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An ADMM + Consensus Based Distributed Algorithm for Dynamic Economic Power Dispatch in Smart Grid

Hao Xing[1,4], Zhiyun Lin[2,4], Minyue Fu[1,3,4]

1. Department of Control Science and Engineering, Zhejiang University, 38 Zheda Road, Hangzhou, 310027 P. R. China
E-mail: haoxing@zju.edu.cn

2. College of Electrical Engineering, Zhejiang University, 38 Zheda Road, Hangzhou, 310027 P. R. China
E-mail: linz@zju.edu.cn

3. School of Electrical Engineering and Computer Science, University of Newcastle, NSW 2308 Australia
E-mail: minyue.fu@newcastle.edu.au

4. State Key Laboratory of Industrial Control Technology, 38 Zheda Road, Hangzhou, 310027 P. R. China

Abstract: In this paper, we propose a distributed algorithm for the dynamic economic dispatch problem (DEDP) in a smart grid scenario. Different from the static economic dispatch problem (SEDP), the DEDP aims at minimizing the aggregate operating costs of a group of generators over a time period with ramp rate constraints. The proposed algorithm is based on the average consensus algorithm on undirected graphs and the alternating direction method of multipliers (ADMM). Our algorithm is distributed in the sense that no leader or master nodes are needed, while all the nodes (generators) conduct local computation and merely communicate with their neighbors. Convergence analysis shows that the proposed algorithm converges to the optimal solution.

Key Words: Economic Dispatch, Dynamic Economic Dispatch, Distributed Convex Optimization, Smart Grid

1 Introduction

The economic dispatch problem (EDP) has occupied an important position in electric power industry and has been intensively investigated in the power systems community [1]. Research on EDP is becoming more meaningful due to the declining petroleum resources. The traditional EDP mainly concerns the economic dispatch of fossil-fired power generation systems to achieve a minimum operational cost within their capacity limits at a single time instant, which we refer to as static economic dispatch problem (SEDP). However, in practice the generators are faced with time-varying power demand, forcing the generators to coordinate their generation output accordingly. The EDP with time-varying demand is termed as the dynamic economic dispatch problem (DEDP), but the main difference between DEDP and SEDP is that DEDP takes ramp rate constraints into consideration [2]. The ramp rate constraints are considered in order to prolong the generators' lifetime, making the generation output at one instant coupled with the outputs at the instants before and after it. The coupling due to the ramp rate constraints makes DEDP much more difficult to solve than SEDP.

The dynamic economic power dispatch problem is usually formulated as an optimization problem, for which many centralized algorithms have been proposed. In [3], a robust heuristic method is proposed to find a feasible and suboptimal solution, while an efficient algorithm for the optimal solution, which is based on the idea of adaptive look-ahead, is presented as well. Reference [4] uses hybrid genetic approach to solve DEDP with ramp rate constraints, where the hybrid genetic algorithm (HGA) is used for a base level search, and then a local search method (gradient search) is used to do the final searching. The authors of [5] propose an algorithm based on evolutionary programming (EP) and sequential quadratic programming (SQP) to solve DEDP. Similar to [4], [5] also designs a double level search scheme, in which EP is adopted as a base level search to estimate the direction of the optimal region, while SQP is applied to determine the optimal solution on a final level.

Future smart grid, which will likely incorporate numerous distributed generation systems, is a typical large scale system [6]. The widely spatial distribution of power generation systems adds extra difficulties in solving EDP. Fortunately, for large scale systems, many decentralized algorithms for control, estimation, and optimization have been proposed [7]. Compared with centralized algorithms, decentralized algorithms exhibit the benefits including the feasibility in large scale systems, the reinforced robustness, and the evenly dispatched computation and communication burdens.

The following references are recent works on decentralized algorithms for SEDP in smart grid. In [8] and [9], the authors propose an incremental cost consensus (ICC) algorithm to solve SEDP, where an average consensus algorithm is used to guarantee the balance between demand and supply. In [10], a decentralized algorithm based on the ratio consensus is developed, which requires that the nodes have enough storage capacities for other nodes' parameters. In [11], the authors propose a consensus based decentralized algorithm, which enables the generators to collectively learn the mismatch between demand and total supply for feedback. Two fully distributed algorithms for SEDP are also proposed in our previous works [12] and [13], respectively. The algorithm proposed in [12] deals with SEDP with quadratic cost functions on connected undirected graphs, and then is extended to deal with SEDP with general convex functions on strongly connected directed graphs in [13]. But to the author's knowledge, no distributed/decentralied algorithms have been proposed for DEDP yet.

In this paper, we propose a distributed algorithm to find the optimal solution of DEDP with ramp rate constraints. With other constraints including spinning reserve, transmis-

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sion losses, and line capacity constraints considered, we formulate the DEDP into a convex optimization problem. Our algorithm is based on the alternating direction method of multipliers (ADMM) and the average consensus algorithm on connected undirected graphs. The proposed algorithm is fully distributed in the sense that it does not rely on any leader or master nodes, and the generators conduct local computation and merely communicate with their neighbors to iteratively find the optimal solution.

2 Preliminaries and Problem Formulation

In this section, we present some basics on graph theory and the average consensus algorithm, then introduce ADMM, and finally give the problem formulation for DEDP.

2.1 Graph Theory

An undirected graph $G = (V, E)$ consists of a non-empty finite set of nodes $V = \{1, 2, \ldots, n\}$ and a finite set of unordered pairs of nodes $E \subseteq V \times V$. For node $i \in V$, its neighbor set is denoted by $N_i = \{j \in V - \{i\} : (j, i) \in E\}$, i.e., node i can bidirectionally communicate with its neighbors. The degree of node i is the cardinality of N_i, denoted by $d_i = |N_i|$. For all $i \in V$ and $j \in N_i$, $(i, j) \in E$ implies $(j, i) \in E$. An undirected graph is connected if there is a path from any node to any other node. It is reasonable to assume that each node can communicate with itself, i.e., $\forall i \in V,\ (i, i) \in E$. We also say that a non-negative matrix $Q \in \mathbb{R}^{n\times n}$ is associated with graph G, where $[Q]_{ij} > 0$ if and only if $(j, i) \in E$.

2.2 Average Consensus

Let us consider the undirected graph $G = (V, E)$, where $V = \{1, \ldots, n\}$. Each node $i \in V$ holds a state denoted by $x \in \mathbb{R}$. Denote by $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ the aggregate state. Define a weight matrix $Q \in \mathbb{R}^{n\times n}$ associated with graph G as

$$q_{ij} = \begin{cases} \dfrac{1}{\max(d_i, d_j) + 1}, & \text{if } j \in N_i, \\ 1 - \sum\limits_{j\in N_i} q_{ij}, & \text{if } i = j, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where q_{ij} is the entry of Q on the ith row and the jth column.

With the iteration index denoted by $\kappa = 0, 1, \ldots$, the average consensus algorithm is given by

$$x(\kappa + 1) = Qx(\kappa), \tag{2}$$

where $x(0)$ is the initial aggregate state at $\kappa = 0$. Rewrite iteration (2) in the following distributed fashion,

$$x_i(\kappa + 1) = q_{ii}x_i(\kappa) + \sum_{j\in N_i} q_{ij}x_j(\kappa),\ \forall i = 1, \ldots, n. \tag{3}$$

It is known that algorithm (2) solves the *average consensus problem* asymptotically [14], i.e., for any initial states $x_i(0)$'s, it follows

$$\lim_{\kappa\to\infty} x_i(\kappa) = \left(\sum_{j=1}^{n} x_j(0)\right)/n,\ \forall i = 1, \ldots, n.$$

It is clear that the iterations in (3) use only local information, thus this algorithm can be implemented in a fully distributed fashion [14].

2.3 Alternating Direction Method of Multipliers

Let us consider the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & f(x) + g(y), \\ \text{subject to} \quad & Ax + By = c, \end{aligned} \tag{4}$$

with optimization variables $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, where $A \in \mathbb{R}^{p\times n}$, $B \in \mathbb{R}^{p\times m}$, and $c \in \mathbb{R}^p$.

We make the following assumptions for the problem (4):

Assumption 1 *The functions* $f(x) : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ *and* $g(y) : \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ *are proper, closed and convex.*

Assumption 2 *The matrices* A *and* B *have full column ranks.*

The augmented Lagrangian of problem (4) is

$$\begin{aligned} L_\rho(x, y, \lambda) =& f(x) + g(y) + \lambda^T(Ax + By - c) \\ &+ \rho/2\|Ax + By - c\|_2^2, \end{aligned} \tag{5}$$

where $\lambda \in \mathbb{R}^p$ is the Lagrange multiplier, and $\rho > 0$ is a penalty parameter. The alternating direction method of multipliers for problem (4) is given as follows [15]:

$$x^{k+1} = \operatorname*{argmin}_x L_\rho(x, y^k, \lambda^k); \tag{6}$$

$$y^{k+1} = \operatorname*{argmin}_y L_\rho(x^{k+1}, y, \lambda^k); \tag{7}$$

$$\lambda^{k+1} = \lambda^k + \rho(Ax^{k+1} + By^{k+1} - c). \tag{8}$$

Define

$$\epsilon^k = Ax^k + By^k - c,\ \varepsilon^k = \rho A^T B(y^{k+1} - y^k),$$

as the primal residual and the dual residual at time k, respectively, both converging to zero as the algorithm converges. The convergence properties of ADMM are given below.

Lemma 1 *[15] If the assumptions 1 and 2 and* $\rho > 0$ *hold, then the ADMM iterations* (6)-(8) *converge to the optimal solution* x^*, y^**, and the optimal Lagrange multiplier* λ^* *of problem* (4)*, with*

$$\lim_{k\to\infty} \|\epsilon^k\|_2 = 0, \text{ and } \lim_{k\to\infty} \|\varepsilon^k\|_2 = 0.$$

Remark 1 *The convergence is guaranteed for any* $\rho > 0$ *under Assumptions 1-2. The impact of* ρ *on the convergence rate follows that a larger* ρ *is inclined to make the primal residual smaller, while on the other hand the penalty on the dual residual is weakened, and vice versa [15].*

2.4 Problem Formulation

The DEDP aims at minimizing the total operational costs of n generators over a given time period, on the premise that, subject to each generator's generation capacity, they cooperatively generate power to meet a time-varying power demand curve. We consider only active power and ignore transmission losses and line capacity constraints in this paper.

We assume that the operational cost of the ith generator is a quadratic function with regard to its power output $P_i(t)$ at time t, given by

$$C_i(P_i(t)) = \alpha_i P_i^2(t) + \beta_i P_i(t) + \gamma_i, \tag{9}$$

where $\alpha_i > 0$, β_i, and γ_i are the cost parameters which can be estimated online [1]. Define $P_i = [P_i(1), \ldots, P_i(T)]^T \in \mathbb{R}^T$. The aggregate operational costs over the time period $t = 1, \ldots, T$ follows

$$C^{total}(P) = \sum_{t=1}^{T} \sum_{i=1}^{n} C_i(P_i(t)), \tag{10}$$

where $P = [P_1^T, \ldots, P_n^T]^T \in \mathbb{R}^{nT}$ is the aggregate variable. The balance between demand and supply yields the equality constraints:

$$\sum_{i=1}^{n} P_i(t) = P_D(t), \ \forall t = 1, \ldots, T, \tag{11}$$

where $P_D(t)$ is the time-varying power demand.
The generation outputs shall not exceed each generator's capacity, which follows

$$\underline{P}_i \leq P_i(t) \leq \bar{P}_i, \ \forall i, \ t, \tag{12}$$

where $\underline{P}_i$ and $\bar{P}_i$ are the lower and upper bound of the ith generator's capacity, respectively.

Since we deal with the time-varying power demand, we also have ramp rate constraints for each generator, given by:

$$\Delta\underline{P}_i \leq P_i(t+1) - P_i(t) \leq \Delta\bar{P}_i, \ \forall t = 1, \ldots, T-1, \tag{13}$$

where $\Delta\underline{P}_i$ and $\Delta\bar{P}_i$ are the lower and upper bounds of the ramp rate of the ith generator.

With other factors (e.g., line capacity, transmission losses) ignored, the dynamic economic dispatch problem can be formulated by (9)-(13).

3 Distributed Algorithm for DEDP

In this section, we present our distributed algorithm for DEDP based on the average consensus algorithm on undirected graphs and ADMM. Our algorithm can be treated as a distributed implementation of ADMM applied to DEDP.

Define two convex sets:

$$\begin{aligned}
\mathcal{S}_1 &= \{P \in \mathbb{R}^{nT} | \sum_{i=1}^{n} P_i(t) = P_D(t), \ \forall 1 \leq t \leq T\}, \\
\mathcal{S}_2 &= \{P \in \mathbb{R}^{nT} | \underline{P}_i \leq P_i(t) \leq \bar{P}_i, \ \forall i, \ 1 \leq t \leq T, \\
&\quad \Delta\underline{P}_i \leq P_i(t+1) - P_i(t) \leq \Delta\bar{P}_i, \ \forall 1 \leq t \leq T-1\}.
\end{aligned}$$

The DEDP formulated as (9)-(13) could be equivalently rewritten as follows:

$$\begin{aligned}
&\text{minimize} \quad C^{total}(P) + g_1(P) + g_2(Q), && (14) \\
&\text{subject to} \quad P - Q = 0, && (15)
\end{aligned}$$

where $Q \in \mathbb{R}^{nT}$, g_1 and g_2 are the indicator functions for the set $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively, i.e.,

$$g_1(P) = \begin{cases} 0, & \text{if } P \in \mathcal{S}_1, \\ +\infty, & \text{otherwise}, \end{cases}$$

$$g_2(Q) = \begin{cases} 0, & \text{if } Q \in \mathcal{S}_2, \\ +\infty, & \text{otherwise} \end{cases}.$$

Note that an indicator function of a convex set is proper, closed, and convex, therefore the functions $C^{total}(P) + g_1(P)$ and $g_2(Q)$ satisfy assumption 1. Also, in this case we have $A = I$ and $B = -I$, where $I \in \mathbb{R}^{nT \times nT}$ is the identify function, so assumption 2 is satisfied as well.

Applying the ADMM to the DEDP yields

$$\begin{aligned}
P^{k+1} &= \operatorname*{argmin}_P L_\rho(P, Q^k, \lambda^k); && (16) \\
Q^{k+1} &= \operatorname*{argmin}_Q L_\rho(P^{k+1}, Q, \lambda^k); && (17) \\
\lambda^{k+1} &= \lambda^k + \rho(P^{k+1} - Q^{k+1}), && (18)
\end{aligned}$$

where $L_\rho(P, Q, \lambda)$ is the augmented Lagrangian for the D-EDP, given by

$$\begin{aligned}
L_\rho(P, Q, \lambda) = &C^{total}(P) + g_1(P) + g_2(Q) \\
&+ \lambda^T(P - Q) + \rho/2\|P - Q\|_2^2.
\end{aligned} \tag{19}$$

However, the iterations given by (16)-(18) are in a centralized fashion. We next present a distributed implementation.

3.1 P-update

Combining (16) and (19), we have

$$\begin{aligned}
P^{k+1} &= \operatorname*{argmin}_P L_\rho(P, Q^k, \lambda^k) \\
&= \operatorname*{argmin}_P (C^{total}(P) + g_1(P) \\
&\quad + (\lambda^k)^T(P - Q^k) + (\rho/2)\|P - Q^k\|_2^2) \\
&= \operatorname*{argmin}_{P \in \mathcal{S}_1} (C^{total}(P) + (\rho/2)\|P - Q^k + u^k\|_2^2) \\
&= \operatorname*{argmin}_{P \in \mathcal{S}_1} (\sum_{t=1}^{T} \sum_{i=1}^{n} (\alpha_i P_i^2(t) + \beta_i P_i(t) + \gamma_i) + \\
&\quad + (\rho/2)(\sum_{t=1}^{T} \sum_{i=1}^{n} P_i^2(t) + 2(-Q^k + u^k)P_i(t) \\
&\quad + (-Q^k + u^k)^2)),
\end{aligned} \tag{20}$$

where $u^k = \lambda^k/\rho$. Then the equivalent problem of (20) is given by:

$$\begin{aligned}
&\text{minimize} \quad \sum_{t=1}^{T} \sum_{i=1}^{n} (\alpha_i' P_i^2(t) + \beta_i' P_i(t) + \gamma_i') \\
&\text{subject to} \quad \sum_{i=1}^{n} P_i(t) = P_D(t), \ \forall t = 1, \ldots, T,
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\alpha_i' &= \alpha_i + \rho/2, \\
\beta_i' &= \beta_i + \rho(-Q_i^k(t) + u_i^k(t)), \\
\gamma_i' &= \gamma_i + (\rho/2)(-Q_i^k(t) + u_i^k(t))^2.
\end{aligned}$$

Note that the optimization problem (21) can be readily decomposed into T subproblems, given by for $t = 1, \ldots, T$,

$$\begin{aligned}
&\text{minimize} \quad \sum_{i=1}^{n} (\alpha_i' P_i^2(t) + \beta_i' P_i(t) + \gamma_i') \\
&\text{subject to} \quad \sum_{i=1}^{n} P_i(t) = P_D(t).
\end{aligned} \tag{22}$$

The optimization problem (22) can be regarded as a unconstrained resource allocation problem [16], of which the optimal condition is given by

$$\sum_{i=1}^{n} P_i^*(t) = P_D(t),\ 2\alpha_i^{'} P_i^*(t) + \beta_i^{'} = \nu^*(t), \forall i, \quad (23)$$

where $P_i^*(t)$ and $\nu^*(t)$ is the optimal solution and the optimal Lagrange multiplier of the tth subproblem. From (23), we further have

$$P_i^*(t) = \frac{\nu^*(t) - \beta_i^{'}}{2\alpha_i^{'}}, \quad (24)$$

which yields

$$\nu^*(t)\sum_{i=1}^{n} 1/2\alpha_i^{'} - \sum_{i=1}^{n} \beta_i^{'}/2\alpha_i^{'} = \sum_{i=1}^{n} P_i^*(t) = P_D(t), \quad (25)$$

$$\nu^*(t) = \frac{P_D(t) + \sum_{i=1}^{n} \beta_i^{'}/2\alpha_i^{'}}{\sum_{i=1}^{n} 1/2\alpha_i^{'}} = \frac{\frac{P_D(t)}{n} + \frac{\sum_{i=1}^{n} \beta_i^{'}/2\alpha_i^{'}}{n}}{\frac{\sum_{i=1}^{n} 1/2\alpha_i^{'}}{n}}.$$

Therefore the optimal Lagrange multiplier is readily derived if we can get $\frac{P_D(t)}{n}$, $\frac{\sum_{i=1}^{n} \beta_i^{'}/2\alpha_i^{'}}{n}$, and $\frac{\sum_{i=1}^{n} 1/2\alpha_i^{'}}{n}$ in a distributed fashion.

We next introduce a distributed demand information gathering technique which is proposed in our previous work [12].

Since the loads are spatially distributed over the grid, the demand information has to be collected in a distributed manner. Suppose there are in total m buses associated with load, generation, or both. Due to the fact that the buses associated with loads tend to be much more than the buses associated with pure generation, we construct two undirected communication networks $G_m = (V_m, E_m)$ and $G_n = (V_n, E_n)$, which involve load & generation buses and pure generation buses, respectively. We hope that after the demand information is collected, the rest of the distributed algorithm only involves pure generation buses. To this end, we design the following procedures.

In graph $G_m = (V_m, E_m)$, define an associated doubly stochastic matrix $Q \in \mathbb{R}^{m\times m}$ using (1) .

where the subscript m indicates that the parameters are defined with regard to graph G_m.

For every node $i \in V_m$, we establish two variables $p_i(\kappa)$ and $s_i(\kappa)$, respectively initialized by $p_i(0) = P_{D,i}(t)$, and

$$s_i(0) = \begin{cases} 1, & i = 1, 2, ..., n, \\ 0, & i = n+1, n+2, ...m, \end{cases}$$

where $P_{D,i}(t)$ is defined as the power demand of the ith bus at time t. If the ith bus is associated with pure generation, we set $P_{D,i}(t) = 0$. We then run the following average consensus algorithms simultaneously until convergence:

$$p_i(\kappa+1) = q_{ii}p_i(\kappa) + \sum_{j\in N_{m,i}} q_{ij}p_j(\kappa), \quad (26)$$

$$s_i(\kappa+1) = q_{ii}s_i(\kappa) + \sum_{j\in N_{m,i}} q_{ij}s_j(\kappa). \quad (27)$$

Defining $p^* = \lim_{\kappa\to\infty} p_i(\kappa)$ and $s^* = \lim_{\kappa\to\infty} s_i(\kappa)$, we have $p^* = P_D(t)/m,\ s^* = n/m$. For every node $i \in V_n$, we have: $P_D(t)/n = p^*/s^*$. To get $\frac{1}{n}\sum_{i=1}^{n} \beta_i^{'}/2\alpha_i^{'}$ and $\frac{1}{n}\sum_{i=1}^{n} 1/2\alpha_i^{'}$, each node in graph G_n establishes two variables $v_i(\kappa)$ and $w_i(\kappa)$, respectively, initialized by

$$v_i(0) = \beta_i^{'}/2\alpha_i^{'},\ w_i(0) = 1/2\alpha_i^{'}.$$

Then run the following average consensus algorithm till convergence:

$$v_i(\kappa+1) = q_{ii}v_i(\kappa) + \sum_{j\in N_{m,i}} q_{ij}v_j(\kappa), \quad (28)$$

$$w_i(\kappa+1) = q_{ii}w_i(\kappa) + \sum_{j\in N_{m,i}} q_{ij}w_j(\kappa). \quad (29)$$

We then have the convergence values

$$v^* = v_i(\infty) = \frac{\sum_{i=1}^{n} \beta_i^{'}/2\alpha_i^{'}}{n},\ w^* = w_i(\infty) = \frac{\sum_{i=1}^{n} 1/2\alpha_i^{'}}{n}.$$

Then the optimal Lagrange multiplier $\nu^*(t)$ is given by:

$$\nu^*(t) = \frac{p^*/s^* + v^*}{w^*}. \quad (30)$$

Combining (30) and (24) yields the optimal solution, and in this way P^{k+1} is computed in a distributed fashion.

3.2 Q-update

Combining (17) and (19), we have

$$\begin{aligned} Q^{k+1} =& \operatorname*{argmin}_{Q} L_\rho(P^{k+1}, Q, \lambda^k) \\ =& \operatorname*{argmin}_{Q} (g_2(Q) + (\rho/2)\|P^{k+1} - Q + u^k\|_2^2) \\ =& \operatorname*{argmin}_{Q\in\mathcal{S}_2} (\rho/2)\|P^{k+1} - Q + u^k\|_2^2 \\ =& \mathcal{P}_{\mathcal{S}_2}(P^{k+1} + u^k), \end{aligned} \quad (31)$$

where $\mathcal{P}_{\mathcal{S}_2}$ is the projection operator onto the set $\mathcal{S}_2$.

Note that the constraints for $\mathcal{S}_2$ only couples the variables temporally. So the Q-update merely relies on local computation. The projection onto a convex set is a convex optimization problem, which can be solved using convex optimization methods, e.g., interior point method [17]. Here we present an iterative algorithm based on the ADMM, which adopts the idea of parallel projection [18].

At node i, denote $x_i = [x_i(1), x_i(2), \ldots, x_i(T)]^T \in \mathbb{R}^T$ and $X = [x_1^T, x_2^T, x_3^T]^T \in \mathbb{R}^{3T}$. We then define 4 convex sets as follows:

$$\mathcal{S}^1 = \{x \in \mathbb{R}^T | \underline{P}_i \le x_i(t) \le \bar{P}_i,\ \forall\, t\},$$

$$\mathcal{S}^2 = \{x \in \mathbb{R}^T | \Delta\underline{P}_i \le x_i(t+1) - x_i(t) \le \Delta\bar{P}_i, \\ \forall t = 1, 3, 5, \ldots\},$$

$$\mathcal{S}^3 = \{x \in \mathbb{R}^T | \Delta\underline{P}_i \le x_i(t+1) - x_i(t) \le \Delta\bar{P}_i, \\ \forall t = 2, 4, 6, \ldots\},$$

$$\mathcal{S}^4 = \mathcal{S}^1 \times \mathcal{S}^2 \times \mathcal{S}^3,$$

$$\mathcal{S}^5 = \{X \in \mathbb{R}^{3T} | x_1 = x_2 = x_3\}.$$

One can easily verify that $\mathcal{S}_2 = \mathcal{S}^1 \cap \mathcal{S}^2 \cap \mathcal{S}^3$. Define $r = P_i^{k+1} + u_i^k$ and $R = [r^T, r^T, r^T]^T$, and then solving the projection (31) is equivalent to solving the following optimization problem for each node (here we omit the subscript i),

$$\begin{aligned} \text{minimize} \quad & \|X-R\|_2^2 + h_a(X) + h_b(Y), \\ \text{subject to} \quad & X - Y = 0, \end{aligned} \tag{32}$$

where $Y \in \mathbb{R}^{3T}$, and h_a, h_b are the indicator functions of the set $\mathcal{S}^4$ and $\mathcal{S}^5$, respectively.

The augmented Lagrangian for problem (32) is

$$\begin{aligned} L_\rho(X, Y, \lambda) = &\|X-R\|_2^2 + h_a(X) \\ &+ h_b(Y) + (\rho/2)\|X - Y + u\|_2^2. \end{aligned}$$

We then apply the ADMM to problem (32) to obtain

$$\begin{aligned} X^{k+1} &= \operatorname*{argmin}_X L_\rho(P, Y^k, u^k); \\ &= \operatorname*{argmin}_X (\|X-R\|_2^2 + h_a(X) \\ &\quad +(\rho/2)\|X - Y^k + u^k\|_2^2), \\ &= \operatorname*{argmin}_{X\in\mathcal{S}^4} (\|X-R\|_2^2 + (\rho/2)\|X - Y^k + u^k\|_2^2) \\ &= \operatorname*{argmin}_{X\in\mathcal{S}^4} (X - \frac{R+Y^k-u^k}{\rho/2+1}) \\ &= \mathcal{P}_{\mathcal{S}^4}(\frac{R+Y^k-u^k}{\rho/2+1}), \end{aligned} \tag{33}$$

$$\begin{aligned} Y^{k+1} &= \operatorname*{argmin}_Y L_\rho(X^{K+1}, Y, u^k) \\ &= \operatorname*{argmin}_Y (h_b(Y) + (\rho/2)\|X^{k+1} - Y + u^k\|_2^2) \\ &= \operatorname*{argmin}_{Y\in\mathcal{S}^5} (\|X^{k+1} - Y + u^k\|_2^2) \\ &= \mathcal{P}_{\mathcal{S}^5}(X^{k+1} + u^k), \end{aligned} \tag{34}$$

$$u^{k+1} = u^k + X^{k+1} - Y^{k+1}. \tag{35}$$

where $\mathcal{P}_{\mathcal{S}^4}$ and $\mathcal{P}_{\mathcal{S}^5}$ are the projection operators onto the set $\mathcal{S}^4$ and $\mathcal{S}^5$, respectively.

The expressions of X and Y updates are given below.

X-update: Denote $Z = \frac{R+Y^k-u^k}{\rho/2+1}$. Since $\mathcal{S}^4 = \mathcal{S}^1 \times \mathcal{S}^2 \times \mathcal{S}^3$, we have

$$\mathcal{P}_{\mathcal{S}^4}(Z) = [(\mathcal{P}_{\mathcal{S}^1}(Z_1))^T, (\mathcal{P}_{\mathcal{S}^2}(Z_2))^T, (\mathcal{P}_{\mathcal{S}^3}(Z_3))^T]^T.$$

Denote $Z_1' = \mathcal{P}_{\mathcal{S}^1}(Z_1)$. We have $Z_1' = [Z_1'(1), \ldots, Z_1'(T)]^T$, where

$$Z_1'(t) = \begin{cases} \underline{P}_i, & \text{for } Z_1(t) \le \underline{P}_i, \\ Z_i(t), & \text{for } \underline{P}_i < Z_i(t) \le \bar{P}_i, \\ \bar{P}_i, & \text{for } \bar{P}_i < Z_i(t). \end{cases}$$

For the projection operator $\mathcal{P}_{\mathcal{S}^2}(Z_2)$, note that the constraints for the set $\mathcal{S}_2$ only couple the variables at $2i-1$ and $2i$, for all $i = 1, 2, \ldots,$while the variables at $2i$ and $2i+1$ are independent. This insight encourages us to further decompose the projection operator $\mathcal{P}_{\mathcal{S}^2}(Z_2)$. Denote $Z_2' = \mathcal{P}_{\mathcal{S}^2}(Z_2)$ and $\Delta Z_2(2i) = Z_2(2i) - Z_2(2i-1)$. We have $Z_2' = [Z_2'(1), \ldots, Z_2'(T)]^T$, where

$$Z_2'(2i-1) = \begin{cases} (1/2)(Z_2(2i) + Z_2(2i-1) - \Delta\underline{P}_i), \\ \qquad \text{for } \Delta Z_2(2i) < \Delta\underline{P}_i, \\ Z_2(2i-1), \text{ for } \Delta\underline{P}_i < \Delta Z_2(2i) \le \Delta\bar{P}_i, \\ (1/2)(Z_2(2i) + Z_2(2i-1) - \Delta\bar{P}_i), \\ \qquad \text{for } \Delta\bar{P}_i < \Delta Z_2(2i), \end{cases}$$

$$Z_2'(2i) = \begin{cases} (1/2)(Z_2(2i) + Z_2(2i-1) + \Delta\underline{P}_i), \\ \qquad \text{for } \Delta Z_2(2i) < \Delta\underline{P}_i, \\ Z_2(2i), \text{ for } \Delta\underline{P}_i < \Delta Z_2(2i) \le \Delta\bar{P}_i, \\ (1/2)(Z_2(2i) + Z_2(2i-1) + \Delta\bar{P}_i), \\ \qquad \text{for } \Delta\bar{P}_i < \Delta Z_2(2i). \end{cases}$$

Similarly, for the projection operator $\mathcal{P}_{\mathcal{S}^3}(Z_3)$, note that the constraints for the set $\mathcal{S}_3$ only couple the variables at $2i$ and $2i+1$, for all $i = 1, 2, \ldots,$while the variables at $2i$ and $2i-1$ are independent.

Applying the same technique for $\mathcal{P}_{\mathcal{S}^3}(Z_3)$ yields: $Z_3' = [Z_3'(1), \ldots, Z_3'(T)]^T$, where

$$Z_3'(2i-1) = \begin{cases} (1/2)(Z_3(2i) + Z_3(2i-1) - \Delta\underline{P}_i), \\ \qquad \text{for } \Delta Z_3(2i) < \Delta\underline{P}_i, \\ Z_3(2i-1), \text{ for } \Delta\underline{P}_i < \Delta Z_3(2i) \le \Delta\bar{P}_i, \\ (1/2)(Z_3(2i) + Z_3(2i-1) - \Delta\bar{P}_i), \\ \qquad \text{for } \Delta\bar{P}_i < \Delta Z_3(2i), \end{cases}$$

$$Z_3'(2i) = \begin{cases} (1/2)(Z_3(2i) + Z_3(2i-1) + \Delta\underline{P}_i), \\ \qquad \text{for } \Delta Z_3(2i) < \Delta\underline{P}_i, \\ Z_3(2i), \text{ for } \Delta\underline{P}_i < \Delta Z_3(2i) \le \Delta\bar{P}_i, \\ (1/2)(Z_3(2i) + Z_3(2i-1) + \Delta\bar{P}_i), \\ \qquad \text{for } \Delta\bar{P}_i < \Delta Z_3(2i). \end{cases}$$

Y-update: For the projection operator $\mathcal{P}_{\mathcal{S}^5}$, denote $U = X^{k+1} + u^k$ and $U' = \mathcal{P}_{\mathcal{S}^5}(X^{k+1} + u^k)$. We have

$$U' = [(U_1')^T, (U_2')^T, (U_3')^T]^T, \; U_1', U_2', \text{ and } U_3' \in \mathbb{R}^T,$$

where $U_1' = U_2' = U_3' = \frac{\sum_{i=1}^3 Z_i'}{3}$. Run the ADMM iterations (34)-(35) until convergence to get Q^{k+1}.

3.3 Overall Sketch of the Algorithm

Running the above iterations till convergence, each generator will get the optimal power assignments $P_i^\star(t)$'s. For clarity, we summarize the above procedures in the following algorithm.

Algorithm 1 Distributed Algorithm for DEDP

Require: $P_{D,i}(t), \forall i \in V_n, \; t$;
Ensure: $P_i^\star(t), \forall i \in V_n, \; t$;

```
1: Gathering demand information using (26)-(30);
2: for k = 0, 1, 2, ... do
3:     Each node performs P-update using (24);
4:     Each node performs Q-update using (33)-(35);
5:     Each node performs λ-update using (18) ;
6: end for
```

Table 1: Generator Parameters

Generator	Bus	α_i	β_i	$\bar{P}_i$	ΔP_i
1	1	0.04	2	80	10
2	2	0.03	3	90	20
3	3	0.035	4	70	10
4	6	0.03	4	70	15
5	8	0.04	2.5	80	10

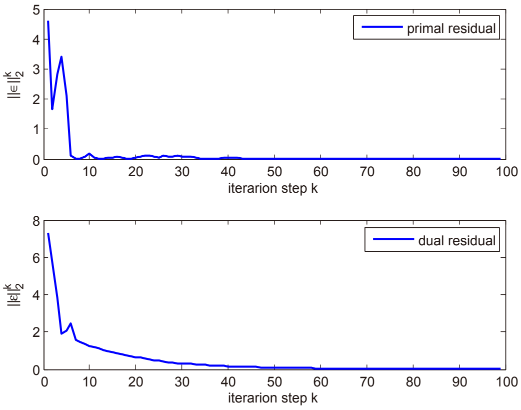

Fig. 1: The convergence result of the proposed algorithm.

4 Simulation

In this section we present a numerical case that is performed on the IEEE 14-bus system using the proposed algorithm. We replace the 3 synchronous condensers with 3 generators, so there are 5 generators in this system, whose parameters are adopted from [13], given in the following table. For all the generators, we set $\underline{P}_i = 10$ MW and $\Delta \underline{P}_i = -\Delta \bar{P}_i = -\Delta P_i$. We take $\rho = 1$, Q^0=0, and u^0=0. For the stopping criteria, we set

$$\|\epsilon^k\|_2 < 0.05, \text{ and } \|\varepsilon^k\|_2 < 0.05.$$

Note that the power transmission network is not necessarily identical with the communication graphs, so we do not assign node to bus 7. Define two corresponding node sets $V_m = \{1, 2, ..., 6, 8, 9, ..., 14\}$, $V_n = \{1, 2, 3, 6, 8\}$. For G_m and G_n, the edge sets E_m and E_n are properly chosen to set up two connected undirected graphs with self-loops.

In this case we consider a time period of 5 instants, and the total power demand at each time instant is given by: $P_D(1) = 380$ MW, $P_D(2) = 330$ MW, $P_D(3) = 270$ MW, $P_D(4) = 295$ MW, $P_D(5) = 340$ MW. The convergence of the proposed algorithm is demonstrated in Fig. 1, while the optimal power assignments are shown in Table 2. From Fig. 1, we can see that both $\|\epsilon^k\|_2$ and $\|\varepsilon^k\|_2$ converge to zero as the iteration proceeds. Note that the ramp rate constraints indeed play a part. Take the 1st generator for example, its optimal power assignments at $t = 2$ and $t = 3$ are 70.46 MW and 60.46MW, respectively, while its ramp rate is limited to be no larger than 10 MW.

Table 2: The optimal power assignments.

Generator \ Time	1	2	3	4	5
1	80.00	70.46	60.46	65.38	73.12
2	90.00	78.08	63.08	70.47	80.80
3	64.00	54.00	44.00	46.16	55.02
4	70.00	61.46	46.47	53.86	64.18
5	76.00	66.00	56.00	59.13	66.88
Total outputs	380.00	330.00	270.01	295.00	340.00

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