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Consensus-based Cooperative Source Localization of Multi-agent Systems

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Abstract: This paper addresses cooperative source localization with a group of mobile agents, for which the goal is to estimate the coordinate of a stationary source for each agent in its own local frame. It is assumed that each agent can only have (possibly failure) range measurements about the source and some neighbors. Collaboration among agents is desired to overcome possible measurement failures and have unintermitted estimates in real time when some agents do not have direct range measurements about the source. The paper develops a continuous time estimation scheme and proposes a cooperative source localization approach for the problem. It is shown that the estimate of every agent is globally asymptotically convergent as long as there is a path from every agent to the source in the sensing graph. Simulation results are also presented to verify the effectiveness of the proposed algorithms.

Key Words: Multi-agent system, Source Localization, Consensus

1 Introduction

Source localization refers to the problem of estimating the precise location of a source, using different kinds of information such as distance, bearing, power level (which is indirectly related to distance), and time difference of arrival information, where two or more agents are involved. It has a lot of applications. For example, a base station in a cellular network may need to estimate the location of a phone in its region of coverage. In sensor networks, groups of sensors may have to position themselves for different purposes such as facilitating routing, target tracking and proper network coverage. Teams of mobile robots may be used in search and rescue where a person or object needs to be located.

There are two research thrusts in this area. In the first, clusters of stationary agents collaborate to localize a source. In two dimensions, this would generically require that at least three distinct non-collinearly situated agents use their distances from the source they seek to localize (see for example, [9, 11, 16, 17]). In the second thrust, a single mobile agent or a team of mobile agents exploits its motion to localize a search target. In [3–5, 13, 15], a single mobile robot is considered to localize an object of interest using different types of measurable information. The main idea of [3, 4, 15] is to design a continuous-time estimator that is able to exponentially converge to a true coordinate of the target. More recently, a probabilistic approach is developed to estimate the location of a target as well as seeking to maximize the usefulness of future measurements by designing information-theoretic control strategies [2, 7].

This paper also studies the cooperative localization problem using a team of collaborated autonomous mobile agents. In contrast with the probabilistic approach of cooperative source localization in [2, 7], the paper aims to develop a deterministic cooperative estimation scheme for all the agents to locate a target of interest in their local frames. It is assumed that the mobile agents can only measure relative distance information, but not all agents have access to the relative distance information with respect to the target due to physical constraints of onboard sensors or complex environments. Thus, collaboration between agents is necessary to have their unintermitted estimations in real time. To address the cooperative localization problem, we first develop a continuous-time observer to estimate the relative coordinate of a stationary source or a mobile neighbor. The proposed observer utilizes only the range measurement and the velocity information of itself and its mobile neighbors, which can be obtained via communication. To avoid calculating the derivative of the range in the estimator, a linear time-varying differentiator is adopted and input-to-state stability theory is recalled to show the asymptotic convergence. Second, a consensus-like fusion scheme is proposed for every agent to fuse the estimates from its neighbors together with its own estimate if it has one. The proposed scheme is fully distributed with only the exchange of the estimates between neighbors. Yet the estimate of every agent asymptotically converges to the true relative coordinate of the source no matter whether the agent has a direct range measurement of the source or not. The proposed cooperative localization strategy also works even when the sensing topology may change over time.

The rest of this paper is organized as follows. In Section II, we formulate the problem we study in this paper. In Section III, an estimator is proposed to localize a stationary source in the local coordinate frame of a mobile agent. In Section IV, a cooperative estimator is developed by introducing a consensus-like fusion scheme. Simulation results are presented in Section V to illustrate the effectiveness of the proposed cooperative localization scheme. In section VI, some conclusions and future directions are discussed.

2 Problem Statement

The paper addresses the *cooperative source localization* problem, for which the goal is to estimate the coordinate

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of a stationary source for each agent in its own local frame based on only range measurements and limited information exchange between neighbors. One stationary source labeled 0 and N mobile agents labeled $1, 2, \ldots, N$ are considered.

Suppose that each agent i is able to access its own velocity v_i in its own inertial frame Σ_i^I. Also, assume that each agent has a moving frame Σ_i^M attached to its body with the orientation the same as that of its inertial frame Σ_i^I. For each agent i, denote by x_{ij}, $j = 0, 1, \ldots, N$, the relative coordinate of the source 0 or agent j in agent i's moving frame Σ_i^M. An illustration is given in Fig. 1. Moreover, with an

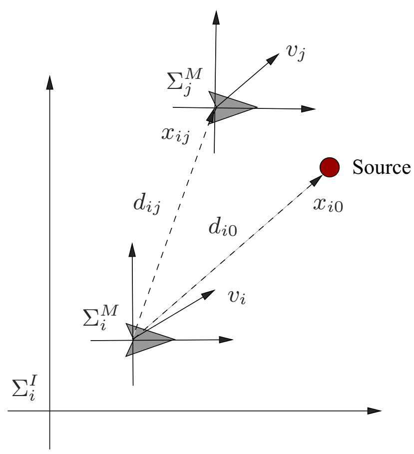

Fig. 1: Local frames and relative states.

onboard sensor each agent i can have range measurements of the source 0 and/or some other agents j, i.e.,

$$d_{ij} = \|x_{ij}\| \tag{1}$$

where $\|\cdot\|$ refers to the Euclidean norm. The objective is to design an estimator for each agent i to estimate the relative coordinate x_{i0} in its local frame. When an agent does not have a direct range measurement d_{i0} about the source, collaboration from others is necessary to help it estimate the coordinate of the source. Instead, when a direct range measurement d_{i0} about the target is available, collaboration from others is still helpful in improving the estimation performance.

In the paper, we make the following assumptions.

Assumption 1. *The velocity $v_i(t)$ of each agent i, $i = 1, \ldots, N$, is continuously differentiable and bounded.*

Assumption 2. *For any i and j, $d_{ij}(t) : \mathbb{R}_{>0} \to \mathbb{R}$ is smooth and any kth ($k = 0, 1, 2, \ldots$) derivative of $d_{ij}(t)$ satisfies*

$$\sup_{t\geq 0} |d_{ij}^{(k)}(t)| \leq M_0 \text{ for a constant } M_0. \tag{2}$$

Assumption 3. *The orientations of reference frames Σ_i^I and Σ_j^I for $i \neq j$ are consistent.*

Assumption 1 ensures that the motion of the agents can be executed by finite force. Assumption 2 can be inferred from Assumption 1 if no two agents diverge away from each other. Assumption 3 can be made satisfied if the agents carry compasses or they initially agree on a common direction via a distributed consensus scheme.

Under Assumption 3, it can be obtained that

$$\dot{x}_{ij}(t) = v_j(t) - v_i(t). \tag{3}$$

On the other hand, since the source is assumed to be stationary, it then follows that

$$\dot{x}_{i0}(t) = -v_i(t). \tag{4}$$

Let $\mathcal{N}_i$ represent the set of agents, $j \in \{1, 2, \ldots, N\}$, called the *neighbor set* of agent i, for which the distance measurement d_{ij} can be measured mutually by agent i and j and furthermore agent i and j can exchange certain information (e.g., their own velocity and their estimate about the coordinate of the source in their local moving frames) via communications. We use an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of $N + 1$ nodes to represent the sensing topology, where $\mathcal{V} = \{0, 1, \ldots, N\}$ corresponds to the set including the source and the N agents. There is an edge (i, j) in $\mathcal{E}$ if j is in $\mathcal{N}_i$ when $j \neq 0$, and agent i has the range measurement about the source when $j = 0$. The graph $\mathcal{G}$ might be time-varying due to possible failures of range measurements about the source or neighbors.

3 Source Localization with a Single Agent

In this section, we assume that the range measurement $d_{i0}(t)$ about the source is available to agent i all the time. We propose an observer to estimate the coordinate of the source in its own frame using only the range measurement $d_{i0}(t)$.

Taking the derivative of both sides of $d_{i0}^2 = \|x_{i0}\|^2$ with respect to time and considering (4), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}[d_{i0}^2(t)] = -2v_i^{\mathrm{T}}(t)x_{i0}$$

i.e.,

$$d_{i0}(t)\dot{d}_{i0}(t) = -v_i^{\mathrm{T}}(t)x_{i0}. \tag{5}$$

Denote by $\hat{x}_{i0}(t)$ the estimate of $x_{i0}(t)$ and denote by $\tilde{x}_{i0}(t) = \hat{x}_{i0}(t) - x_{i0}(t)$ the estimation error. We consider the following estimator

$$\dot{\hat{x}}_{i0}(t) = -v_i(t) - v_i(t)[d_{i0}(t)\dot{d}_{i0}(t) + v_i^{\mathrm{T}}(t)\hat{x}_{i0}(t)]. \tag{6}$$

Combining (5) and (6), one obtains the dynamics of the estimation error as follows.

$$\dot{\tilde{x}}_{i0}(t) = -v_i v_i^{\mathrm{T}}\tilde{x}_{i0}(t). \tag{7}$$

The limit of $\hat{x}_{i0}(t)$ gives an estimate for the coordinate of the source when $\tilde{x}_{i0}(t)$ converges to zero. We then have the following result.

Theorem 1. *The zero solution of (7) is exponentially stable if and only if there exist $\mu_1 > 0$, $\mu_2 > 0$ and $T > 0$ such that for all $t \geq 0$*

$$\mu_1 I \leq \int_t^{t+T} v_i(\tau)v_i^{\mathrm{T}}(\tau)\mathrm{d}\tau \leq \mu_2 I \tag{8}$$

Proof. The proof is a direct consequence of Theorem 1 in [1]. □

Next we come to understand the condition (8). We recall the notion of linear independent functions here.

Definition 1 ([14]). *The n functions $f_1(t), f_2(t), \ldots, f_n(t)$ are* linearly dependent *if, for some $c_1, c_2, \ldots, c_n \in \mathbb{R}$ not all zero,*

$$\sum_{i=1}^{n} c_i f_i(t) = 0 \tag{9}$$

for all t in some interval $\mathcal{I}$. Otherwise, they are said to be linearly independent.

Let $v_i(t) = [v_{i_x}(t)\ v_{i_y}(t)]^{\mathrm{T}}$. Then

$$v_i v_i^{\mathrm{T}} = \begin{bmatrix} v_{i_x}^2(t) & v_{i_x}(t)v_{i_y}(t) \\ v_{i_x}(t)v_{i_y}(t) & v_{i_y}^2(t) \end{bmatrix}. \tag{10}$$

Define

$$\begin{aligned} A(t) &= \int_t^{t+T} v_i(\tau)v_i^{\mathrm{T}}(\tau)\mathrm{d}\tau \\ &= \begin{bmatrix} \int_t^{t+T} v_{i_x}^2(\tau)\mathrm{d}\tau & \int_t^{t+T} v_{i_x}(\tau)v_{i_y}(\tau)\mathrm{d}\tau \\ \int_t^{t+T} v_{i_x}(\tau)v_{i_y}(\tau)\mathrm{d}\tau & \int_t^{t+T} v_{i_y}^2(\tau)\mathrm{d}\tau \end{bmatrix}. \end{aligned}$$

By the Cauchy-Bunyakovsky inequality [6], we derive that

$$\int_t^{t+T} v_{i_x}^2(\tau)\mathrm{d}\tau \int_t^{t+T} v_{i_y}^2(\tau)\mathrm{d}\tau \geq \left(\int_t^{t+T} v_{i_x}(\tau)v_{i_y}(\tau)\mathrm{d}\tau\right)^2$$

stands for any given $T \geq 0$. The Cauchy-Bunyakovsky inequality becomes an equality if and only if $v_{i_x}(t)$ and $v_{i_y}(t)$ are linearly dependent. Hence, if $v_{i_x}(t)$ and $v_{i_y}(t)$ are linearly independent, then $A(t)$ is positive definite. That is, as long as the two components $v_{i_x}(t)$ and $v_{i_y}(t)$ of the velocity are linearly independent, the estimation error exponentially converges to zero. However, adopting such an approach in practice would require the explicit differentiation of the measured signal $d_{i0}(t)$. In the sequel, we provide a linear time-varying (LTV) differentiator to estimate the derivative of $d_{i0}(t)$.

Ibrir [8] proposed an exact linear time-varying differentiator whose state converges asymptotically to the successive higher derivatives of a given input signal. This differentiator does not need any information about the signal to be differentiable, like the nature of the signal or *a prior* knowledge of the upper bounds of its higher derivatives.

Lemma 1 ([8]). *Let $\eta(t) : \mathbb{R}_{>0} \to \mathbb{R}$ be a scalar function of class C^∞ and let $(\rho_k,\ k = 0, 1, 2, \cdots)$ be a sequence of positive real numbers. If the higher derivatives of $\eta(t)$ satisfy*

$$\sup_{t\geq 0} |\eta^{(k)}(t)| \leq \rho_k, \quad k = 0, 1, 2, \ldots$$

then the state of the following time-varying system

$$\dot{z}(t) = A(t)z(t) + B(t)\eta(t) \tag{11}$$

asymptotically converges to $[\eta(t)\ \dot{\eta}(t)]^T$. We note

$$A(t) = \begin{bmatrix} 0 & 1 \\ -\alpha^2 t^2 & -2\alpha t \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \alpha^2 t^2 \end{bmatrix},\ \alpha \in \mathbb{R}_{>0}.$$

Example: Here we give a simple example to illustrate the LTV differentiator. We take $\eta(t) = \sin(t) + \cos(5t)$. The estimation error of the derivative of $\eta(t)$ for the LTV differentiator (11) is depicted in Fig. 2, which asymptotically converges to zero.

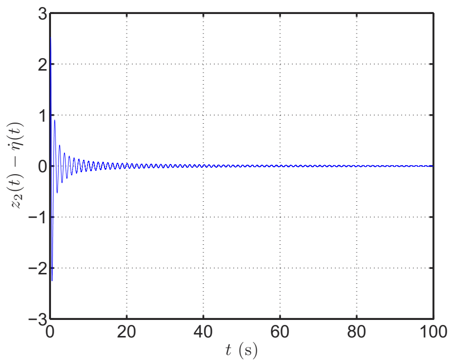

Fig. 2: The estimation error of the derivative of $\eta(t)$.

Considering the LTV differentiator, we propose the following estimator that uses only the distance measurement and its own velocity.

$$\begin{cases} \dot{\eta}_{i0}(t) &= \xi_{i0}(t) \\ \dot{\xi}_{i0}(t) &= -\alpha^2 t^2 \eta_{i0}(t) - 2\alpha t \xi_{i0}(t) + \alpha^2 t^2 d_{i0}(t) \\ \dot{\hat{x}}_{i0}(t) &= -v_i(t) - v_i(t)[d_{i0}(t)\xi_{i0}(t) + v_i^T(t)\hat{x}_{i0}(t)]. \end{cases} \tag{12}$$

We define

$$\begin{aligned} \varrho_{i0}(t) &= \eta_{i0}(t) - d_{i0}(t), \\ \delta_{i0}(t) &= \xi_{i0}(t) - \dot{d}_{i0}(t). \end{aligned} \tag{13}$$

Then we obtain the following estimation error dynamics for $\tilde{x}_{i0}$.

$$\dot{\tilde{x}}_{i0}(t) = -v_i(t)v_i^{\mathrm{T}}(t)\tilde{x}_{i0}(t) - v_i(t)d_{i0}(t)\delta_{i0}(t). \tag{14}$$

The convergence property of the modified estimator (12) is given below.

Theorem 2. *Suppose Assumption 1 and Assumption 2 hold. If* (8) *holds, then $\tilde{x}_{i0}(t)$ in* (14) *asymptotically converges to* 0.

The proof of the theorem uses input-to-state stability theory. The notion of input-to-state stability and a related result will be presented first.

Definition 2 ([10]). *Consider a nonlinear dynamical system*

$$\dot{x} = f(t, x, u) \tag{15}$$

with the state $x(t)$ and inputs $u(t)$. It is said to be input-to-state stable *if for every initial state $x(t_0) \in \mathbb{R}^n$ and every continuous and bounded input $u(t) \in \mathbb{R}^m$, the solution $x(t)$ exists for all $t \geq t_0$ and satisfies*

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|), \tag{16}$$

where $\beta(s,t)$ is a class $\mathcal{KL}$ function and $\gamma(s)$ is a class $\mathcal{K}$ function.

The inequality (16) guarantees that for a bounded input $u(t) \in \mathbb{R}^m$, the state $x(t)$, $t \geq t_0$, is bounded. In particular, as t increases, the state $x(t)$, $t \geq t_0$, is bounded by a class $\mathcal{K}$ function of $\sup_{t\geq t_0} \|u(t)\|$.

Lemma 2 ([10]). *Suppose $f(t,x,u)$ is continuously differentiable and globally Lipschitz in (x,u), uniformly in t. If the unforced system $\dot{x} = f(t,x,0)$ has a globally exponentially stable equilibrium at the origin $x = 0$, then the system (15) is input-to-state stable.*

Consider system (14) and take $-v_i(t)d_{i0}(t)\delta_{i0}(t)$ as the input. Since system (7) is exponentially stable, then by Lemma 2, we conclude that (14) is input-to-state stable. Furthermore, since $v_i(t)$ and $d_{i0}(t)$ are bounded and $\delta_{i0}(t) \to 0$ as $t \to 0$ according to Lemma 1, we come to the result that $v_i(t)d_{i0}(t)\delta_{i0}(t) \to 0$ as $t \to \infty$. Thus, we can show that $\tilde{x}_{i0}(t) \to 0$ as $t \to \infty$ by the input-to-state stability theory. The following is the rigorous proof of Theorem 2.

Proof of Theorem 2: Denote $u(t) = -v_i(t)d_{i0}(t)\delta_{i0}(t)$. Since $v_i(t)$ and $d_{i0}(t)$ are bounded (Assumptions 1-2) and $\delta_{i0}(t) \to 0$ as $t \to \infty$, it follows from Lemma 1 that $\lim_{t\to\infty} u(t) = 0$. On the other hand, from Theorem 1, it follows that

$$\dot{\tilde{x}}_{i0}(t) = -v_i v_i^{\mathrm{T}} \tilde{x}_{i0}(t)$$

is globally exponentially stable. Denote

$$f(t,\tilde{x}_{i0},u) = -v_i(t)v_i^{\mathrm{T}}(t)\tilde{x}_{i0}(t) + u.$$

Then it can be checked that there exists L such that

$$\|f(t,\tilde{x}_{i0},u) - f(t,\tilde{x}_{i0}',u')\| \le L \left\| \begin{bmatrix} u - u' \\ \tilde{x}_{i0} - \tilde{x}_{i0}' \end{bmatrix} \right\| \quad (17)$$

because $v_i(t)$ is bounded by Assumption 1. That is, $f(t,\tilde{x}_{i0},u)$ is globally Lipschitz in $(\tilde{x}_{i0},u)$, uniformly in t. By lemma 2, one knows that (14) is input-to-state stable, i.e., for all $t \ge t_0$,

$$\|\tilde{x}_{i0}(t)\| \le \beta(\|\tilde{x}_{i0}(t_0)\|, t - t_0) + \gamma(\sup_{t_0 \le \tau \le t} \|u(\tau)\|).$$

For any given $\varepsilon > 0$, choose $\mu > 0$ such that $\gamma(\mu) \le \varepsilon/2$. Since $\lim_{t\to\infty} u(t) = 0$, it follows that there exists $t_1 > 0$ such that $\|u(t)\| \le \mu$ when $t \ge t_1$. Now, since $\tilde{x}_{i0}(t)$ is bounded, suppose the bound of $\tilde{x}_{i0}(t)$ is ρ. Then, it follows that

$$\begin{aligned} \|\tilde{x}_{i0}(t)\| &\le \beta(\|\tilde{x}_{i0}(t_1)\|, t - t_1) + \gamma(\mu) \\ &\le \beta(\rho, t - t_1) + \varepsilon/2 \end{aligned} \quad (18)$$

for $t \ge t_1$. Since $\beta(\rho, t - t_1) \to 0$ as $t \to \infty$, there exists $t_2 > 0$ such that $\beta(\rho,t) \le \varepsilon/2, t \ge t_2$. Thus, it follows from (18) that $\|\tilde{x}_{i0}(t)\| < \varepsilon$, $t > \mathcal{T}$, where $\mathcal{T} = \mathtt{max}(t_1,t_2)$, which implies that

$$\lim_{t\to\infty} \|\tilde{x}_{i0}(t)\| = 0,$$

i.e., $\tilde{x}_{i0}(t)$ converges to 0 as $t \to \infty$. □

4 Cooperative Source Localization with Multiple Agents

In this section, we investigate the cooperative source localization problem using multiple mobile agents.

First we consider how to estimate the coordinate of a neighbor mobile agent in order to use the estimation information about the source passed from the neighbor. The estimation scheme is quite similar to the one of estimating the source developed in the preceding section, except that the object being localized is not stationary now.

Taking the derivative of both sides of $d_{ij}^2 = \|x_{ij}\|^2$ and following the same procedure as for the stationary source, we obtain an estimator of the following form

$$\dot{\hat{x}}_{ij}(t) = -\nu_{ij}(t) - \nu_{ij}(t)[d_{ij}(t)\dot{d}_{ij}(t) + \nu_{ij}^{\mathrm{T}}(t)\hat{x}_{ij}(t)] \quad (19)$$

where $\hat{x}_{ij}$ is the estimate of the coordinate of agent j in the moving frame Σ_i^M of agent i and $\nu_{ij} = v_i - v_j$.

Notice that in (19), $\dot{d}_{ij}(t)$ is not available to agent i. So we again consider the LTV differentiator to have $\dot{d}_{ij}(t)$. Moreover, the moving velocity v_j of agent j can be communicated to agent i as agent j is a neighbor of agent i. Thus, the following estimator for agent i uses only its range measurement and the information from its neighbors obtained via communication to estimate the relative coordinate of agent $j \in \mathcal{N}_i$.

$$\begin{cases} \dot{\eta}_{ij}(t) = \xi_{ij}(t) \\ \dot{\xi}_{ij}(t) = -\alpha^2 t^2 \eta_{ij}(t) - 2\alpha t \xi_{ij}(t) + \alpha^2 t^2 d_{ij}(t) \\ \dot{\hat{x}}_{ij}(t) = -\nu_{ij}(t) - \nu_{ij}(t)[d_{ij}(t)\xi_{ij}(t) + \nu_{ij}^{\mathrm{T}}(t)\hat{x}_{ij}(t)]. \end{cases} \quad (20)$$

Following almost the same argument for the case of estimating the stationary source, the following result can be obtained.

Corollary 1. *Suppose Assumption 1 and Assumption 2 hold. If there exist $\mu_1 > 0$, $\mu_2 > 0$, and $T > 0$ such that for all $t \ge 0$*

$$\mu_1 I \le \int_t^{t+T} \nu_{ij}(\tau)\nu_{ij}^T(\tau)d\tau \le \mu_2 I, \quad (21)$$

then $\hat{x}_{ij}(t)$ in (20) asymptotically converges to $x_{ij}(t)$.

Next, we develop a cooperative estimator for each agent i to estimate the relative coordinate of the stationary source though it may not have a direct range measurement about the source or may have measurement failures over time. As illustrated in Fig. 3, if agent i is able to estimate agent j's relative coordinate and agent j is able to estimate and communicate to agent i the relative coordinate of the source, denoted as z_j, in its local frame Σ_j^M, then the estimate of the source can be indirectly obtained by agent i as

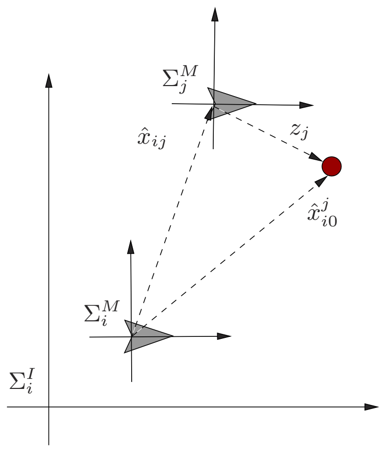

Fig. 3: Indirect estimation of the relative coordinate of the source.

$$\hat{x}_{i0}^j = \hat{x}_{ij} + z_j. \quad (22)$$

Let $\sigma_i(t)$ be 1 if agent i has the direct range measurement about the source and 0 otherwise. Then we consider a consensus-like estimation fusion scheme for agent i, i.e.,

$$\dot{z}_i(t) = -v_i(t) + \sigma_i(t)\left[\hat{x}_{i0}(t) - z_i(t)\right] + \sum_{j\in\mathcal{N}_i(t)} \left[\hat{x}_{i0}^j(t) - z_i(t)\right] \tag{23}$$

where z_i represents the estimate of the relative coordinate of the source in the local frame Σ_i^M, and $\hat{x}_{i0}$ and $\hat{x}_{i0}^j$ are the direct and indirect estimate of the source obtained in (12) and (22), respectively.

Then we have the following result.

Theorem 3. *Suppose* (8) *and* (21) *hold. If at any time t there is a path from any agent i to the source node in $\mathcal{G}$, then the estimate $z_i(t)$ in* (23) *asymptotically converges to the relative coordinate $x_{i0}(t)$ of the source.*

Proof. For $i = 1, \ldots, N$, we let $y_i = z_i - x_{i0}$, $\tilde{x}_{i0} = \hat{x}_{i0} - x_{i0}$, and $\tilde{x}_{ij} = \hat{x}_{ij} - x_{ij}$. Then (23) can be transformed to

$$\dot{y}_i = \sum_{j\in\mathcal{N}_i} [y_j - y_i] + \sigma_i\left[\tilde{x}_{i0} - y_i\right] + \sum_{j\in\mathcal{N}_i} \tilde{x}_{ij}. \tag{24}$$

This is a typical consensus system, in which y_i ($i = 1, \ldots, N$) are the individual states of N agents, $\tilde{x}_{i0}$ ($i = 1, \ldots, N$) can be treated as the states of N external reference agents, and $\sum_{j\in\mathcal{N}_i} \tilde{x}_{ij}$ can be thought as an external perturbation. If at any time t there is a path from any agent i to the source node in $\mathcal{G}$, it means that at least one σ_i in (24) is nonzero and also $\tilde{x}_{i0}(t)$ tends to zero followed from Theorem 1. Thus, from the consensus result [12], it can be known that without the perturbation term $\sum_{j\in\mathcal{N}_i} \tilde{x}_{ij}$, all the individual states y_i's asymptotically converge to the external reference signal $\tilde{x}_{i0}(t)$ and therefore approaches 0. In addition, notice that

$$\sum_i \sum_{j\in\mathcal{N}_i} \tilde{x}_{ij}(t) = 0$$

always holds, which means this perturbation term does not shift the centroid of the consensus states $y_1, \ldots, y_N$. Also, it vanishes as time goes to infinity according to Corollary 1. Therefore, all y_i's still converge to 0 even in the presence of the term $\sum_{j\in\mathcal{N}_i} \tilde{x}_{ij}$ in (24). Thus, it can be concluded that $z_i(t) \to x_{i0}(t)$ as $t \to \infty$. □

5 Simulation

In this section, we present a simulation of five mobile agents achieving cooperative source localization. To demonstrate the success of the proposed estimation scheme, we purposely set the stationary source at the origin and let each of the five mobile agents labeled $1, 2, \ldots, 5$ take an uniform circular motion around the stationary source labeled 0 along the counterclockwise direction. Both the radius of the circles and the moving speeds are 1, 2, 3, 4, 5 for agent $1, 2, \ldots, 5$, respectively. The setup is illustrated in Fig. 4. The sensing graph of the five agents together with the source is depicted in Fig. 5. That is, agent 1, 2 and 3 have direct range measurements about the source while agent 4 and agent 5 do not and they can only have indirect estimate through their neighbors. It can be checked that for the sensing graph there is a path from every agent to the source. Moreover, it can be

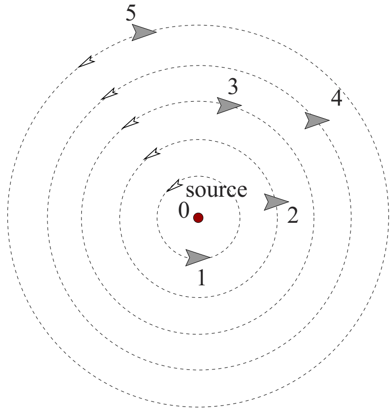

Fig. 4: An example of cooperative localization.

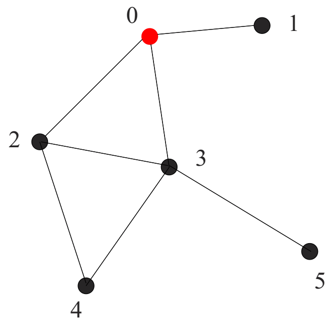

Fig. 5: The sensing graph.

verified that for each agent, (8) and (21) hold. We adopt the estimator described in (12), (20) and (23) to estimate the coordinate of the source for every agent. Then, by Theorem 3, every agent has its estimate $z_i(t)$ converging to relative coordinate of the source in its local frame. We choose $t = 40s$. The evolution curves of the estimation errors $\|z_i(t) - x_{i0}\|$ ($i = 1, \ldots, 5$) are shown in Fig. 6, which also validates our theoretic results. To see the estimates more visually, we plot two trajectories of the estimates in the plane, namely, $-z_2(t)$ and $-z_5(t)$, (see Fig. 7).

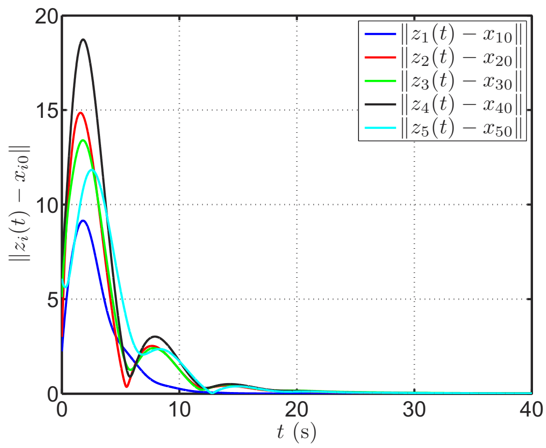

Fig. 6: The evolution of $\|z_i(t) - x_{i0}\|$, $i = 1, 2, \ldots, 5$.

In addition, we carry out a simulation for a scenario where some agents lose the range measurements about the source during the movement. In the simulation, agent 3 loses its range measurement during the time interval $[20, 25]$ temporarily due to measurement failures. The evolution of the

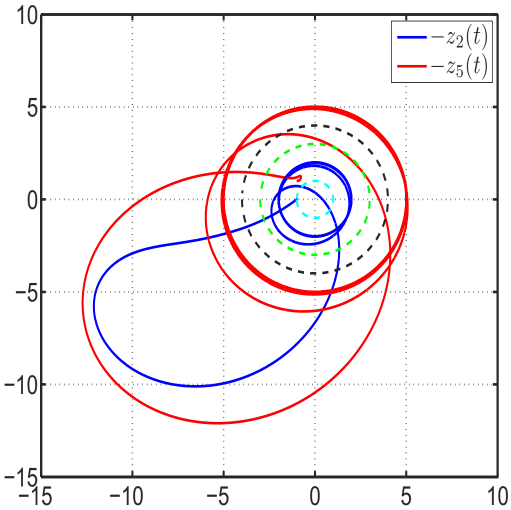

Fig. 7: The estimates $-z_2(t)$ and $-z_5(t)$ in the plane.

estimation errors for all the agents is shown in Fig. 8. It can be observed from Fig. 8 that though agent 3 loses its range measurement during the time interval $[20, 25]$, the estimate about the source does not become worse at all (compared to Fig. 6) because of the contribution from its collaborative neighbors. However, when agent 3 recovers its measurement about the source starting at $t = 25s$ and its own direct estimate based the measured information using (12), due to the initial estimation error of the estimator (12), the estimates of itself and also its neighbor $2, 4, 5$ will be affected, which can be noticed by comparing Fig. 8 and Fig. 6 at the interval after $t = 25s$. However, the estimation errors will converge to zero again.

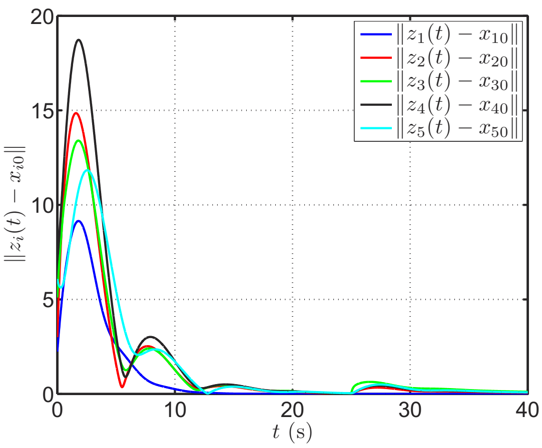

Fig. 8: The estimation errors when agent 3 loses the range measurement about the source during $t \in [20,\ 25]$.

6 Conclusion

The paper studies the localization problem for a stationary source based on only the range measurement information. First, an estimator is developed to estimate the relative coordinate of a stationary or moving object in its local frame with the help of a linear time-varying differentiator. Second, a consensus-like fusion scheme is proposed for cooperative localization by fusing the estimation from the neighboring agents. The proposed estimation scheme only requires the exchange of agents' velocities and also their estimates, yet the estimate of every agent is globally asymptotically convergent as long as there is a path from every agent to the source in the sensing graph. There are quite a few interesting research problems arising from the setup introduced in this paper such as robustness to measurement noises, communication delays, etc.

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