



## Brief paper

# Mean-square stabilizability via output feedback for a non-minimum phase networked feedback system<sup>☆</sup>



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## ARTICLE INFO

## Article history:

Received 18 August 2015

Received in revised form 1 November 2018

Accepted 3 March 2019

Available online 5 April 2019

## Keywords:

Networked control system

Output feedback

Mean-square stabilization

Non-minimum phase zero

## ABSTRACT

This work studies mean-square stabilizability via output feedback for a networked linear time invariant (LTI) feedback system with a non-minimum phase plant. In the feedback system, the control signals are transmitted to the plant over a set of parallel communication channels with possible packet dropout. Our goal is to analytically describe intrinsic constraints among channel packet dropout probabilities and the plant's characteristics in the mean-square stabilizability of the system. It turns out that this is a very hard problem. Here, we focus on the case in which the plant has relative degree one and each non-minimum zero of the plant is only associated with one of control input channels. Then, the admissible region of packet dropout probabilities in the mean-square stabilizability of the system is obtained. Moreover, a set of hyper-rectangles in this region is presented in terms of the plant's non-minimum phase zeros, unstable poles and Wonham decomposition forms which is related to the structure of controllable subspace of the plant. A numerical example is presented to illustrate the fundamental constraints in the mean-square stabilizability of the networked system.

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## 1. Introduction

In the last two decades, stabilization problems for networked feedback systems have attracted a great amount of research interests (for example, see Fu and Xie (2005), Ishii and Francis (2003), Nair and Evans (2002), Nair, Fagnani, Zampieri, and Evans (2007), Vargas, Chen, and Silva (2014) and the references therein). These works mainly focus on coping with new challenges caused by limited resources, uncertainties/unreliability in communication channels. Great success has been achieved in this research area, in particular, for stabilization via state feedback. In Elia (2005), networked multi-input multi-output (MIMO) LTI feedback systems are studied where control signals are transmitted to actuators over fading channels. Uncertainties in the channels are modeled as multiplicative noises and a design scheme

is presented for mean-square stabilization via state feedback. Moreover, fundamental constraints in mean-square stabilizability caused by channel uncertainties are studied for the networked systems (Elia, 2005). It is shown for a networked single-input feedback system that the minimum capacity required for mean-square stabilization via state feedback is determined by the product of all the unstable poles of the plant. In Xiao, Xie, and Qiu (2009), this problem is studied for a networked MIMO system where the total capacity of the feedback control channels is given. It is found that the minimum total channel capacity for the mean-square stabilization problem is also determined by the product of all the unstable poles of the plant. Some new developments in stabilization and state estimation for networked systems over packet dropping channels, where both actuators and sensors are connected to controllers over communication channels, are presented in Elia and Eisenbeis (2011).

In this work, we study the mean-square stabilizability via output feedback for a networked MIMO LTI system where the control signals are transmitted over packet dropping channels. The channel uncertainties are also modeled as multiplicative noises. The difficulties for mean-square stabilization with multiplicative noises are well recognized (see e.g. Lu and Skelton (2002)), especially for the case with non-minimum phase zeros (see e.g., Qi, Chen, Su, and Fu (2017)). Here, we attempt to explore

<sup>☆</sup> This work was supported in part by the National Natural Science Foundation of China under grants NSFC 61673183 and 61273109. The material in this paper was presented at the 2012 International Conference on Information and Automation (ICIA), June 6–8, 2012, Shenyang, China. This paper was recommended for publication in revised form by Associate Editor Claudio De Persis under the direction of Editor Christos G. Cassandras.

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fundamental constraints among channel packet dropout probabilities and plant's characteristics and structure in mean-square stabilizability of the networked system with a non-minimum phase plant. With this purpose, our study focuses on the case in which the plant is with relative degree one and each non-minimum phase zero is associated with one of control input channels. The largest admissible region of packet dropout probabilities for mean-square stabilizability of the system is presented. Moreover, a set of hyper-rectangles in this region is found in terms of plant's nonminimum phase zeros, unstable poles and Wonham decomposition forms (Wonham, 1967). The boundaries of these hyper-rectangles describe the interactions between channel packet dropout probabilities and the plant's characteristics and structure in this problem. Moreover, to explain the features of this admissible region comprehensively, we introduce a concept, blocking packet dropout probability with which data transmitted over all channels are lost. An upper bound of this probability allowed to the mean-square stabilizability is presented for the non-minimum phase networked system.

The remainder of this paper is organized as follows. We proceed in Section 2 to formulate the problem under study. A useful tool, upper triangular coprime factorization, is developed in Section 3. Section 4 presents our main results on mean-square stabilizability via output feedback for the networked systems. Section 5 concludes the paper.

The notation used throughout this paper is fairly standard. For any complex number  $z$ , we denote its complex conjugate by  $\bar{z}$ . For any vector  $u$ , we denote its transpose by  $u^T$ , conjugate transpose by  $u^*$  and Euclidean norm by  $\|u\|$ . For any matrix  $A$ , the transpose, conjugate transpose, spectral radius and trace are denoted by  $A^T$ ,  $A^*$ ,  $\rho(A)$  and  $\text{Tr}(A)$ , respectively. Denote a state–space model of an LTI system by  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ . For any real rational function matrix  $G(z)$ ,  $z \in \mathbb{C}$ , define  $\tilde{G}(z) = G^T(1/z)$ . Denote the expectation operator by  $\mathbf{E}\{\cdot\}$ . Let the open unit disc be denoted by  $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ , the closed unit disc by  $\bar{\mathbb{D}} := \{z \in \mathbb{C} : |z| \leq 1\}$ , the unit circle by  $\partial\mathbb{D}$ , and the complements of  $\mathbb{D}$  and  $\bar{\mathbb{D}}$  by  $\mathbb{D}^c$  and  $\bar{\mathbb{D}}^c$ , respectively. The space  $\mathcal{L}_2$  is a Hilbert space. For  $F, G \in \mathcal{L}_2$ , the inner product is defined as

$$\langle F, G \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{Tr} [F^*(e^{j\theta})G(e^{j\theta})] d\theta \quad (1)$$

and the induced norm is defined by  $\|G\|_2 = \sqrt{\langle G, G \rangle}$ . It is well-known that  $\mathcal{L}_2$  admits an orthogonal decomposition into the subspaces  $\mathcal{H}_2$  and  $\mathcal{H}_2^\perp$ . Note that for any  $F \in \mathcal{H}_2^\perp$  and  $G \in \mathcal{H}_2$ ,  $\langle F, G \rangle = 0$  (see e.g. Zhou, Doyle, and Glover (1995)). Define the Hardy space  $\mathcal{H}_\infty := \{G : G(z) \text{ bounded and analytic in } \mathbb{D}^c\}$ . A subset of  $\mathcal{H}_\infty$ , denoted by  $\mathcal{RH}_\infty$ , is the set of all proper stable rational transfer function matrices in the discrete-time sense. Note that we have used the same notation  $\|\cdot\|_2$  to denote the corresponding norm for spaces  $\mathcal{L}_2$ ,  $\mathcal{H}_2$  and  $\mathcal{H}_2^\perp$ .

## 2. Problem formulation

The networked feedback system under study is depicted in Fig. 1. The plant  $G$  in the system is a MIMO LTI system and the signal  $y(k)$  is the measurement. The control signal  $u(k)$  for the plant is generated by the feedback controller  $K$ . It includes  $r$  entries  $u_1(k), \dots, u_r(k)$  which are sent to the plant  $G$  over  $r$  parallel packet dropping channels, respectively. The signal  $v(k) = [v_1(k), \dots, v_r(k)]^T$  is the received control signal at the plant side.

Let  $\{\alpha_j(k), k = 0, 1, 2, \dots, \infty\}$ ,  $j = 1, \dots, r$  be random processes with independent identical Bernoulli probability distributions, respectively. It indicates the receipt of the control signal  $u(k)$ , i.e.,  $\alpha_j(k) = 1$  if  $u_j(k)$  is received, otherwise  $\alpha_j(k) = 0$ . Let the

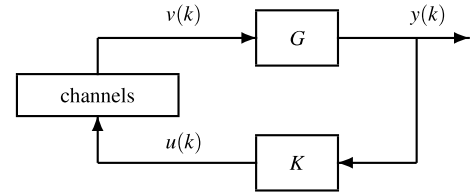


Fig. 1. A networked feedback system.

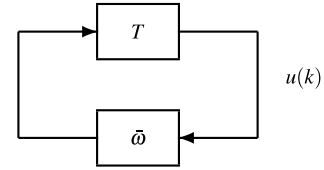


Fig. 2. An LTI system with a multiplicative noise.

probability of  $\alpha_j(k) = 0$  be  $p_j$ ,  $p_j \in (0, 1)$ . The averaged receiving rate of data packets is  $\mathbf{E}\{\alpha_j(k)\} = 1 - p_j$  in the  $j$ th channel. Let  $\omega_j(k) = \alpha_j(k) - (1 - p_j)$ . Subsequently, the received control signal  $v_j(k)$  is written as:

$$v_j(k) = \alpha_j(k)u_j(k) = (1 - p_j)u_j(k) + \omega_j(k)u_j(k). \quad (2)$$

It is clear that  $\{\omega_j(k), k = 0, 1, 2, \dots, \infty\}$ ,  $j = 1, \dots, r$  have independent identical probability distributions, referred to as *i.i.d* random processes, respectively. The *i.i.d* random process  $\{\omega_j(k), k = 0, 1, 2, \dots, \infty\}$  has zero mean and variance  $(1 - p_j)p_j$ . Now, it is assumed that  $\{\alpha_j(k)\}$ ,  $j = 1, \dots, r$  are mutually independent. Then, it holds for any  $i, j \in \{1, \dots, r\}$ ,  $i \neq j$  that  $\mathbf{E}\{\omega_i(k_1)\omega_j(k_2)\} = 0$ ,  $\forall k_1, k_2 > 0$ .

Denote the averaged channel gain by  $\mu = \text{diag}\{1 - p_1, \dots, 1 - p_r\}$  and the multiplicative noise in the channels by

$$\omega(k) = \text{diag}\{\omega_1(k), \dots, \omega_r(k)\}. \quad (3)$$

It follows from the discussion above that  $\mathbf{E}\{\omega(k)\} = 0$  and  $\mathbf{E}\{\omega(k)\omega^T(k)\} = \text{diag}\{p_1(1 - p_1), \dots, p_r(1 - p_r)\}$ . Let  $\tilde{\omega}(k) = \mu^{-1}\omega(k)$ . From (2), the packet dropout channels in the system shown in Fig. 1 are modeled as follows (also see Elia (2005)):

$$v(k) = \mu u(k) + \mu \tilde{\omega}(k)u(k). \quad (4)$$

It is verified from mean and variance of  $\omega(k)$ ,  $k = 0, 1, 2, \dots$  that  $\mathbf{E}\{\tilde{\omega}(k)\} = 0$ ,  $\mathbf{E}\{\tilde{\omega}(k)\tilde{\omega}^T(k)\} = \Sigma$  and  $\Sigma = \left\{ \frac{p_1}{1 - p_1}, \dots, \frac{p_r}{1 - p_r} \right\}$ .

**Definition 1** (See Willems and Blankenship (1971)). For any initial state, if it holds for the control signal and the output that  $\lim_{k \rightarrow \infty} \mathbf{E}\{u(k)u^T(k)\} = 0$  and  $\lim_{k \rightarrow \infty} \mathbf{E}\{y(k)y^T(k)\} = 0$ , then the feedback system in Fig. 1 is said to be mean-square stable.

To study the mean-square stability for the networked feedback system in Fig. 1, it is re-diagrammed as an LTI system with a multiplicative noise as shown in Fig. 2. Let  $\Delta(k) = \tilde{\omega}(k)u(k)$ . The channel model (4) is rewritten as  $v(k) = \mu u(k) + \mu \Delta(k)$ . Thus, the transfer function  $T$  from  $\Delta(k)$  to  $u(k)$  in the nominal system is given by

$$T = (I - KG\mu)^{-1}KG\mu \quad (5)$$

where  $G\mu$  is considered as a new plant involved with the averaged gain of the channel. Let  $T_{ij}$ ,  $i, j = 1, \dots, r$  be the  $\{i, j\}$ th entry of the transfer function matrix  $T$  and

$$\hat{T} = \begin{bmatrix} \|T_{11}\|_2^2 & \cdots & \|T_{1r}\|_2^2 \\ \vdots & \ddots & \vdots \\ \|T_{r1}\|_2^2 & \cdots & \|T_{rr}\|_2^2 \end{bmatrix}. \quad (6)$$

**Lemma 1** (See Lu and Skelton (2002)). The LTI system with a multiplicative noise in Fig. 2 is mean-square stable if and only if it holds that

$$\rho(\hat{T}\Sigma) < 1. \quad (7)$$

To design an output feedback controller  $K$  which stabilizes the system in Fig. 2 in the mean-square sense is referred to as mean-square stabilization via output feedback. If this problem is solvable, the system is referred to as mean-square stabilizable. Denote the packet dropout probability vector by  $p = (p_1, \dots, p_r)$  and the mean-square stabilizable region of  $p$  for the closed-loop system by  $\mathcal{P}$ , referred to as the admissible region of the packet dropout probabilities. Here, we attempt to describe the admissible region in terms of the characteristics of the plant  $G$ .

### 3. Upper triangular coprime factorization

To study the mean-square stabilizability of the networked system in Fig. 2, we consider the set of all possible stabilizing controllers for the plant  $G\mu$ , which is described by Youla parametrization in terms of its coprime factorizations. A useful tool for the mean-square stabilization design, referred to as upper triangular coprime factorization, is introduced in this section.

Suppose that the state-space model of the plant  $G\mu$  is given by

$$G\mu = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}, \text{ and } \{A, B\} \text{ is controllable, } \{A, C\} \text{ is detectable.}$$

Let the right coprime factorization of the plant  $G\mu$  be  $NM^{-1}$ , where the factors  $N$  and  $M$  are from  $\mathcal{RH}_\infty$ . Moreover,  $N$  and  $M$  are given by

$$M = I - F(zI - A + BF)^{-1}B, \quad (8)$$

$$N = C(zI - A + BF)^{-1}B, \quad (9)$$

where  $F$  is any stabilizing state feedback gain (for details, see e.g. Zhou et al. (1995)).

It is shown in Wonham (1967) that, with certain state transformation, the state-space model of  $G\mu$  can be transformed into so-called Wonham decomposition form  $\begin{bmatrix} A_w & B_w \\ C_w & 0 \end{bmatrix}$  with

$$A_w = \begin{bmatrix} A_1 & \star & \cdots & \star \\ 0 & A_2 & \cdots & \star \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_r \end{bmatrix}, \quad B_w = \begin{bmatrix} b_1 & \star & \cdots & \star \\ 0 & b_2 & \cdots & \star \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_r \end{bmatrix},$$

where

$$A_j = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{jl_j} & -a_{j(l_j-1)} & -a_{j(l_j-2)} & \cdots & -a_{j1} \end{bmatrix}, \quad b_j = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Since the pairs  $\{A_j, b_j\}$ ,  $j = 1, \dots, r$ , are all controllable, it is always possible to find row vectors  $f_j$  such that  $A_j + b_j f_j$  is stable for all  $j = 1, \dots, r$ . Now, we select a block diagonal state feedback gain  $F = \text{diag}\{f_1, f_2, \dots, f_r\}$ . Applying Wonham decomposition forms and the state feedback gain  $F$  into (8) and (9) yields a right coprime factorization  $G\mu = NM^{-1}$  in which the factor  $M$  is an upper triangular matrix. In this work, this coprime factorization is referred to as *upper triangular coprime factorization*. It is summarized in the following result.

**Lemma 2.** For a given plant  $G\mu$ , there exist coprime matrices  $N$  and  $M \in \mathcal{RH}_\infty$  such that  $G\mu = NM^{-1}$  and the matrix  $M$  is an upper triangular matrix. Furthermore, the diagonal elements  $m_{jj}$ ,  $j = 1, \dots, r$  of  $M$  are given by

$$m_{jj} = 1 - f_j(zI - A_j + b_j f_j)^{-1} b_j.$$

Taking account of the structures of  $A_j$  and  $b_j$ , we can see that the numerator polynomial of  $m_{jj}$  is the characteristic polynomial of  $A_j$ . Denote the unstable poles of  $A_j$  by  $\lambda_{j1}, \dots, \lambda_{jj}$ . Note the fact that  $\{A_j, b_j\}$  is controllable. By selecting a proper  $f_j$ , the poles of  $m_{jj}$  are assigned as  $1/\lambda_{j1}, \dots, 1/\lambda_{jj}$  and all stable poles of  $A_j$ . This yields that the diagonal elements  $m_{jj}$  are given by  $m_{jj} = \frac{(z - \lambda_{j1}) \cdots (z - \lambda_{jj})}{(\lambda_{j1}^* z - 1) \cdots (\lambda_{jj}^* z - 1)}$ . It is clear that  $m_{jj}$  is an inner, i.e.,  $m_{jj}^*(z) m_{jj}(z) = 1$  (for definition of an inner, see e.g. Zhou et al. (1995)). Denote it by  $m_{j,in}$ . For this particular upper triangular coprime factorization, let  $M_{in} = \text{diag}\{m_{1,in}, \dots, m_{r,in}\}$  be referred to as diagonal inner. Moreover, a balanced realization of  $m_{j,in}$ , which is used in remainder of this work, is denoted by

$$m_{j,in} = \begin{bmatrix} A_{j,in} & B_{j,in} \\ C_{j,in} & D_{j,in} \end{bmatrix}. \quad (10)$$

In general, for a given plant  $G\mu$ , there is a finite number of Wonham decomposition forms to  $G\mu$  in which poles of the plant could be assigned to different diagonal sub-matrices in the state matrix  $A_w$ , respectively. This gives a set of upper triangular coprime factorizations and associated diagonal inners  $M_{in}$  for the plant, which depend on the unstable poles in the diagonal sub-matrices in the Wonham decomposition forms. It will be shown in next section that the interaction between this feature and non-minimum phase zeros of the plant leads to non-convexity in analyzing the mean-square stabilizability for a non-minimum phase system.

### 4. Mean-square stabilizability

In this section, the mean-square stabilizability via output feedback in terms of the admissible region of the packet dropout probabilities is studied for the system in Fig. 1. In general, this is a very hard problem since non-minimum phase zeros make the mean-square stabilization via output feedback to be a non-convex problem (see for, example Qi et al. (2017)). Our study focuses on a non-minimum phase plant under Assumption 1.

**Assumption 1.** The plant  $G$  has non-minimum phase zeros  $z_1, \dots, z_r$ . Each of them is associated with a column of  $G$ , i.e.  $G = G_0 \text{diag}\{1 - z_1 z^{-1}, \dots, 1 - z_r z^{-1}\}$  where  $G_0$  is a minimum phase system and with relative degree one, i.e.,  $\lim_{|z| \rightarrow \infty} zG(z)$  is invertible.

At first glance, this assumption is quite artificial. However, due to multi-path transmission in wireless communication, multiple paths with different propagation lengths yield a channel with finite impulse response (FIR) which may include a non-minimum phase zero. In general, there is as called ‘‘common sub-channel zero’’ induced by multi-path transmission which is a difficult issue in channel identification and estimation (for example see Liang and Ding (2003) and Tugnait (1995)). This is a case which fits Assumption 1. On the other hand, we attempt to analytically investigate inherent constraints on the mean-square stabilizability imposed by interaction between Wonham decomposition forms and non-minimum phase zeros of the plant for the networked system. To seek simplicity, the plants under this assumption are studied. However, the results in this work can be extended to the case  $G = G_0 \text{diag}\{z^{-\tau_1} g_1, \dots, z^{-\tau_r} g_r\}$  where scale transfer functions  $g_j$ ,  $j = 1, \dots, r$  have more than one non-minimum phase zeros and relative degree zero,  $\tau_j$ ,  $j = 1, \dots, r$  are positive integers,  $G_0$  is a minimum phase system and with relative degree one, as explained in Remark 2 later.

Now, we consider all stabilizing controllers for the nominal closed-loop system  $T$ . Let  $NM^{-1}$  be a right coprime factorization of the plant  $G\mu$ . And let  $\tilde{M}^{-1}\tilde{N}$ , with  $M, \tilde{N} \in \mathcal{RH}_\infty$ , be the left

coprime factorization of the plant  $G\mu$  associated with  $NM^{-1}$ . It is well known (see Zhou et al., 1995 for details) that the factors  $N, M, \tilde{N}, \tilde{M}$  with some  $X, Y, \tilde{X}, \tilde{Y} \in \mathcal{RH}_\infty$  satisfy the Bezout Identity below:

$$\begin{bmatrix} M & Y \\ N & X \end{bmatrix} \begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = I. \quad (11)$$

All stabilizing controllers for the nominal system are given

$$K = (\tilde{X} - Q\tilde{N})^{-1}(\tilde{Y} - Q\tilde{M}), \quad (12)$$

where  $Q \in \mathcal{RH}_\infty$  is a parameter to be designed. Applying the controller (12) to the system in Fig. 2, we obtain the nominal closed-loop system  $T$  in (5) as follows:

$$T = (Y - MQ)\tilde{N}. \quad (13)$$

According to Lemma 1, the system is mean-square stabilizable if and only if there exists a  $Q$  satisfying the inequality  $\rho(\hat{T}\Sigma) < 1$ .

To this end, we need the following result (see Horn and Johnson (1985) for details),

**Lemma 3.** Suppose  $W$  is an  $r \times r$  positive matrix and  $w_{ij}$  is the  $\{i, j\}$ th entry of  $W$ . Then, it holds that

$$\rho(W) = \inf_{\Gamma} \max_j \sum_{i=1}^r \frac{\gamma_i^2}{\gamma_j^2} w_{ij}$$

where  $\Gamma = \text{diag} \{ \gamma_1^2, \dots, \gamma_r^2 \}$ , with  $\gamma_i > 0, i = 1, \dots, r$ .

It holds from Lemma 3 and the definition of  $\mathcal{L}_2$ -norm that

$$\rho(\hat{T}\Sigma) = \inf_{\Gamma} \max_j \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2 \frac{p_j}{1 - p_j} \quad (14)$$

where  $T_j$  is the  $j$ th column of  $T$ . According to Lemma 1 and the spectral radius given in (14), we have the next result straightforwardly.

**Lemma 4.** The closed-loop system in Fig. 2 is mean-square stabilizable if and only if it holds for some  $\Gamma$  and  $Q$  that

$$0 \leq p_j \leq \frac{1}{1 + \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2}, \quad j = 1, \dots, r. \quad (15)$$

For any given  $\Gamma$  and  $Q$ , the inequalities in (15) describe an admissible hyper-rectangle of the probabilities  $p_1, \dots, p_r$  to the mean-square stabilizability of the system. Denote this hyper-rectangle by  $\mathcal{P}_\Gamma(Q)$ . Now, we study how to find the hyper-rectangle for a given  $\Gamma$  with the largest volume.

Let  $M_\Gamma = \Gamma^{1/2} M \Gamma^{-1/2}, \tilde{N}_\Gamma = \Gamma^{1/2} \tilde{N} \Gamma^{-1/2}, \tilde{X}_\Gamma = \Gamma^{1/2} \tilde{X} \Gamma^{-1/2}$ , and  $Q_\Gamma = \Gamma^{1/2} Q \Gamma^{-1/2}$ . Let the inner-outer factorization of  $M_\Gamma$  be given by  $M_\Gamma = M_{\Gamma in} M_{\Gamma out}$  where  $M_{\Gamma in}, M_{\Gamma out}$  are inner and outer, respectively (see e.g. Zhou et al., 1995).

**Lemma 5.** For a given  $\Gamma$ , it holds that

$$\begin{aligned} \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2 &= \left\| \left[ M_{\Gamma out} (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - M_{\Gamma in}^{-1}(\infty) \right] e_j \right\|_2^2 \\ &\quad + \left\| \left[ M_{\Gamma in}^{-1} - M_{\Gamma in}^{-1}(\infty) \right] e_j \right\|_2^2. \end{aligned} \quad (16)$$

**Proof.** From (13), it holds for the system that

$$\Gamma^{1/2} T_j \gamma_j^{-1} = \Gamma^{1/2} (Y - MQ) \tilde{N} \Gamma^{-1/2} e_j \quad (17)$$

where  $e_j$  is the  $j$ th column of the  $r \times r$  identity matrix  $I$ . Applying Bezout identity (11) into (17) leads to

$$\Gamma^{1/2} T_j \gamma_j^{-1} = \Gamma^{1/2} [M(\tilde{X} - Q\tilde{N}) - I] \Gamma^{-1/2} e_j. \quad (18)$$

We rewrite (18) as  $\Gamma^{1/2} T_j \gamma_j^{-1} = [M_\Gamma (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - I] e_j$ . Noting the identity  $M_{\Gamma in}^{-1} M_{\Gamma in} = I$  and the definition of  $\mathcal{L}_2$  norm, we have that

$$\left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2 = \left\| \left[ M_{\Gamma out} (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - M_{\Gamma in}^{-1} \right] e_j \right\|_2^2. \quad (19)$$

Due to the facts that  $M_{\Gamma in}^{-1} - M_{\Gamma in}^{-1}(\infty) \in \mathcal{H}_2^\perp$  and  $M_{\Gamma out} (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - M_{\Gamma in}^{-1}(\infty) \in \mathcal{H}_2$ , it holds

$$\langle M_{\Gamma in}^{-1} - M_{\Gamma in}^{-1}(\infty), M_{\Gamma out} (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - M_{\Gamma in}^{-1}(\infty) \rangle = 0. \quad (20)$$

Hence, (16) follows from (19) and (20).

For a non-minimum phase plant,  $\tilde{N}_\Gamma$  is not invertible in  $\mathcal{RH}_\infty$ . This leads to certain coupling among  $\left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2, j = 1, \dots, r$ , which makes maximizing the volume of  $\mathcal{P}_\Gamma(Q)$  to be a very hard problem. However, this problem is solvable under Assumption 1.

**Lemma 6.** Suppose that the plant  $G$  satisfies Assumption 1. For a given  $\Gamma > 0$ , there exists an optimal  $Q_\Gamma$  to minimize  $\left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2, j = 1, \dots, r$ , simultaneously. Moreover, it holds that

$$\begin{aligned} \min_{Q_\Gamma} \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2 &= \left\| \left[ M_{\Gamma in}^{-1} - M_{\Gamma in}^{-1}(\infty) \right] e_j \right\|_2^2 \\ &\quad + \left\| \left[ M_{\Gamma out}(z_j) \tilde{X}_\Gamma(z_j) - M_{\Gamma in}^{-1}(\infty) \right] e_j \frac{1 - z_j^* z_j}{z - z_j} \right\|_2^2. \end{aligned} \quad (21)$$

**Proof.** From Assumption 1, an inner-outer factorization of  $\tilde{N}_\Gamma$  is given by  $\tilde{N}_\Gamma = \tilde{N}_{\Gamma out} \text{diag} \{ n_{1,in}, \dots, n_{r,in} \}$  where  $\tilde{N}_{\Gamma out}$  is an outer of  $\tilde{N}_\Gamma$  and  $n_{j,in} = \frac{z - z_j}{z_j^* z - 1}, j = 1, \dots, r$  are inner factors. Thus, from  $n_{j,in}^* n_{j,in} = 1$ , we obtain that

$$\begin{aligned} &\left\| \left[ M_{\Gamma out} (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - M_{\Gamma in}^{-1}(\infty) \right] e_j \right\|_2^2 \\ &= \left\| M_{\Gamma out} Q_\Gamma \tilde{N}_{\Gamma out} e_j - \left[ M_{\Gamma out} \tilde{X}_\Gamma - M_{\Gamma in}^{-1}(\infty) \right] e_j n_{j,in}^{-1} \right\|_2^2. \end{aligned} \quad (22)$$

Subsequently, it follows from fraction decomposition that

$$\begin{aligned} &\left[ M_{\Gamma out} \tilde{X}_\Gamma - M_{\Gamma in}^{-1}(\infty) \right] e_j n_{j,in}^{-1} \\ &= \left[ M_{\Gamma out}(z_j) \tilde{X}_\Gamma(z_j) - M_{\Gamma in}^{-1}(\infty) \right] e_j \frac{1 - z_j^* z_j}{z - z_j} + L_j, \end{aligned} \quad (23)$$

where  $L_j$  is the remainder part of this fraction decomposition which belongs to  $\mathcal{H}_2$ . Note the fact that

$$\left[ M_{\Gamma out}(z_j) \tilde{X}_\Gamma(z_j) - M_{\Gamma in}^{-1}(\infty) \right] e_j \frac{1 - z_j^* z_j}{z - z_j} \in \mathcal{H}_2^\perp.$$

Then, substituting (23) into (22) leads to

$$\begin{aligned} &\left\| \left[ M_{\Gamma out} (\tilde{X}_\Gamma - Q_\Gamma \tilde{N}_\Gamma) - M_{\Gamma in}^{-1}(\infty) \right] e_j \right\|_2^2 \\ &= \left\| M_{\Gamma out} Q_\Gamma \tilde{N}_{\Gamma out} e_j - L_j \right\|_2^2 \\ &\quad + \left\| \left[ M_{\Gamma out}(z_j) \tilde{X}_\Gamma(z_j) - M_{\Gamma in}^{-1}(\infty) \right] e_j \frac{1 - z_j^* z_j}{z - z_j} \right\|_2^2. \end{aligned} \quad (24)$$

Let  $L = [L_1 \dots L_r]$ . Select  $Q_\Gamma = \hat{Q}_\Gamma = M_{\Gamma out}^{-1} L \tilde{N}_{\Gamma out}^{-1}$  or  $Q = \hat{Q}(\Gamma) = \Gamma^{-1/2} \hat{Q}_\Gamma \Gamma^{1/2}$ . It is clear from (16) and (24) that  $\hat{Q}_\Gamma$  minimizes  $\left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2, j = 1, \dots, r$  simultaneously and (21) holds.

**Remark 1.** For a given  $\Gamma$ , the optimal  $\hat{Q}_\Gamma$  (or  $\hat{Q}(\Gamma)$ ) yields the largest admissible hyper-rectangle. Denote this hyper-rectangle by  $\hat{\mathcal{P}}_\Gamma$ . It holds from (15) and Lemma 6 that for any  $Q \in \mathcal{RH}_\infty, \mathcal{P}_\Gamma(Q) \subseteq \hat{\mathcal{P}}_\Gamma$ .

**Remark 2.** The key to the proof for Lemma 6 is to decompose  $[M_{\Gamma_{out}}\tilde{X}_{\Gamma} - M_{\Gamma_{in}}^{-1}(\infty)]e_j n_{j,i}^{-1}$  into two terms: One belongs to  $\mathcal{H}_2$  and the other belongs to  $\mathcal{H}_2^{\perp}$  as shown by (23). This decomposition also holds for the case when each channel has more than one non-minimum phase zero and a relative degree greater than one. Hence, the result in Lemma 6 can be extended to this case.

The following result is straightforwardly from Lemma 6.

**Lemma 7.** Suppose that the plant  $G$  satisfies Assumption 1. The admissible region of the packet dropout probabilities is given by  $\mathcal{P} = \cup_r \hat{\mathcal{P}}_r$ . For given packet dropout probabilities  $p_1, \dots, p_r$ , the optimal solution  $Q^*$  in minimizing  $\rho(\hat{T}\Sigma)$  belongs to the set  $\{\hat{Q}(\Gamma) : \Gamma > 0\}$ .

Now, we are ready to discuss the admissible region  $\mathcal{P}$  in terms of the plant's characteristics. Denote a balanced realization of  $M_{\Gamma_{in}}$  by  $M_{\Gamma_{in}} = \begin{bmatrix} A_{\Gamma_{in}} & B_{\Gamma_{in}} \\ C_{\Gamma_{in}} & D_{\Gamma_{in}} \end{bmatrix}$ .

**Theorem 1.** Suppose that the plant  $G$  satisfies Assumption 1. The system in Fig. 1 is mean-square stabilizable if and only if the packet dropout probability vector  $p = (p_1, \dots, p_r) \in \mathcal{P}$  and  $\mathcal{P}$  is given by

$$\mathcal{P} = \left\{ p = (p_1, \dots, p_r) \mid p_j < (e_j^T \Phi_{\Gamma,j} e_j + 1)^{-1} \right. \\ \left. j = 1, \dots, r, \quad \Gamma > 0 \right\} \quad (25)$$

where  $\Phi_{\Gamma,j} = D_{\Gamma_{in}}^{*-1} B_{\Gamma_{in}}^* N_{j,in}^* (A_{\Gamma_{in}}^{*-1}) N_{j,in} (A_{\Gamma_{in}}^{*-1}) B_{\Gamma_{in}} D_{\Gamma_{in}}^{-1}$  and  $N_{j,in} (A_{\Gamma_{in}}^{*-1}) = (z_j^* A_{\Gamma_{in}}^{*-1} - I)(z_j I - A_{\Gamma_{in}}^{*-1})^{-1}$ .

The proof of this theorem is given in Appendix.

With a given coprime factorization of the plant  $G\mu$ , Lemma 7 and Theorem 1 describe the admissible region  $\mathcal{P}$  of the packet dropout probabilities for mean-square stabilizability of the system. Since for all individual coprime factorizations of the plant, the controller sets given by (12) are equivalent up to an invertible factor of  $Q$ , these results are independent of the coprime factorization of the plant. In general, the admissible region  $\mathcal{P}$  is non-convex. Now, a convex sub-region of  $\mathcal{P}$  is studied by using the structural information of a particular upper triangular coprime factorization of the plant  $G\mu$ . As studied in the preceding section, this coprime factorization is generated from one of the plant's Wonham decomposition forms and its diagonal inner  $M_{in}$  describes the key features of the Wonham decomposition form. With balanced realizations of  $M_{in}$ 's components given in (10), this subregion is described below.

**Theorem 2.** Suppose that the plant  $G$  satisfies Assumption 1. Then, the system in Fig. 1 is mean-square stabilizable if, for all  $j = 1, \dots, r$ , the packet dropout probability  $p_j$  in  $j$ th channel satisfies:

$$p_j \leq \hat{p}_j \quad (26)$$

where  $\hat{p}_j^{-1} = D_{j,in}^{*-1} B_{j,in}^* N_{j,in}^* (A_{j,in}^{*-1}) N_{j,in} (A_{j,in}^{*-1}) B_{j,in} D_{j,in}^{-1} + 1$ .

The proof of this theorem is presented in Su, Lu, Wu, Fu, and Chen (2018).

In general, there are more than one Wonham decomposition form for the plant. Denote the diagonal inner associated with the  $s$ th Wonham decomposition form by  $M_{s,in}$  and its diagonal entries by  $m_{s1,in}, \dots, m_{sr,in}$ , i.e.,  $M_{s,in} = \text{diag}\{m_{s1,in}, \dots, m_{sr,in}\}$ . Let  $\begin{bmatrix} A_{sj,in} & B_{sj,in} \\ C_{sj,in} & D_{sj,in} \end{bmatrix}$  be a balance realization of  $m_{sj,in}$ ,  $j = 1, \dots, r$ .

Applying Theorem 2 with the diagonal inner yields an admissible hyper-rectangle  $\mathcal{P}_s \subseteq \mathcal{P}$  for the packet dropout probabilities.

**Corollary 1.** If the packet dropout probability vector  $(p_1, \dots, p_r)$  is in the union of all  $\mathcal{P}_s$ , i.e.,

$$(p_1, \dots, p_r) \in \cup_s \mathcal{P}_s, \quad (27)$$

then the networked feedback system in Fig. 1 is mean-square stabilizable.

**Proof.** Since the admissible region  $\mathcal{P}$  is independent of coprime factorization  $NM^{-1}$  of the plant, repeatedly applying Theorem 2 with balanced realizations of the diagonal inners yields a set of hyper-rectangles. Each of these hyper-rectangles is associated with one of the plant's Wonham decomposition forms and belongs to  $\mathcal{P}$ . So, the union of these hyper-rectangles belongs to  $\mathcal{P}$ .

If the plant  $G$  has only one Wonham decomposition form, the mean-square stabilizable hyper-rectangles merge to one hyper-rectangle. Eq. (27) becomes the necessary and sufficient condition for the mean-square stabilizability of the system. In particular, for a SIMO plant  $G$ , there is only one Wonham decomposition form, the admissible region and hyper-rectangle studied in Theorems 1 and 2, respectively, degrade to a common interval in one dimension space. In this case, Theorem 2 presents a necessary and sufficient condition for the mean-square stabilizability of the system, i.e.,  $\hat{p}_1$  given by the theorem is the supremum of the packet dropout probability which is allowed for the mean-square stabilizability of the network feedback system. For a SISO plant with one unstable pole  $\lambda_1$  and one non-minimum phase zero  $z_1$ , this supremum is given by  $\hat{p}_1 = [(\lambda_1^2 - 1)(z_1 \lambda_1 - 1)^2 / (z_1 - \lambda_1)^2 + 1]^{-1}$ .

Notice the fact that the product  $\prod_{j=1}^r p_j$  is the probability with which data packets over all channels are dropped simultaneously. In this work, it is referred to as the blocking packet dropout probability. The volume of a hyper-rectangle  $\mathcal{P}_s$  is the maximum of the blocking packet dropout probability for all  $(p_1, \dots, p_r) \in \mathcal{P}_s$ . Thus, it leads to:

**Corollary 2.** If the blocking packet dropout probability  $\prod_{j=1}^r p_j$  of the channels satisfies the inequality

$$\prod_{j=1}^r p_j < \max_s \left\{ \prod_j \hat{p}_{s,j} \right\}, \quad (28)$$

then, there exists a set of data dropout probabilities  $p_1, \dots, p_r$  with which the networked feedback system in Fig. 1 is mean-square stabilizable.

**Remark 3.** For a minimum phase plant, Corollary 2 is a necessary and sufficient condition and the upper bound of the blocking packet dropout probability is determined by the product of the plant's unstable poles (see Su et al. (2018) for more details).

**Example 1.** Suppose that the plant in the networked feedback system shown in Fig. 1 is a two-input two-output system. The transfer function of the plant is given as below:

$$G = \begin{bmatrix} \frac{(z - 0.25)(z + 2)}{z(z - 2)(z + 1.5)} & \frac{z - 1.5}{z(z + 1.5)} \\ \frac{z + 2}{z(z - 2)} & \frac{(2z - 2.75)(z - 1.5)}{z(z - 0.25)(z - 2.5)} \end{bmatrix}.$$

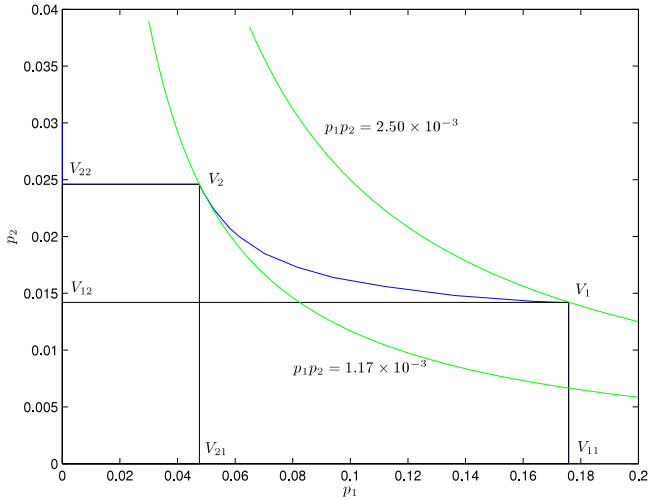


Fig. 3. Mean-square stabilizable region for data dropout rate.

Let  $p_1$  and  $p_2$  be packet dropout probabilities of two channels, respectively. Applying Theorem 1, we obtain the admissible region of the packet dropout probabilities, enclosed by the blue curve  $V_{11}V_1V_2V_{22}$  and axes as shown in Fig. 3, numerically.

There are two Wonham decomposition forms for the plant. Two diagonal inners associated with these forms are  $M_{1,in} = \text{diag} \left\{ \frac{z-2}{2z-1}, \frac{(z+1.5)(z-2.5)}{(-1.5z-1)(2.5z-1)} \right\}$  and  $M_{2,in} = \text{diag} \left\{ \frac{(z-2)(z+1.5)}{(2z-1)(-1.5z-1)}, \frac{z-2.5}{2.5z-1} \right\}$ , respectively. According to Theorem 2, the admissible subregion of  $(p_1, p_2)$  is obtained from a balance realization of  $M_{1,in}$  for  $p_1$  and  $p_2$ , which is the rectangle  $OV_{11}V_1V_{12}$  shown in Fig. 3. Similarly, from  $M_{2,in}$ , the other admissible rectangle  $OV_{21}V_2V_{22}$  shown in Fig. 3 is obtained for the packet dropout probability vector. Areas of these two rectangles are  $2.50 \times 10^{-3}$ ,  $1.17 \times 10^{-3}$ , respectively. It is worth noting that, all rectangles with area  $2.50 \times 10^{-3}$  are bounded by the green curve  $p_1p_2 = 2.50 \times 10^{-3}$ . While, all rectangles with area  $1.17 \times 10^{-3}$  are bounded by the green curve  $p_1p_2 = 1.17 \times 10^{-3}$ . The upper bound of the blocking packet dropout probability for mean-square stabilizability of the system is  $2.50 \times 10^{-3}$ . If the plant had only one Wonham decomposition form, these two green curves would merge to one curve and the two rectangles would merge to one rectangle as well.

### 5. Conclusion

This work studies the mean-square stabilizability via output feedback for a networked MIMO feedback system over several parallel packet dropping communication channels. The admissible region of packet dropout probabilities is discussed for the mean-square stabilizability of a non-minimum phase networked system. The trade-off among these packet dropout probabilities, plant's characteristics and structure in the mean-square stabilizability of the system is presented by an upper bound of blocking packet dropout probability in the region.

### Appendix. Proof of Theorem 1

Taking account to (15) and Lemma 7, we can see that the key in proving this theorem is to find the expression of  $\min_{Q_r} \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2$  in terms of the balance realization of  $M_{r,in}$  and the non-minimum phase zeros. Now, we consider the first term in the right side of (21). Since  $M_{r,in}$  is an inner, it holds that

$$\left\| [M_{r,in}^{-1} - M_{r,in}^{-1}(\infty)] e_j \right\|_2^2 = \left\| [I - M_{r,in} M_{r,in}^{-1}(\infty)] e_j \right\|_2^2.$$

Applying the balanced realization, we have  $[M_{r,in} M_{r,in}^{-1}(\infty) - I] e_j = C_{r,in} (zI - A_{r,in})^{-1} B_{r,in} D_{r,in}^{-1} e_j$ . According to Corollary 21.19 and Remark 21.6 in Zhou et al. (1995), it holds that

$$\left\| [I - M_{r,in} M_{r,in}^{-1}(\infty)] e_j \right\|_2^2 = e_j^T D_{r,in}^{*-1} B_{r,in}^* B_{r,in} D_{r,in}^{-1} e_j. \quad (A.1)$$

On the other hand, it follows from Bezout identity (11) and Assumption 1 that  $M_r(z) \tilde{X}_r(z) e_j = e_j$ . Applying the inner-outer factorization  $M_r(z) = M_{r,in}(z) M_{r,out}(z)$ , we have  $M_{r,out}(z) \tilde{X}_r(z) e_j = M_{r,in}^{-1}(z) e_j$ . Hence, the second term of the right hand side in (21) is written as follows:

$$\begin{aligned} & \left\| [M_{r,in}^{-1}(z) - M_{r,in}^{-1}(\infty)] e_j \frac{1 - z_j^* z_j}{z - z_j} \right\|_2^2 \\ &= (z_j^* z_j - 1) \left\| [M_{r,in}^{-1}(z) - M_{r,in}^{-1}(\infty)] e_j \right\|_2^2. \end{aligned} \quad (A.2)$$

By applying Corollary 21.19 and Lemma 3.15 in Zhou et al. (1995), we have that

$$M_{r,in}^{-1}(z) - M_{r,in}^{-1}(\infty) = -D_{r,in}^{-1} C_{r,in} (zI - A_{r,in}^*)^{-1} B_{r,in} D_{r,in}^{-1}. \quad (A.3)$$

Substituting (A.1), (A.2), (A.3) into (21) leads to

$$\begin{aligned} & \min_{Q_r} \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2 \\ &= e_j^T D_{r,in}^{*-1} B_{r,in}^* (z_j^* I - A_{r,in}^{-1})^{-1} [(z_j^* z_j - 1) C_{r,in}^* D_{r,in}^{*-1} D_{r,in}^{-1} C_{r,in} \\ & \quad + (z_j^* I - A_{r,in}^{-1})(z_j I - A_{r,in}^*)^{-1}] (z_j I - A_{r,in}^*)^{-1} B_{r,in} D_{r,in}^{-1} e_j \end{aligned}$$

It follows from Corollary 21.19 in Zhou et al. (1995) that  $C_{r,in}^* D_{r,in}^{*-1} D_{r,in}^{-1} C_{r,in} + I = A_{r,in}^{-1} A_{r,in}^*$ . This leads that

$$\begin{aligned} & \min_{Q_r} \left\| \Gamma^{1/2} T_j \gamma_j^{-1} \right\|_2^2 = e_j^T D_{r,in}^{*-1} B_{r,in}^* (z_j^* I - A_{r,in}^{-1})^{-1} (z_j A_{r,in}^{-1} - I) \\ & \quad \times (z_j^* A_{r,in}^* - I) (z_j I - A_{r,in}^*)^{-1} B_{r,in} D_{r,in}^{-1} e_j. \end{aligned} \quad (A.4)$$

Consequently, from (15) and (A.4), we obtain that the system is mean-square stabilizable if and only if  $(p_1, \dots, p_r) \in \mathcal{P}$ .

### References

Elia, N. (2005). Remote stabilization over fading channels. *Systems & Control Letters*, 54(3), 237–249.

Elia, N., & Eisenbeis, J. N. (2011). Limitations of linear control over packet drop networks. *IEEE Transactions on Automatic Control*, 56(4), 826–841.

Fu, M., & Xie, L. (2005). The sector bound approach to quantized feedback control. *IEEE Transactions on Automatic Control*, 50(11), 1698–1711.

Horn, R., & Johnson, C. (1985). *Matrix analysis*. Cambridge University Press.

Ishii, H., & Francis, B. (2003). Quadratic stabilization of sampled-data systems with quantization. *Automatica*, 39(10), 1793–1800.

Liang, J., & Ding, Z. (2003). Nonminimum-phase fir channel estimation using cumulant matrix pencils. *IEEE Transaction on Signal Processing*, 51(9), 2310–2320.

Lu, J., & Skelton, R. E. (2002). Mean-square small gain theorem for stochastic control: discrete-time case. *IEEE Transactions on Automatic Control*, 47(3), 490–494.

Nair, G., & Evans, R. (2002). Mean square stabilizability of stochastic linear systems with data rate constraints. *Proc. 41st IEEE conference on decision and control*, 1632–1637.

Nair, G., Fagnani, F., Zampieri, S., & Evans, R. (2007). Feedback control under data rate constraints: an overview. *Proceedings of the IEEE*, 95(1), 108–137.

Qi, T., Chen, J., Su, W., & Fu, M. (2017). Control under stochastic multiplicative uncertainties: Part I, fundamental conditions of stabilizability. *IEEE Transactions on Automatic Control*, 62(3), 1269–1284.

Su, W., Lu, J., Wu, Y., Fu, M., & Chen, J. (2018). Mean-square Stabilizability via Output Feedback for a Non-minimum Phase Networked Feedback System, <http://arxiv.org/abs/1810.12818>.

Tugnait, J. K. (1995). On blind identifiability of multipath channels using fractional sampling and second-order cyclostationary statistics. *IEEE Transaction on Information Theory*, 41(1), 308–311.

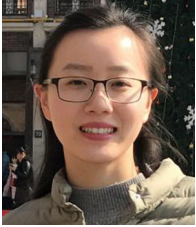
Vargas, F. J., Chen, J., & Silva, E. I. (2014). On stabilizability of mimo systems over parallel noisy channels. *Proc. 53rd IEEE conference on decision and control*, 6074–6079.

Willems, J., & Blankenship, G. (1971). Frequency domain stability criteria for stochastic systems. *IEEE Transactions on Automatic Control*, 16(4), 292–299.

Wonham, W. M. (1967). On pole assignment in multi-input controllable linear systems. *IEEE Transactions on Automatic Control*, 12(6), 660–665.

Xiao, N., Xie, L., & Qiu, L. (2009). *Proceedings of the 28th IEEE conference on decision and control, Mean square stabilization of multi-input systems over stochastic multiplicative channels.*

Zhou, K., Doyle, J., & Glover, K. (1995). *Robust and optimal control.* Prentice Hall.



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