has $\vartheta_i = 1$ when $|\hat{x}_i|$ is the largest element of $\big\{|\hat{x}_1|, |\hat{x}_2|, \ldots, |\hat{x}_N|\big\}$. In particular, the ϑ_i in (22) will approach 1 since the states of all agents will reach an agreement following the control protocol (8).

Remark 10. Specifically in Theorem 8, the condition (14) gives $0 < 1 - N^{\frac{1-\mu}{2}}\big(\frac{\lambda_N^3\sigma_{\max}^2}{\lambda_2}\big)^{\frac{\mu+1}{2}} < 1$, implying that $0 < \hat{\alpha} < \alpha$. The event-driven control protocol (8) then has a larger settling time than the continuous-time controller (9), and this phenomenon can be verified by comparing the two time estimates respectively given by (11) and (15). As σ_i decreases near enough to 0, the event-driven time estimate (15) also decreases near to that given by (11). Letting $\sigma_i = 0$, the first inequality in (17) becomes

$$\begin{aligned}\mathrm{D}^{+}W(x(t))\big|_{(13)} &\le -\alpha\sum_{i=1}^{N}\hat{x}_i(t)\big[\hat{x}_i(t)\big]^{[\mu]} \\ &= -\alpha\sum_{i=1}^{N}\Big|\sum_{j=1}^{N}a_{ij}\big(x_i(t) - x_j(t)\big)\Big|^{\mu+1}. \end{aligned}$$

Then the result of Theorem 8 conforms to that developed in Lemma 7. In contrast, the advantage of the event-driven control protocol (8) is a reduction in the number of control updates, i.e., a decrease in control effort, and a tradeoff between the settling time and the triggering condition.

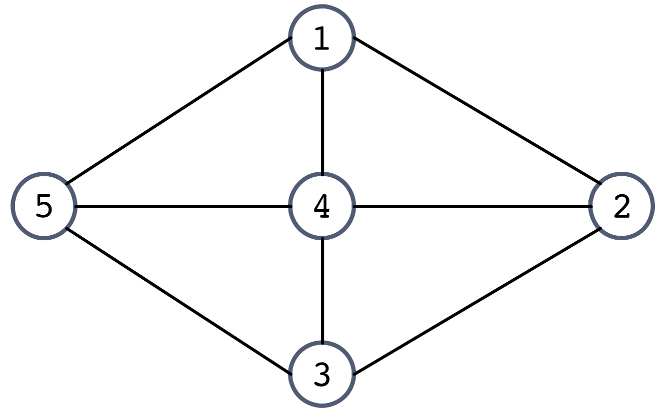


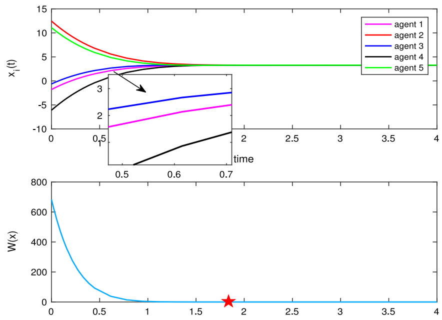
Fig. 1. Communication graph.

Remark 11. Theorem 8 provides an appropriate unified framework for synthesizing hybrid control systems consisting of nonsmooth control and event-driven data. Differing from the existing work, e.g., Dimarogonas et al. (2012), Li et al. (2015), Liu et al. (2016) and Meng et al. (2016), the trick here is using the inequalities given in Lemma 3 to deal with the event-triggering condition in the Lyapunov analysis, though the agent model is limited to single integrator and scalar state. It can be verified that Lemma 3 holds for the multidimensional case, i.e., $[z]^{[\mu]} = \mathrm{col}\big(\mathrm{sign}(z_1)|z_1|^{\mu}, \ldots, \mathrm{sign}(z_n)|z_n|^{\mu}\big)$, $z \in \Re^n$. In this case, combining the consensus control methods developed in Guan et al. (2014, 2012) and Lu et al. (2017), the proposed event-driven control scheme would work also well on complicated MANs, e.g., with double integrator dynamics, vector-valued states and switching topologies. In addition, using the sampled-data-event based method from Meng and Chen (2013), the proposed control approach can be further applied to address the finite-time consensus or optimization problems for discrete-time MANs.

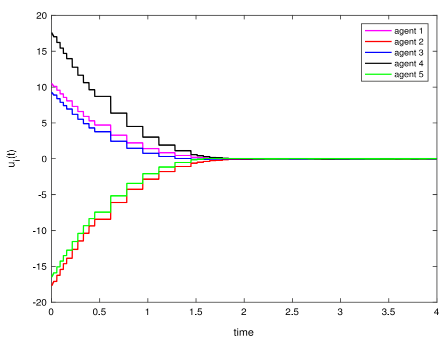
4. Simulations

In this section, simulation results are provided and compared to illustrate the above theoretical results.

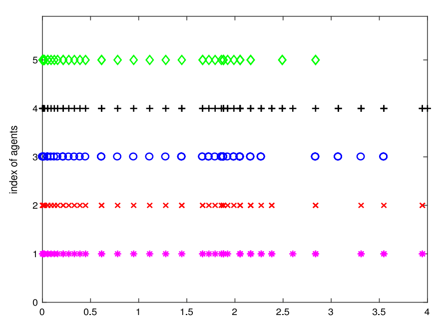
Consider the MAN (6) consisting of five agents, with the communication graph shown in Fig. 1. The adjacency matrix $\mathcal{A}$ is an $0-1$ matrix of dimensions 5×5. The second-smallest and largest eigenvalues of the corresponding Laplacian matrix L are respectively $\lambda_2 = 3$ and $\lambda_N = 5$. The initial condition of agents is given by $x(0) = \mathrm{col}(-1.8074, 12.4584, -0.6378, -6.1134, 11.0932)$.



(a) State trajectories of agents and evolution of the Lyapunov function: The zoom in curves show the indentations in state response

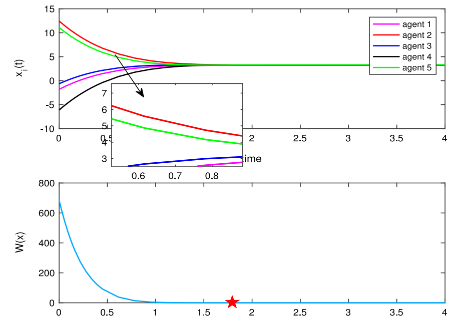


(b) Control inputs of agents

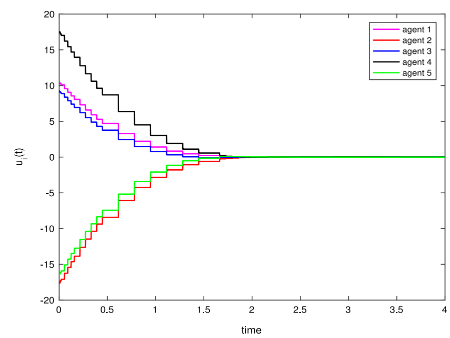


(c) Triggering instants versus time

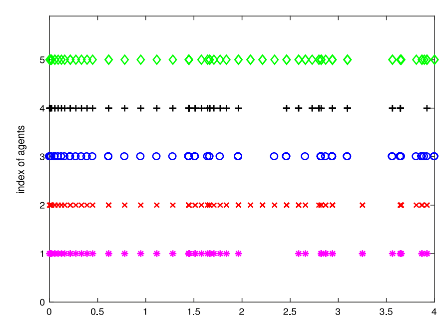
Fig. 2. Under the event-driven control protocol (8): σ_i is given by (23a), and the red star represents the settling time.



(a) State trajectories of agents and evolution of the Lyapunov function



(b) Control inputs of agents



(c) Triggering instants versus time

Fig. 3. Under the event-driven control protocol (8): σ_i is given by (23b), and the red star represents the settling time.

Consider the event-driven control protocol (8) and the event condition (7). The following simulation results demonstrate the finite-time consensus result developed in Theorem 8. Choose $\alpha = 1$ and $\mu = 3/4$. For comparison, two types of event thresholds σ_i are considered:

$$\sigma_1 = 0.1, \sigma_2 = 0.12, \sigma_3 = 0.106,$$
$$\sigma_4 = 0.09, \sigma_5 = 0.098; \tag{23a}$$
$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0.006\,. \tag{23b}$$

Then the triggering threshold satisfies constraint (14), i.e., $0 < \sigma_{\max} < \sqrt{\lambda_2/(\lambda_N^3 N^{\frac{1-\mu}{1+\mu}})} = 0.1381$.

In the case (23a), Fig. 2(a) shows the state trajectories of MAN (6) under the event-driven controller (8) and the Lyapunov function evolution, where the real simulation settling time, denoted by the red star, is 1.8377, denoting the first time visiting $W(x) \leq 10^{-3}$. According to Theorem 8, an estimate for the settling time is $\hat{T}_S = 9.5324$. The control inputs and triggering instants versus time are illustrated respectively by Figs. 2(b) and 2(c). Hence, comparing with the continuous-time controller, the event-driven controller (8) has the advantage in reducing the number of control updates. Moreover, Fig. 3 shows the state trajectories, the control inputs and agents' triggering instants for MAN (6) under the case (23b), where the real settling time is 1.7942 and the calculated estimate for the settling time is $\hat{T}_S = 4.1309$.

From Figs. 2 and 3 it suggests that a smaller event threshold σ_i brings a smaller settling time, implying a better convergence performance. This observation also is in accord with the results developed in Theorem 8. In this sense, a tradeoff can be found between the control updating cost and the time performance while the proposed event-driven control protocol is used for finite-time consensus. In terms of the settling time, the theoretical estimate may be bigger than the real simulation one (highlighted by the red stars in Figs. 2(a) and 3(a)). This time discrepancy results largely from the conservativeness along the convergence analysis given in Theorem 8, which requires further investigation.

5. Conclusion

In this paper, the finite-time consensus problem of multi-agent networks has been studied. A distributed event-driven control protocol has been presented for finite-time consensus, and sufficient conditions involving a model-based triggering scheme have been established. Comparing with the continuous-time controller, the proposed event-driven control scheme has proven capable of reducing the number of control updates. More importantly, a link between the settling time and the event-triggering condition has been derived, showing that the event threshold brings a tradeoff between the control updating cost and the time performance. The theoretical results have been illustrated by a simulation example. Future work includes developing event-driven control for multi-agent networks with more complicated models.

Appendix

Proof of Lemma 3.

(i) For simplicity, we only prove the inequality (5) in the following while the inequality (4) can be similarly verified.

(ii) Consider $|\tilde{z}| = |\tilde{y}|$. It is obvious that the inequality (5) holds in the case of $\tilde{z} = \tilde{y}$ since the left-side is no greater than zero while the right-side equals zero, and particularly the equality in (5) holds when $\tilde{z} = \tilde{y} = 0$. Moreover, the inequality (5) with $\tilde{z} = -\tilde{y}$ is also true since both sides of (5) equal zero. Consider $|\tilde{z}| < |\tilde{y}|$ ($\tilde{y} \neq 0$). Assume that there exists a real number θ: $-1 < \theta < 1$, such that $\tilde{z} = \theta\tilde{y}$. With $1 + \theta > 0$, the left-hand side of (5) can be written as

$$-\tilde{y}[\tilde{y} + \tilde{z}]^{[\mu]} = -\tilde{y}[\tilde{y} + \theta\tilde{y}]^{[\mu]} = -(1+\theta)^{\mu}\tilde{y}[\tilde{y}]^{[\mu]} - (1+\theta)^{\mu}|\tilde{y}|^{1+\mu}.$$

On the other hand, the right-hand side of (5) satisfies

$$-\tilde{y}[\tilde{y}]^{[\mu]} + |\tilde{y}|\,|\tilde{z}|^{\mu} = -\tilde{y}[\tilde{y}]^{[\mu]} + |\theta|^{\mu}|\tilde{y}|\,|\tilde{y}|^{\mu} = \left(-1 + |\theta|^{\mu}\right)|\tilde{y}|^{1+\mu}.$$

Note that $0 < \mu \leq 1$. For $0 \leq \theta < 1$, one has

$$(1+\theta)^{\mu} + |\theta|^{\mu} - 1 \geq 1^{\mu} + \theta^{\mu} - 1 = \theta^{\mu} \geq 0.$$

For $-1 < \theta < 0$, one gets

$$(1+\theta)^{\mu} + |\theta|^{\mu} - 1 \geq (1+\theta) - \theta - 1 = 0.$$

Thus, for $-1 < \theta < 1$, it follows that

$$\left(-1 + |\theta|^{\mu}\right)|\tilde{y}|^{1+\mu} \geq -(1+\theta)^{\mu}|\tilde{y}|^{1+\mu},$$

which yields the desired result. □

Proof of Lemma 4.

(i) Define vector $\xi = \mathrm{col}(\xi_1, \xi_2, \dots, \xi_N)$ and the norm $\|\xi\|_s = \left(\sum_{i=1}^{N} \xi_i^s\right)^{1/s}$. Using the norm equivalence property (Parsegov et al., 2013):

$$\|\xi\|_s \leq \|\xi\|_r \leq N^{\frac{1}{r}-\frac{1}{s}}\|\xi\|_s, \quad 0 < r < s,$$

the inequalities can be verified. Details are omitted.

(ii) By the definition of 2-norm, it follows that

$$\|\xi(t)\|^2 = \frac{N}{N}\sum_{i=1}^{N}|\xi_i(t)|^2 \leq N\max_i\{M_i^2\} \leq N\max_i\{M_i^2\}\frac{|\xi_i(t)|^2}{m_i^2} = N\theta_i^2|\xi_i(t)|^2,$$

which gives the desired results. □

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