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Blind System Identification and Channel Equalization for IIR Systems

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Abstract

A novel approach is proposed to blindly identify an unknown IIR system. The methods presented are linear in parameters of the unknown system so that many standard recursive algorithms can readily apply. It is also shown in the paper that under a generic condition, any finite order IIR system is identifiable provided the over-sampling ratio is high enough.

1 Introduction

The blind system identification (BSI) and blind channel equalization (BCE) problems addressed in this paper can be formulated as follows: A sequence of input signal $u[kh_i]$ is transmitted at sampling rate $f_i = 1/h_i$ to a continuous time system via a zero-order-hold (ZOH) or an impulse generator. The received signal $y[nh_o]$ is sampled at the rate $f_o = 1/h_o$. The BSI and BCE problems are to identify from $y[nh_o]$ both the system transfer function and the input $u[kh_i]$. These problems have received a lot of attention in recent years for applications in digital image processing, speech coding and mobile communications such as cellular and cordless telephony where communication channels drift constantly.

A traditional way to solve the BSI problem is to use statistical properties of the unknown input signal, e.g., high order statistics. Although the method provides satisfactory results in many cases, it does require enough *a priori* statistical information on the unknown input, an assumption that may not be valid in certain applications. An alternative approach is to use training signals. For example, in the GSM standard currently adopted in Europe, every 28 bits in a 116-bit sequence are used for the receiver to identify the channel. This method, though being simple, significantly reduces the transmission efficiency.

To overcome this difficulty, the so-called over-sampling (or multipath) approach has been proposed recently. It is shown that equalization can be achieved without training signals by using a multiple sampling rate for the receiver, or equivalently, using multi-channels. This new approach, however, has to assume that the channel transfer function is a finite-impulse-response (FIR). Although it may often be reasonable to approximate a communication channel by an FIR function, such approximation is often of very high order (up to 70th order FIRs are used. Further, the order of the filter tends to increase as the sampling rate increases. In contrast, infinite-impulse-response (IIR) representation often requires much less number of parameters. Also, the equalization part can be simpler. For instance, if the channel is modeled as an all-pole system, the equalizer becomes a FIR filter. Note that the equalizer is an all-pole filter if the channel is modeled as a FIR system. An all-pole equalizer is not easy to construct, especially the identified FIR channel is unstable or has high order.

In this paper, we present a novel approach to the BSI and BCE problems which allows us to develop efficient algorithms without the FIR assumption on the channel, any statistical information on the input signal, or training signals. We also use the idea of oversampling, i.e., f_o is faster than f_i. The main results of the paper can be summarized as follows:

1. First, we show that double sampling rate for the receiver is sufficient for BSI and BCE when the channel is an all-pole filter. An algorithm is presented for blind identification of these systems. The algorithm is least-squares or LMS based and is convergent if the input is sufficiently rich.
2. Secondly, we show that BSI and BCE with an IIR channel can always be achieved when

$f_o/f_i \geq n+1$, where n is the order of the channel transfer function. Since IIR models of the channel can often be of low order, this result offers an attractive alternative for the BSI and BCE problems. Again, least-squares and LMS based algorithms are given which guarantee convergence under some persistent excitation (PE) conditions.

3. Finally, we address a general identifiability issue and give some necessary and sufficient conditions for unique identification of both the channel transfer function and the input signal. We show that these conditions are actually generically satisfiable.

2 Problem Statement

In this paper, two types of sampled data systems are considered. The first one uses a Zero-Order-Hold (ZOH) as shown in Figure 1. This model is very common in the control literature. The second type of sampled data systems consists of an ideal impulse generator as shown in Figure 2. The input to the continuous time system is not piecewise constant but a sequence of δ-functions whose magnitudes vary according to the input sequence. This model is very common in digital signal processing and communication systems. The input to either of the systems is a discrete sequence $u[mh_i]$ with the time interval h_i and the output $y[kh_o]$ is also a discrete sequence with the sampling interval h_o. The sampling intervals h_i and h_o are usually different. It is assumed in the paper that $h_o = h_i/p$ for some positive integer $p \geq 1$, referred to as the over-sampling ratio. The channel is assumed to be a linear time-invariant continuous-time system with matrices A, B, C and possibly some delay τ, known or unknown. The purpose of the paper is to present a novel approach to solving the BSI and BCE problems for an IIR system. To focus on this point, we do not include any noise at the channel outputs. However, analysis of the noise effect is standard and can be found in many control and signal processing textbooks. By the same token, we will concentrate on the zero delay ($\tau = 0$) case. Extensions to the case with unknown delay τ will be discussed, but briefly. Now, if $\tau = 0$, it can be verified easily that the transfer function considered at the output sampling period h_o, is given by an nth order rational transfer function:

$$G(z) = \frac{b(z)}{a(z)} = \frac{b_1 z^{-1} + \dots + b_{n-1} z^{-n}}{1 - a_1 z^{-1} - \dots - a_n z^{-n}} \quad (2.1)$$

for some constants a_i and b_i. In fact, the order of the system is independent of the sampling period h_o, except in some pathological cases for which the degree reduces. Our goal is to identify the parameters in (2.1) as well as the input signal $u[mh_i]$ based on the output measurements $y[kh_o]$ with as little knowledge as possible on the unknown input sequence. Let $Y(z)$ and $U(z)$ be the z-transforms of output and input sequences, respectively. Then $Y(z) = G(z)U(z) = (\alpha G(z))(\frac{1}{\alpha}U(z))$ for any non-zero constant α. Therefore, the best we can do is to identify $G(z)$ and $U(z)$ up to a scaling constant. The BSI and BCE problems are formally defined as:

BSI and BCE: *Consider the sampled data systems in Figures 1 and 2, assuming that the input is an unknown non-zero bounded sequence. Identify $G(z)$ and $U(z)$, up to a constant, for some over-sampling ratio $p \geq 1$ based only on the output observation $y[kh_o]$.*

Let $Y(z)$ and $U(z)$ denote the z-transforms of $y[kh_o]$ and $u[kh_o]$ respectively. For $h_i/h_o = p$, the input to the system in Figure 1 is held to be a constant for every p samples and thus $U(z)$ can be written as

$$U(z) = U_0(z^p)(1 + \dots + z^{-p+1}) \quad (2.2)$$

where $U_0(z) = \sum_{k=0}^{\infty} u[kh_i] z^{-k}$. That is, all the p-polyphase components of $U(z)$ are identical. Consequently,

$$Y(z) = G(z)U_0(z^p)(1 + \dots + z^{-p+1}). \quad (2.3)$$

Define $\bar{U}_0(z^p) = U_0(z^p)(1 - z^{-p})$ and $\bar{Y}(z) = Y(z)(1 - z^{-1})$, we have $\bar{Y}(z) = G(z)\bar{U}_0(z^p)$. Therefore, the BSI and BCE problems become to identify both $G(z)$ and $\bar{U}_0(z^p)$.

Assumption 1: It is assumed through out the paper that the discrete time system corresponding to the system in Figure 2 is minimal when the over-sampling ratio $p = 1$, i.e. when $h_i = h_o$.

3 All-Pole Systems

We first consider a simple case of IIR systems, namely an all-pole filter, to convey the idea. Let the over-sampling ratio $p = 2$, i.e., $h_i = 2h_o$. Suppose the resultant sampled data system is an all-pole system, in terms of the output sampling interval h_o, i.e.,

$$G(z) = \frac{b_1}{1 - a_1 z^{-1} - \dots - a_n z^{-n}} \quad (3.1)$$

We have dropped the delay z^{-1} in the numerator for notational simplicity. The result in this section remains valid with some minor notational changes if z^{-1} is present. The time domain expression of (3.1) is

$$y[kh_o] = \sum_{i=1}^{n} a_i y[(k-i)h_o] + b_1 u[kh_o].$$

Observe that $p = 2$ and the input $u[kh_o]$ to the impulse generator as shown in Figure 2 is zero for odd k. It follows that

$$y[kh_o] = \sum_{i=1}^{n} a_i y[(k-i)h_o] \tag{3.2}$$

for $k = 2l+1,\ l = 0, 1, 2, \cdots$ In other words, the coefficients a_i's are the solution of the following linear equation

$$\Phi[a_1 \ \ a_2 \ \ \cdots \ \ a_n]' = q \tag{3.3}$$

where

$$q = (\, y[(2l+1)h_o], \dots)' \tag{3.4}$$

and

$$\Phi = \begin{pmatrix} y[(2l)h_o] & \dots & y[(2l+1-n)h_o] \\ \vdots & \ddots & \vdots \end{pmatrix}. \tag{3.5}$$

Thus, a_i's can be uniquely solved by many standard methods if the matrix Φ has full column rank. a_i's can also be calculated on-line recursively by employing recursive least squares or LMS algorithms.

Theorem 3.1 *Consider the system in Figure 2 with $p = 2$ and $G(z)$ given by (3.1). Then (1) $\phi[l]$ is PE if the spectral measure of the input $u[kh_i]$ (in h_i not in h_o, a weaker condition) is not concentrated on $k < n$ points. (2) The recursive least squares (or LMS) estimates $\hat{a}_i$'s of equation (3.3) converge asymptotically (or exponentially) to the true value a_i's, if the PE condition in 1 above is satisfied.*

It should be pointed out that Theorem 3.1 assumes that the input and the output are synchronized (i.e., the channel has zero delay). The case where synchronization is not achieved can be handled similarily by slightly modifying the above algorithm.

4 IIR Systems

We now consider arbitrary IIR systems characterized by (2.1). Two algorithms will be provided. The first algorithm requires $p = 2(n+1)$ and identifies a set of parameters α_i and β_i which are equivalent to a_i and b_i. The identification of α_i and β_i will be done separately with the advantage that estimate errors of α_i and β_i are independent. The second algorithm uses $p = n+1$. This algorithm identifies a_i first, and then uses the estimate of a_i to identify b_i. With the lower oversampling ratio, the tradeoff of this algorithm is that estimation error in a_i affects the estimate for b_i.

Algorithm 4.1. Let $p = 2(n+1)$. The input $U(z)$ is given by $U(z) = U_0(z^{2(n+1)})$. Decompose the output $Y(z) = Y_0(z^2) + z^{-1}Y_1(z^2)$ and let the transfer functions $G_0(z)$ and $G_1(z)$ be the transfer functions from $U_0(z^{n+1})$ to $Y_0(z)$ and $Y_1(z)$, respectively. Then, we have

$$Y_0(z) = G_0(z)U_0(z^{n+1}); Y_1(z) = G_1(z)U_0(z^{n+1}).$$

$$Y(z) = G(z)U(z^{2(n+1)}) = \frac{b(z)a(-z)}{a(z)a(-z)}U(z^{2(n+1)})$$

Note that $a(z)a(-z)$ is an even polynomial. Denote $\alpha(z^2) = a(z)a(-z)$ and decompose $b(z)a(-z) = \beta_0(z^2) + z^{-1}\beta_1(z^2)$ we obtain the following equations:

$$G_0(z) = \frac{\beta_0(z)}{\alpha(z)}; \quad G_1(z) = \frac{\beta_1(z)}{\alpha(z)} \tag{4.1}$$

In fact, $G_0(z)$ and $G_1(z)$ are the transfer functions from $U(z^{(n+1)})$ to the outputs $Y_0(z)$ and $Y_1(s)$, respectively, in terms of the sampling period $h = 2h_o$. Thus, $G_0(z)$ and $G_1(z)$ share the common nth order denominator. Also, as far as equalization is concerned, either $G_0(z)$ or $G_1(z)$ is sufficient to recover the input because $U(z^{(n+1)}) = G_0^{-1}(z)Y_0(z) = G_1^{-1}(z)Y_1(z)$.

Note that the equations are expressed in terms of the sampling interval $h = 2h_o$. Its time domain representation is given by

$$y[kh] = \sum_{i=1}^{n} \alpha_i y[(k-i)h] + \sum_{i=1}^{n} \beta_i u[(k-i)h]. \tag{4.2}$$

The input sequence $u[kh]$ is non-zero only if $k = l(n+1)$, i.e.,

$$u[(l(n+1)-1)h] = \cdots = u[(l(n+1)-n)h] = 0$$

for all l. Now consider equation (4.2) at the sampling interval h for $k = l(n+1)$,

$$y_0[l(n+1)h] = \sum_{i=1}^{n} \alpha_i y_0[(l(n+1) - i)h]. \quad (4.3)$$

This equation is linear in α_i and can be written as follows (similar to (3.3)):

$$\Phi[\alpha_1 \ \ \alpha_2 \ \ \cdots \ \ \alpha_n]' = q \quad (4.4)$$

for some appropriately defind q and Φ. Again any standard identification method applies for finding α_i's, e.g., least squares and LMS.

To identify $\beta_0(z)$ and $\beta_1(z)$, note the following:

$$\beta_1(z)Y_0(z) - \beta_0(z)Y_1(z) = 0. \quad (4.5)$$

Let $\beta_0(z) = \beta_{01}z^{-1} + \cdots + \beta_{0n}z^{-n}$ and $\beta_1(z) = \beta_{11}z^{-1} + \cdots + \beta_{1n}z^{-n}$. Then, we have, in time domain,

$$\bar{\Phi}[\beta_{11} \ \vdots \ \beta_{1n} \ \ \beta_{01} \ \ \cdots \ \ \beta_{0n}]' = 0 \quad (4.6)$$

for some $\bar{\Phi}$. Again, β_{0i} and β_{1i} appear linearly in (4.6). The difference between this equation and equation (4.4) is that the solution to this equation is unique only up to a scaling constant, provided that $\bar{\Phi}$ has full column rank. This is consistent with our goal of blind system identification. There are many ways to solve this equation. The simpliest one is probably to normalize one of the components of $\beta_0(z)$ or $\beta_1(z)$. For instance, let $\beta_{11} = 1$, then we have

$$\phi'[kh][\beta_{12} \cdots \beta_{1n}\beta_{01} \cdots \ \beta_{0n}]' = y[kh+h_o] \quad (4.7)$$

with

$$\phi'[kh] = (\, y[kh], ..., -y[(k-n+1)h + h_o]\,)$$

or in a compact form

$$\tilde{\Phi}[\beta_{12} \cdots \beta_{1n}\beta_1 \cdots \beta_n]' = q \quad (4.8)$$

where q and $\tilde{\Phi}$ are similarily defined as before. Thus, β_{0i} and β_{1i} can be solved in a similar way as for equation (4.4).

Theorem 4.1 *Consider the sampled data system in Figure 2 with $p = 2(n+1)$. Let $G_0(z)$ and $G_1(z)$ be the transfer functions described above. Then, (1) The recursive least squares (or LMS) estimates $\hat{\alpha}_i$'s from equation (4.4) converge asymptotically (or exponentially) to α_i's if the spectral measure of the input $u[kh_i]$ is not concentrated on $k < n$ points. (2) The recursive least squares (or LMS) estimate $(\hat{\beta}_{01}, \ldots, \hat{\beta}_{0n})$ and $(\hat{\beta}_{11}, \ldots, \hat{\beta}_{1n})$ from equation (4.8) converges asymptotically (or exponentially) to $(\beta_{01}, \ldots, \beta_{0n})$ and $(\beta_{11}, \ldots, \beta_{1n})$ up to a scaling constant, provided that the spectral measure of the input $u[kh_i]$ is not concentrated on $k < 2n-1$ points and that the numerator of the transfer functions $G(z)$ has coprime even and odd components.*

Algorithm 4.2. Let $p = n+1$. Our first step is to identify $a(z)$. To this end, we note that $a(z)Y(z) = b(z)U(z^{n+1})$. The time domain expression for the above is

$$y[kh_o] = \sum_{i=1}^{n} a_i y[(k-i)h_o] + \sum_{i=0}^{n-1} b_i u[(k-i)h_o]. \quad (4.9)$$

Taking $k = lp+n$, we have $u[(k-i)h_o] = 0, \forall i = 0, \cdots n-1$. Hence,

$$y[(lp+n)h_o] = \sum_{i=1}^{n} a_i y[(lp+n-i)h_o] \quad (4.10)$$

Once again, we have

$$\Phi[a_1 \ \ a_2 \ \ \cdots \ \ a_n]' = q \quad (4.11)$$

for some appropriately defined q and Φ. So, recursive least-square algorithms or LMS algorithms can be used to estimate a_i with asymptotic or exponential convergence, provided Φ is PE. Once the estimate $\hat{a}(z)$ of $a(z)$ are obtained, our next step is to use them to further estimate b_i. For this purpose, we compute $V(z) = \hat{a}(z)Y(z)$ and decompose it into $V(z) = V_0(z^p) + z^{-1}V_1(z^p) + \cdots z^{-n}V_n(z^p)$. Assume now $\hat{a}_i = a_i$ for all i. Then, $V(z) = b(z)U_0(z^p)$, meaning $V_0(z^p) = 0, V_i(z) = b_iU_0(z)$. Normalizing $b_1 = 1$, the above becomes $V_(z) = U_0(z)$ and $V_i(z) = b_iV_1(z)$. The time domain expression of the above gives a set of linear equations for b_i:

$$q_1 b_i = q_i, \quad i = 2, \cdots, n \quad (4.12)$$

Subsequently, $b_i = (q_1'q_1)^{-1}q_1'q_i$.

Theorem 4.2 *Consider the sampled data system in Figure 2 with $p = n+1$. Then, (1) The* *recursive least squares (or LMS) estimates $\hat{a}_i$'s from equation (4.11) converge asymptotically (or exponentially) to a_i's if the spectral measure of the input $u[kh_i]$ is not concentrated on $k < 2n-1$ points. (2) Suppose the PE condition above holds. Then, the recursive least squares (or LMS) estimate $\hat{b}_i$ from equation (4.12) converges asymptotically (or exponentially) to b_i up to a scaling constant.*

5 Identifiability

We have seen in the previous sections that $p = 2$ is sufficient for all-pole or FIR systems and $p = n+1$ for IIR systems with order equal to n, provided that some very mild PE conditions are satisfied. In this section, we study a more general identifiability problem: *Under what conditions can $G(z)$ be uniquely identified based on the observation of $\{y(kh_o)\}_0^\infty$ or equivalently its z-transform $Y(z)$, regardless of the input sequence $\{u(kh_i)\}_0^\infty$?* Here, unique identifiability, or simply identifiability, means the following: Suppose there exists $G(z)$, $\bar{G}(z)$, $U_0(z^p)$ and $\bar{U}_0(z^p)$ such that the degree and relative degree of $\bar{G}(z)$ are the same as those of $G(z)$ and that

$$Y(z) = G(z)U_0(z^p) = \bar{G}(z)\bar{U}_0(z^p)$$

then

$$G(z) = \alpha \bar{G}(z), \qquad U_0(z^p) = \bar{U}_0(z^p)/\alpha$$

for some $\alpha \neq 0$. Implicitly assumed above is that $Y(z) \neq 0$ because otherwise the problem of identifiability can not be defined.

The main results of this section provide necessary and sufficient conditions for the identifiability problem. Note that the setting of the identifiability problem requires the availability of the whole sequence of $\{y[kh_o]\}_0^\infty$. So the necessary and sufficient condition for the identifiability problem will become necessary only for the identifiability of $G(z)$ when a finite sequence of $y[kh_o]$ is available. Nevertheless, the identifiability conditions will help us understand and possibly generalize the results in previous sections.

Since $Y(z) = G(z)U_0(z^p)$, roughly speaking, $G(z)$ can be identified if and only if $G(z)$ is not confused with the input $U_0(z^p)$ and that the numerator $b(z)$ and the denominator $a(z)$ are coprime. In the sequel, we will address these two issues separately.

(p-factor and p-cofactor) A polynomial (resp. rational function) $f(z)$ is called a p-factor if it can be written as $g(z^p)$ for some nontrivial polynomial (resp. rational function) $g(\cdot)$. Given a polynomial $f(z)$ which does not contain any p-factor, its p-cofactor, denoted by $f^c(z)$, is a polynomial such that $f^c(z)$ contains no p-factor, $f(z)$ and $f^c(z)$ are coprime, and that $f(z)f^c(z)$ is a p-factor.

Note that the p-cofactor is unique up to a constant. Also, $(f^c(z))^c = f(z)$.

Theorem 5.1 *Consider the system in Figure 2. (1) Suppose $p = 2$ and $G(z)$ is an all-pole system, i.e. $G(z) = b_1/a(z)$. Then $G(z)$ is identifiable if and only if the even and odd components of $a(z)$ are coprime. (2) Similarly, suppose $p = 2$ and $G(z)$ is an FIR system, i.e. $G(z) = b(z)$. Then $G(z)$ is identifiable if and only if the even and odd components of $b(z)$ are coprime.*

Unfortunately, it is not possible to identify a general nth order IIR system with the over-sampling ratio equal to 2. To illustrate this point, we consider the sampled data system in Figure 2 with $p = 2$ and $G(z) = (1-\alpha z^{-1})/(1-\beta z^{-1})(1+\gamma z^{-1})$, then

$$Y(z) = \frac{(1-\alpha z^{-1})}{(1-\beta z^{-1})(1+\gamma z^{-1})} U_0(z^2)$$

$$= \frac{(1+\beta z^{-1})}{(1+\alpha z^{-1})(1+\gamma z^{-1})} \left[\frac{(1-\alpha^2 z^{-2})}{(1-\beta^2 z^{-2})} U_0(z^2) \right].$$

Since the input is unknown, the transfer function $G(z)$ is clearly not identifiable.

To find out the necessary and sufficient conditions for the identifiability of a general IIR system, we define a notion of Q_p set:

(Q_p set) For any over-sampling ratio $p \geq 2$, let the set Q_p be all nth order strictly proper and stable transfer functions $G(z) = b(z)/a(z)$, in terms of the output sampling interval h_o, satisfying: (1) $a(z)$ and $b(z)$ are coprime. (2) $a(z)$ and $b(z)$ do not contain any p-factor. (3) Given any $G(z) = b(z)/a(z)$ and $\bar{G}(z) = \bar{b}(z)/\bar{a}(z)$ in Q_p, express them as follows:

$$b(z) = b_1(z)b_2(z); \bar{b}(z) = b_1(z)\bar{b}_2(z)$$

$$a(z) = a_1(z)a_2(z); \bar{a}(z) = a_1(z)\bar{a}_2(z)$$

where $b_2(z)$ and $\bar{b}_2(z)$ are coprime, and so are $a_2(z)$ and $\bar{a}_2(z)$. Then, the following condition is not satisfied: $\deg a_2(z) = \deg \bar{a}_2(z)$ and

$$\bar{a}_2(z) = b_2^c(z); \quad \bar{b}_2(z) = a_2^c(z) \tag{5.1}$$

modulo a constant.

With the above definition, we have the following result.

Theorem 5.2 *Consider the systems in Figure 2. We have the following: (1) For any oversampling ratio $p \geq 2$, $G(z)$ is identifiable if and only if $G(z) \in Q_p$. (2) An n-th order IIR $G(z)$ is always identifiable for $p \geq 2n$, provided that $G(z)$ is coprime. (3) An n-th order IIR $G(z)$ is always identifiable for $p \geq n+1$, under Assumption 1.*

Note that the identifiability conditions above impose no assumptions on the input signal. If some *a priori* information on the input signal is available, i.e, the input signal is restricted to be in a certain class, then the result above can be made stronger. For example, $p = 2$ is sufficient provided that the input signal is such that it does not cancel the zeros and poles of the the channel transfer function. For $p \geq 3$, the possibility that $G(z) \notin Q_p$ (therefore the system being identifiable) is pathological, i.e., basically $p = 3$ is sufficient for the identifiability condition, although we do not have a practical algorithm for it. To argue that $G(z) \notin Q_p$ is pathological for $p = 3$, we note first that the possibility that $G(z)$ is coprime is generic. Similarly, the possibility that $a(z)$ and $b(z)$ contain a p-factor is also generic. Further, if Condition (3) in the definition of Q_p is violated, then $b_2^c(z) = \bar{a}_2(z)$ in particular. Then, either $b_2(z)$ or $\bar{a}_2(z)$ will have two roots with the same magnitude but phase separated by $2\pi k/p$ for some integer $k > 0$. Such a chance is pathological. This means that p must be pathological if both $G(z)$ and $\bar{G}(z)$ can be admitted as possible transfer functions for the channel. Hence, the satisfaction of Condition (3) is also nonpathological for $p \geq 3$.

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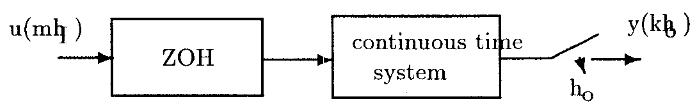

Fig. 2.1

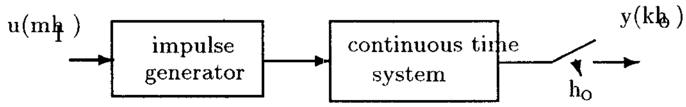

Fig. 2.2