A new approach to distributed charging control for plug-in hybrid electric vehicles

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Abstract: Plug-in hybrid electric vehicles (PHEV) tend to become more widespread in the next decades. However, large penetration of PHEVs will overload the distribution system. In smart grid, the charging of PHEVs can be controlled to reduce the peak load, which is known as demand side management (DSM). In this paper, we focus on the impact of PHEV charging on low-voltage transformers. The objective is to flatten the load curve of low-voltage transformers, while each consumer's requirement for their PHEV to be charged to the required level by their specified time is satisfied. We first formulate it as a convex optimization problem and then propose a distributed water-filling-based algorithm to solve it. Proofs of optimality and numerical simulations are given to demonstrate the effectiveness of our algorithm.

Key Words: Smart Grid, Demand Side Management, Plug-in Hybrid Electric Vehicle, Water Filling, Distributed Convex Optimization

1 Introduction

Road traffic is known as one of the main causes of greenhouse gas emission. Together with the rising fuel price and high energy efficiency, electric vehicles tend to become more widespread in the next decades. To satisfy the need of long distance travel, plug-in hybrid electric vehicles (PHEVs) are more desirable. A PHEV has both an electric and a combustion engine, so short drives can use the onboard battery while fuel is used for long drives. A PHEV is charged when it is plugged into the charger located at home or a public charging station. However, this may pose challenge to the electric grid's distribution system [1]. Large penetration of PHEVs will add to the current peak load or create new peak load. It can cause serious voltage deviation and overloading of transformers. Excessive voltage deviation can cause damage to electrical appliances while persistent overloading can overheat transformers, which may result in a blackout.

Fortunately, with the development of smart grid, advanced metering and communication systems enable us to develop better algorithms to deal with these problems. So the timing and rate of PHEV charging can be controlled to reduce the peak load, which is called demand side management (DSM). DSM refers to programs that attempt to influence customer consumption patterns of electricity to match current or projected capabilities of the power supply system [2]. DSM is surely an critical part of smart grid, as it makes the grid more economical, reliable and eco-friendly.

There have been a number of studies on DSM of PHEVs. Reference [3] presents a hierarchical control algorithm to realize the synergy between PHEV charging and wind power. The three-level controller proposed in this paper utilizes PHEV charging to compensate wind power fluctuation and thus indirectly regulate the grid frequency. References [4] and [5] both deal with the valley-filling problem by controlling the charging of a large population of PHEVs. In [4], a decentralized algorithm is developed based on Nash equilibrium, which proves optimality in the homogeneous case where all PHEVs have the same exit time, energy need and maximum charging power. In [5], a control signal from the utility company is altered to guide the updates of PHEVs' charging profiles. This algorithm converges to optimal charging profiles in both homogeneous and non-homogeneous cases. References [6] and [7] are more relevant to this paper since they both tackle the problem of distribution transformer overheating. In [6], peak load shedding is utilized to do load shaping, with consideration of consumers' preferences and load priorities. A multi-agent system solution (MAS) is adopted in [7], which features high adaptability and scalability. Reference [8] gives a comprehensive review on PHEV charging problems.

In this paper, we focus on the impact of PHEV charging on low-voltage transformers. The objective is to flatten the load curve of every low-voltage transformer, while each consumer's requirement for their PHEV to be charged to the required level by their specified time is satisfied. Inspired by the *water filling principle* in information theory, we first formulate it as a convex optimization problem and then propose a distributed water-filling-based algorithm to solve it. In the algorithm, a low-voltage transformer is only responsible for communication and requiring very little computation while the PHEVs connected to it share the computation in a distributed fashion. Proofs of optimality and numerical simulations are given to demonstrate the effectiveness of our algorithm.

The rest of the paper is organized as follows. In Section 2, the model of PHEV charging is introduced and the problem formulation is presented. In Section 3, we give a water-filling algorithm and its modified version to address this problem. In Section 4, numerical simulations are given to illustrate the algorithm. Concluding remarks are given in Section 5.

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2 Preliminaries and Problem Formulation

In this section, we first give the dynamic model of PHEV charging and then introduce the charging methods of Lithium-ion batteries briefly. Finally, the DSM problem for PHEV charging at low-voltage transformers is formulated as an optimization problem.

2.1 Dynamic Model of PHEV Charging

Nowadays, almost all PHEVs use Lithium-ion batteries because of its advantages, e.g., high energy density, good load characteristics and low maintenance.

The state of charge (SOC) is the equivalent of a fuel gauge for the battery in a PHEV and is defined as:

$$SOC = \frac{C_R}{C} \times 100\% \tag{1}$$

with C representing the battery energy capacity (kWh) and C_R is the remaining battery energy capacity (kWh).

Suppose there are n households, each having a PHEV. Suppose we start at time 0 and the sampling period is set to be ΔT. For the i-th PHEV, the initial SOC, target SOC, exit time, battery capacity, charging efficiency coefficient and maximum charging power are denoted by $x_i(0), x_i^\star$, $T_i = K_i \Delta T$, C_i, $\eta_i (\in (0,\ 1))$ and $p_{i,\max} (\geq 0)$, respectively. For households without active PHEVs, their capacities and maximum charging powers will be set to zero. The dynamic model for the i-th charging PHEV is given by

$$x_i(k+1) = x_i(k) + \frac{\eta_i \Delta T}{C_i} p_i(k) \tag{2}$$

where $x_i(k)$ and $p_i(k)$ are the SOC and charging power, respectively, at time k. The charging power is assumed to be kept constant in each sampling interval.

Denoting

$$a_i = \begin{cases} \eta_i \Delta T / C_i & C_i > 0 \\ 0 & C_i = 0 \end{cases}$$

equation (2) can be rewritten as

$$x_i(k+1) = x_i(k) + a_i p_i(k) \tag{3}$$

which is valid for all PHEVs, whether charging or not. The constraint on $p_i(k)$ is given by

$$0 \leq p_i(k) \leq p_{i,\max}(k) \tag{4}$$

with

$$p_{i,\max}(k) = \begin{cases} p_{i,\max} & k < K_i \text{ and } C_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

2.2 Charging Method

Traditionally, constant current constant voltage (CC-CV) charging method is used to charge PHEVs, shown in Fig. 1. First, the PHEV is charged with a constant current and when the battery voltage limit is reached, Stage 2 begins. In terms of SOC, the best functional range of Lithium-ion batteries is from 20% to 85%. Actually, when Stage 1 terminates, SOC can reach 85%. Thus, it is recommended to ignore Stage 2 [9]. In this paper, PHEVs are charged with variable power. Regarding the effects of variable charging current on Lithium-ion battery, [10] has given a detailed description in terms of capacity fade and efficiency, from which, we can conclude that Lithium-ion batteries can be charged with dynamic power without adversary effects. Therefore, PHEVs are assumed be charged by a *smart charger*, which could determine the charging current of PHEV based on the power allocated and terminal voltage.

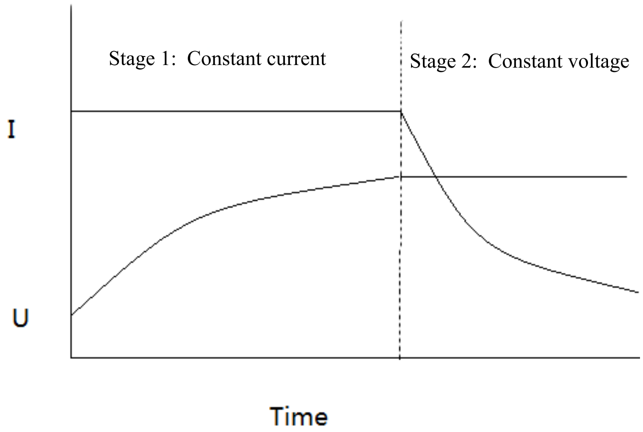

Fig. 1: CC-CV charging process of Lithium-ion batteries

2.3 Problem Formulation

Consider a distribution grid as in Fig. 2 where the distribution grid is a hierarchical structure involving a high-voltage transformer (HVT) connecting to a set of low-voltage transformers (LVT) each of which in turn connecting to a number of households with PHEV chargers [7]. The objective is to apply DSM on all PHEV chargers to maximally flatten the power demand curve at the LVT connected to the PHEVs, while satisfying each consumer's requirement for their PHEV to be charged to the required level by their specified time. As the HVT and LVT can not get the flattest demand curve at the same time and LVTs tend to get overloaded sooner than HVTs [7], we only consider LVTs in this paper.

Consider the given time horizon from $k = 0$ to $N-1$, where $N \geq K_i, i = 0, 1, \ldots, n$. It is assumed that the forecast non-PHEV power consumption $q_i(k), k = 0, 1, \ldots, N-1$, for each household i is known to the household i. The objective of DSM can be formulated as follows:

$$\begin{gathered} \min f(p,\eta) = \textstyle\sum_{k=0}^{N-1} \left(\sum_{i=1}^{n} (p_i(k) + q_i(k)) - \eta\right)^2 \\ \text{subject to} \begin{cases} x_i(K_i) = x_i^\star \\ 0 \leq p_i(k) \leq p_{i,\max}(k) \end{cases} \end{gathered} \tag{5}$$

The variables are $p_i(k)$ and η. The physical meaning of η is that this is the "ideal" flat power curve. Indeed, $f(p,\eta) = 0$ if and only if the aggregate power curve $\sum_{i=1}^{n}(p_i(k) + q_i(k))$ is flat over k. The solution for η is easily given by differentiating $f(p,\eta)$ with respect to η and setting the result to zero, which gives

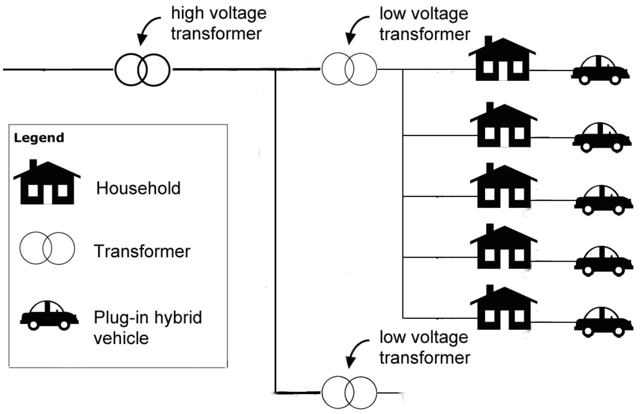

Fig. 2: Schematic description of the structure of a distribution grid [7]

$$\sum_{k=0}^{N-1}\left(\sum_{i=1}^{n}(p_i(k)+q_i(k))-\eta\right)=0 \tag{6}$$

which yields

$$\eta=\frac{1}{N}\left(\sum_{i=1}^{n}b_i+\sum_{k=0}^{N-1}\sum_{i=1}^{n}q_i(k)\right) \tag{7}$$

where $b_i=(x_i^\star-x_i(0))/a_i$. The physical meaning of b_i is the energy need of the i-th PHEV (normalized by ΔT).

3 Main Results

In this section, we first give a water filling algorithm for a single PHEV and then develop it into a multiple-PHEV one. A modified version of the second algorithm is also presented.

3.1 Water Filling for a Single PHEV

We drop the subscript because there is only one PHEV. Without loss of generality, we assume the required exit time $K=N$. Using the Lagrangian multiplier [11], the Lagrangian is given by

$$\sum_{k=0}^{K-1}(p(k)+q(k)-\eta)^2+2\lambda(\sum_{k=0}^{K-1}p(k)-b) \tag{8}$$

where $b=(x^\star-x(0))/a$. The factor of 2 above is for convenience. Differentiating the Lagrangian with respect to $p(k)$ and setting the result to zero yields

$$p(k)+q(k)-\eta+\lambda=0 \tag{9}$$

Denoting $\alpha=\eta-\lambda$ (which is independent of k), equation (9) can be rewritten as

$$p(k)+q(k)=\alpha \tag{10}$$

The above is the optimality condition without considering the constraints $0\le p(k)\le p_{\max}$. When these constraints are considered, the optimality condition becomes:

There exists a constant α (called *equip-power level*) such that for any $0\le k<K$, either (10) holds or

$$p(k)=0 \text{ and } p(k)+q(k)>\alpha \tag{11}$$

or

$$p(k)=p_{\max} \text{ and } p(k)+q(k)<\alpha \tag{12}$$

Remark 1 *The optimal solution above can be simply interpreted as the following water filling principle: Initialize $\alpha=\min_k q_k$. Then, gradually raise α. Each time α is raised a bit, compute the $p(k)=\alpha-q(k)$ and project it to the feasible region $[0,p_{\max}(k)]$, then compute $\sum_k p(k)$. Gradually raise α until the sum equals b.*

It is easy to see that the optimal value of α can be found using a bi-section method. See algorithm 1.

Algorithm 1 Water filling for a single PHEV

Input: $p_{\max},\ b$ and $q(k), k=0,1,\dots,N-1$
Output: α and $p(k), k=0,1,\dots,N-1$

```
Initialize α_min = min_k q(k) and α_max = max_k q(k) + p_max
while α_max − α_min > ε do
    Choose α = (α_min + α_max)/2
    Compute p(k) = P[α − q(k)]
    if Σ_{k=0}^{N−1} p(k) > b then
        set α_max = α
    else if Σ_{k=0}^{N−1} p(k) < b then
        set α_min = α
    end if
end while
```

In the above algorithm, ε is a very small number and $\mathcal{P}[\cdot]$ is the projection operation, i.e.,

$$\mathcal{P}[x(k)]=\begin{cases}p_{\max}(k) & x(k)>p_{\max}(k)\\ x(k) & 0\le x(k)\le p_{\max}(k)\\ 0 & x<0\end{cases}$$

3.2 Water Filling for Multiple PHEVs

For the case of multiple PHEVs, the optimal solution is not unique. This is because two PHEVs can "trade" their charging times without affecting the total power consumption. In the following, we give an optimal solution. Without loss of generality, we assume that $K_1\le K_2\le\ \dots\ K_n\le N$. The main idea is that we do water filling to all the PHEVs one by one from 1 to n until all PHEVs are done.

Algorithm 2 Water filling for Multiple PHEVs

Input: $p_{i,\max},\ b_i$ and $q_i(k), k=0,1,\dots,N-1, i=1,2,\dots,n$
Output: $p_i(k), k=0,1,\dots,N-1, i=1,2,\dots,n$

```
Every PHEV reports non-PHEV power demand q_i(k) to LV,
  k = 0,1,...,N−1, i = 1,2,...,n
LV computes the aggregate demand q(k) = Σ_{i=1}^{n} q_i(k)
for i = 1,2,...,n do
    PHEV_i gets q(k) from LV
    PHEV_i uses water filling to compute p_i(k) (Algorithm 1)
    PHEV_i reports p_i(k) to LV
    LV computes q(k) ⇐ q(k) + p_i(k)
end for
```

Theorem 1 *The solution given by Algorithm 2 is an optimal solution.*

To prove Theorem 1, we first examine some properties of the water filling principle and then give some preliminary results.

Denote the set $\{0, 1, \ldots, N-1\}$ by $\mathcal{N}$ and decompose it into three disjoint subsets: $\mathcal{N}_1 = \{k \in \mathcal{N} : p(k) = p_{\max}\}$, $\mathcal{N}_2 = \{k \in \mathcal{N} : 0 < p(k) < p_{\max}\}$; $\mathcal{N}_3 = \{k \in \mathcal{N} : p(k) = 0\}$. Then the following properties of the water filling solution $p(k)$ hold:

Property 1 *For any $k_1 \in \mathcal{N}_1$, $k_2 \in \mathcal{N}_2$, $k_3 \in \mathcal{N}_3$ (if the subsets are not empty), $q(k_1) \leq q(k_2) \leq q(k_3)$ and $q(k_1) + p(k_1) \leq q(k_2) + p(k_2) \leq q(k_3) + p(k_3)$.*

Property 2 *$p(k) + q(k)$ are at equal level for all $k \in \mathcal{N}_2$, if this subset has multiple elements.*

Property 3 *If $q(k) = q(\tilde{k})$ for some $k \neq \tilde{k}$, then $q(k) + p(k) = q(\tilde{k}) + p(\tilde{k})$.*

Lemma 1 *Consider the case $n = 2$ and $K_1 = K_2 = N$. Then, the solution of $p_i(k)$, $i = 1, 2, 1 \leq k < N$, given by Algorithm 2 is optimal.*

Proof 1 *Without loss of generality, we assume that $q(k)$ is monotonically non-decreasing, i.e., $q(0) \leq q(1) \leq \ldots \leq q(N-1)$. If this were not the case, we could sort $q(k)$ and relabel the time indices to make $q(k)$ monotonically non-decreasing. It is clear that the constraints for $p_i(k)$ remain the same after the sorting and relabeling.*

Denote $p_i = [p_i(0)\ p_i(1)\ \ldots\ p_i(N-1)]'$ and $p = [p_1\ p_2]$. We proceed by contradiction. Let $p^\star$ be the solution given by Algorithm 2 and suppose this is not an optimal solution. Note that the constrained optimization problem for $f(p, \eta)$ is convex, thus any stationary point of $f(p, \eta)$ is an optimal solution. Since $p_2^\star$ is optimised using the water filling principle for the given $p_1^\star$, $p^\star$ not being optimal means that we can perturb $p_1^\star$ without violating its constraints to further reduce $f(p, \eta)$. We show below that this is not possible.

Equivalently, this is to say that $p_1^\star$ remains optimal when q is replaced with $q + p_2^\star$. Indeed, using the properties of the water filling solution, we know that $q(k) + p_1^\star(k)$ and $q(k) + p_1^\star(k) + p_2^\star(k)$ are also monotonically non-decreasing. Use our notation of $\mathcal{N}_1, \mathcal{N}_2$ and $\mathcal{N}_3$ before. It is clear that $k_1 < k_2 < k_3$ for $k_1 \in \mathcal{N}_1, k_2 \in \mathcal{N}_3, k_3 \in \mathcal{N}_3$ (if the subsets are non-empty). Suppose we try to perturb $p_1(k_2)$ for some $k_2 \in \mathcal{N}_2$, then the water filling principle (Property 2) dictates that $p_1(k)$ must change by the same amount for all $k \in \mathcal{N}_2$. If the change is positive, $p_1(k_1)$ must decrease for some $k_1 \in \mathcal{N}_1$ and thus $q(k_1) + p_1(k_1) + p_2^\star(k_1)$ will decrease; If the change is negative, $p_1(k_3)$ must increase for some $k_3 \in \mathcal{N}_3$ and thus $q(k_3) + p_1(k_3) + p_2^\star(k_3)$ will increase. Either way, this goes against the water filling principle and the resulting value of $f(p, \eta)$ will increase. This is a contradiction, which implies that $p_1^\star$ is indeed optimal for $q + p_2^\star$. Q.E.D.

Then we give the proof of Theorem 1 on the basis of Lemma 1.

Proof 2 *We now consider the general case of any $n \geq 2$. Recall that $K_1 \leq K_2 \leq \ldots \leq K_n \leq N$. Similar to the proof of Lemma 1, we proceed by contradiction. Let $p^\star = [p_1^\star\ p_2^\star\ \ldots\ p_n^\star]$ be obtained by Algorithm 2 and suppose that this is not optimal. This implies that $[p_1^\star\ p_2^\star \ldots\ p_{n-1}^\star]$ can be perturbed to reduce $f(p, \alpha)$ further. Since the constraints for each p_i are independent, the above further implies that there exists some $i < n$ such that $p_i^\star$ can be perturbed to reduce $f(p, \alpha)$. It is clear that the choice of p_i does not affect those $p(k)$ with $k \geq K_i$. Therefore, it suffices to consider the following optimization problem:*

$$\min_{p_i, \eta} \sum_{k=0}^{K_i - 1} (Q(k) + p_i(k) + P_2(k) - \eta)^2$$

subject to the given constraints on p_i, where $Q(k) = q(k) + \sum_{j=1}^{i-1} p_j^\star(k)$, $P_2(k) = \sum_{j=i+1}^{n} p_j^\star(k)$. Following the same argument as in the proof of Lemma 1, we conclude that the optimal p_i for solving the above is still $p_i^\star$. This contradicts the earlier assumption that $p^\star$ is not optimal for minimizing $f(p, \eta)$. Hence, $p^\star$ must be an optimal solution. Q.E.D.

Corollary 1 *It is inferred from the Proof of Theorem 1 that if $K_1 = K_2 \ldots = K_n = N$, the result is optimal no matter in which order we do water filling.*

3.3 Modified Water Filling for Multiple PHEVs

The solution given by Algorithm 2 tends to meet the demand of PHEV$_i$ earlier than that of PHEV$_j$ for any $i < j$. This can create a problem if the future supply and demand forecasts become inaccurate or the exit times get altered. It would be more desirable to have a somewhat balanced dispatch of power to all PHEVs. This can be achieved by the following modified version of Algorithm 2. To describe the modified algorithm more clearly, we give the definition of *Circular Order*.

Definition 1 *Given n numbers, $x_1 \leq x_2 \leq \ \ldots\ x_n$, then Circular Order includes n different orders and the i-th one is $x_i, x_{i+1}, \ \ldots\ x_n, x_1 \ldots\ x_{i-1}$*

Here we still assume that $K_1 \leq K_2 \leq \ \ldots\ K_n \leq N$. The idea is to do power allocation in Circular Order and allocate power according to the average. More specifically, firstly, do water filling as in Algorithm 2 in the first Circular Order and get the intermediate energy need at $K_1, K_2, \ldots, K_{n-1}$; then do water filling in other orders in the interval from 0 to K_1, K_1 to K_2,... K_{n-1} to K_n. See Algorithm 3 for details.

Algorithm 3 Modified water filling for multiple PHEVs

Input: $p_{i,\max}$, b_i , $q_i(k), k = 0, 1, \ldots, N-1$ and $K_i, i = 1, 2, \ldots, n$, $K_0 = 0$
Output: $\bar{p}_i(k), k = 0, 1, \ldots, N-1$

Do water filling as in Algorithm 2 in the first Circular Order
Every PHEVi gets its intermediate energy need at time $K_1, K_2, \ldots K_{i-1}$
for $i = 1, 2, \ldots, n-1$ **do**
    Do water filling as in Algorithm 2 all Circular Orders during the interval from K_{i-1} to K_i
    Compute $\bar{p}_i(k), k = K_{i-1}, \ldots, K_i$
end for
Get $\bar{p}_i(k), k = 0, 1, \ldots, N-1$

Theorem 2 *The solution given by Algorithm 3 is an optimal solution.*

To prove Theorem 2, we first give the following lemma.

Lemma 2 *Consider the case $K_1 = K_2 \ldots = K_n = N$, do water filling according to Algorithm 2 n times in Circular Order. The average of the n optimal results is still optimal.*

Proof 3 *Optimal solution to the i-th PHEV in the j-th Circular Order is denoted by* $p_{i,j}^{\star} = [p_{i,j}^{\star}(0)\ p_{i,j}^{\star}(1)\ \ldots\ p_{i,j}^{\star}(K_i - 1)]'$ *and the average* $1/n \sum_{j=1}^{n} p_{i,j}^{\star}$ *is denoted by* $\bar{p_i}^{\star} = [\bar{p_i}^{\star}(0)\ \bar{p_i}^{\star}(1)\ \ldots \bar{p_i}^{\star}(K_i)]'$*. First, we prove* $\bar{p_i}^{\star}$ *still satisfy the two constraints.*

$$p_{i,j}^{\star}(k) \leq p_{i,max}(k) \Rightarrow$$

$$\bar{p_i}^{\star}(k) = \frac{1}{n}\sum_{j=1}^{n} p_{i,j}^{\star}(k) \leq \frac{1}{n}\sum_{j=1}^{n} p_{i,max}(k) = p_{i,max}(k)$$

Similarly, $\bar{p_i}(k) \geq 0$*. So* $0 \leq p_i(k) \leq p_{i,\max}(k)$ *is satisfied.*

$$\sum_{k=0}^{N-1} p_{i,j}^{\star}(k) = b_i \Rightarrow$$

$$\sum_{k=0}^{N-1} \bar{p_i}^{\star}(k) = \sum_{k=0}^{N-1} \frac{1}{n}\sum_{j=1}^{n} p_{i,j}^{\star}(k) = \frac{1}{n}\sum_{j=1}^{n}\sum_{k=0}^{N-1} p_{i,j}^{\star}(k) = b_i$$

Thus the equality constraint is also satisfied. We proceed with the proof of optimality. Recall the objective function in (5), $p_{i,j}^{\star}(k)$ *being optimal means*

$$\sum_{i=1}^{n}(p_{i,j}^{\star}(k) + q_i(k)) - \eta = M^{\star}(k), \quad j = 1, 2, \ldots, n$$

So

$$\begin{aligned}
& \textstyle\sum_{i=1}^{n}(\bar{p_i}^{\star}(k) + q_i(k)) - \eta \\
= & \textstyle\sum_{i=1}^{n}(\frac{1}{n}\sum_{j=1}^{n} p_{i,j}^{\star}(k) + q_i(k)) - \eta \\
= & \textstyle\sum_{i=1}^{n}\frac{1}{n}\sum_{j=1}^{n} p_{i,j}^{\star}(k) + \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{n} q_i(k) \\
& \textstyle -\frac{1}{n}\sum_{j=1}^{n}\eta \\
= & \textstyle\frac{1}{n}\sum_{j=1}^{n}(\sum_{i=1}^{n} p_{i,j}^{\star}(k) + \sum_{i=1}^{n} q_i(k) - \eta) \\
= & \textstyle\frac{1}{n}\sum_{j=1}^{n}[\sum_{i=1}^{n}(p_{i,j}^{\star}(k) + q_i(k)) - \eta] \\
= & \textstyle\frac{1}{n}\sum_{j=1}^{n} M^{\star}(k) = M^{\star}(k)
\end{aligned}$$

As a result, $\bar{p_i}^{\star}$ *is also optimal. Q.E.D.*

Then we give the proof of Theorem 2 on the basis of Lemma 2.

Proof 4 *We consider the general case where* $K_1 \leq K_2 \leq \ldots \leq K_n \leq N$*. After doing water filling as in Algorithm 2 in the first Circular Order every PHEVi gets its intermediate energy needs at times* $K_1, K_2, \ldots K_{i-1}$*. Then, for every time interval* K_i *to* K_{i+1}*, following the same argument as in the proof of Lemma 2,* $\bar{p}_i$ *is optimal. Q.E.D.*

4 Numerical Simulations

In this section, we first give a numerical simulation to illustrate Algorithm 2 and Algorithm 3. Then, realistic power consumption data are used to demonstrate the effectiveness of Algorithm 3 in flattening low-voltage transformer load curve.

4.1 Illustration of Algorithm 2 and Algorithm 3

We consider 5 households here and the simulation parameters are given in Table 1. The aggregate non-PHEV power is given by

$$q(k) = \begin{cases} 25 & k = 1, 2 \ldots 10 \\ q(k) = \frac{23}{400}(k-31)^2 + 2 & k = 11, 12 \ldots 50 \end{cases}$$

Table 1: Parameters of the five households

NO.	Max Power	Energy Need	Exit Time	non-PHEV
1	4	20	38	$0.2q(k)$
2	3.5	15	50	$0.3q(k)$
3	3.9	20	50	$0.1q(k)$
4	4.6	25	50	$0.25q(k)$
5	4	20	50	$0.15q(k)$

Both Algorithm 2 and Algorithm 3 can minimize the objective function and flatten the aggregate power curve; see Fig. 3.

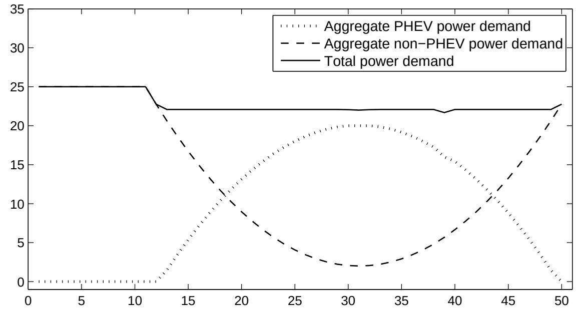

Fig. 3: Power curve from Algorithm 2 and Algorithm 3

However, the power allocated to each PHEV is completely different for different algorithms. As we have mentioned before, the solution given by Algorithm 2 tends to meet the demand of PHEV_i earlier than that of PHEV_j for any $i < j$. We can see from Fig. 4 and Fig. 5 that Algorithm 3 gives more balanced charging curves than given by Algorithm 2.

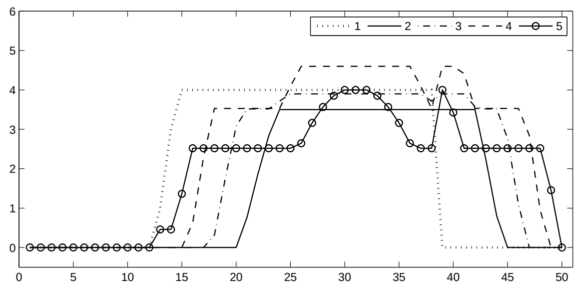

Fig. 4: Power allocation by Algorithm 2

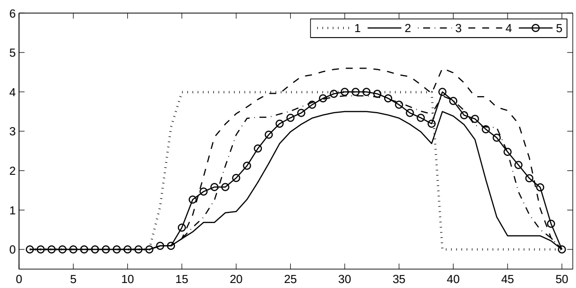

Fig. 5: Power allocation by Algorithm 3

4.2 Simulation Using Realistic Data

The realistic household non-PHEV demand curve has similar pattern to the curve in [6] as described in Fig. 6-a.

Fig. 6-b is regarded as the predicted household non-PHEV demand curve and we emulate it by processing the data in Fig. 6-a with a low-pass filter. Parameters of the five PHEVs are shown in Table 2. These data are from [6, 12, 13].

Table 2: Parameters of the five PHEVs

Model	Battery Size	Energy Need	Max Power
GM-Chevy Volt	16	8	3.84
Nissan-Leaf	24	12	6.6
Tesla-MODEL S	60	30	10
Volvo-C30	24	12	3.52
BMW-Mini E	35	17	11.52

In the simulation, suppose PHEVs are not at home from 7:30 to 17:30 (450-1050 minutes), so they are not allowed to be charged during this interval. Fig. 6-c describes the power allocated to this PHEV using Algorithm 3 and Fig. 6-d shows the total power curve of this household. It can be seen that no new peak load is created.

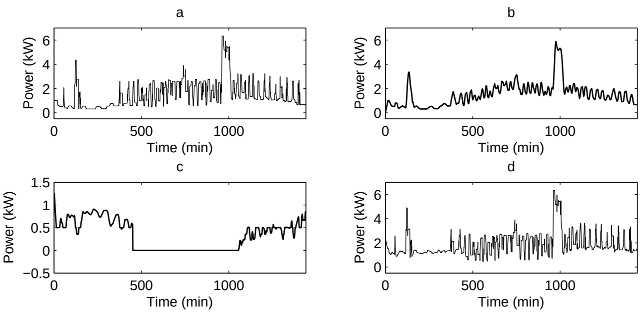

Fig. 6: Power curve of a household

Fig. 7 shows the total power demand curve with and without DSM. Without DSM, every PHEV starts charing with maximum power as soon as it arrives home. Obviously, a new peak load is created and it can be devastating to the low-voltage transformer. DSM helps to shift the charing timing of PHEVs to when non-PHEV power demand is relatively small.

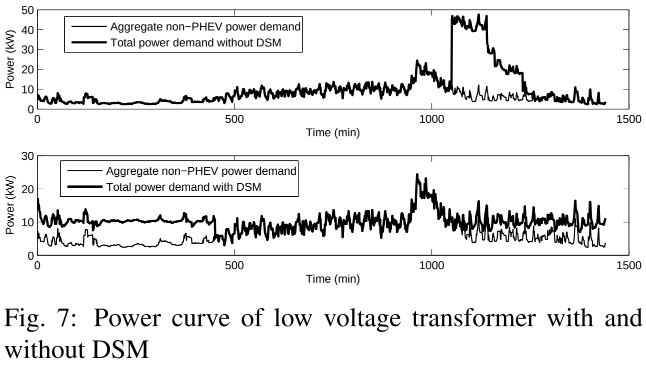

Fig. 7: Power curve of low voltage transformer with and without DSM

5 Conclusion

Demand side management of PHEVs will become necessary to reduce peak loads as the penetration of PHEVs become greater. Trying to flatten power demand curve at transformers will avoid overloading and defer investment. In this paper, we formulate this problem as a convex optimization problem and propose a distributed algorithm, in which the water filling principle plays a pivotal role. Simulation results show that the proposed algorithm can efficiently fulfill the task of flattening power demand curve.

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