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A New Power-efficient Distributed Method for Clock Synchronization in Sensor Networks

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Abstract: This paper presents a new distributed power-efficient method consisting of clock synchronization achieving and maintaining protocols. The achieving protocol can realize synchronization of time variations and initial time errors independently and simultaneously with an energy-efficient communication scheme designed in the paper. Then a stopping criterion is proposed to degrade the protocol into a maintaining one which is synthesized under the general strongly connected digraph to achieve more energy conservation. Numerical simulations are given to illustrate the performance of the method.

Key Words: Distributed method, Power-efficient, Clock Synchronization

1 Introduction

Nowadays low-cost and low-power programmable sensors are capable of sensing, processing and storing data, and communicating via wireless channel. Wireless sensor networks (WSNs) are made up of such kind of agents, which are distributed and dedicated to observing and monitoring certain concerned phenomena of the physical world. In such networks, sensors may be static or embedded in the environment. Or they are enabled with mobility by being attached to physical artifacts.

WSNs are able to be applied in a wider range of cases only on the condition that local clocks of all sensors can achieve synchronization. However, successful clock synchronization protocols for wired networks are unsuitable for a wireless-sensor environment for three main reasons all concerned with power. First, WSNs have a wider deployment of sensors, which makes it impossible to provide a power source for each node in the vast network. Additionally, node failure and packet losses caused by low power may lead to the change of the communication topology. Furthermore, the energy conservation becomes a significant concern for the fact that the smaller-size sensors are almost battery-based with more difficulty in power storing and procuring.

1.1 Literature Review

Traditional protocols for wireless communication synchronize all local clocks to a reference time source according to an external standard. However, usually such kind external standard requires a high energy level unsuitable for nowadays WSNs due to above statements. As for the modern researches on clock synchronization for WSNs, protocols can be roughly classified into master-to-slave versus peer-to-peer synchronization patterns [1]. The latter pattern is more power-resilient than the former one, for the master node poses a high demand for CPU processing speed and resources, which is proportional to the number of slaves [2]. And once the power supply of that master was unstable, it would be difficult for slaves to maintain a common notion of time for the fact that they take the reading of master clock as the reference time. Among the peer-peer pattern, protocols can be divided into realizing internal or external synchronization. *Time-Diffusion Protocol* (TDP) [3] methods belong to the former kind, in which the local physical clocks have to be corrected. However, resetting or correcting each sensor's local clock to a global timescale consumes much more energy than just external synchronization does. *RBS* [4] is one recognized peer-peer protocol without physical clock correction, however, it increases message complexity for dividing the whole net into clusters and selecting reference nodes, which in return increases energy consumption.

1.2 Statement of Contribution

According to the statements and related work mentioned above, our paper attaches more weights on the following innovations as far as we know. For static WSNs, the paper presents two light-weight linear protocols to achieve and maintain the clock synchronization respectively without physical clock correction. This allows a tradeoff between the synchronization accuracy and energy consumption. One achieving protocol, instead of two coupled algorithms like [5], achieves the consensus of time variations and that of initial time errors simultaneously and independently. This ensures lower power consumption caused by message complexity. The maintaining protocol further decreases the number and complexity of messages needed transmitting and computing by degrading the communication topology required in the previous protocol to an unbalanced or even unidirectional one as long as it is strongly connected. Additionally, unlike other distributed ways [5, 6] only proving asymptotical consensus, this paper figures out how to establish a stopping criterion for the achieving protocol to conserve energy for data's transmission and computation. On the other hand, the achieving protocol's implementation adopts a specially designed reactive flooding communication scheme by using MAC-layer. In this way, a whole strategy for the process of achieving and maintaining clock synchronization is completed.

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2 Notation and Graph Theory

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is composed of a non-empty node set $\mathcal{V} = \{1, 2, \cdots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ where each directed edge (j, i) of $\mathcal{G}$ corresponding with a weight ξ_{ij} represents that node j can send messages to node i. The undirected graph is a special case of bidirectional graph, whose edge can also be viewed as a pair of un-ordered nodes. In unbalanced digraphs, we let $\mathcal{N}_i^+$ denote the in-neighbor set of node i, i.e., $\mathcal{N}_i^+ = \{j : (j, i) \in \mathcal{E}\}$. Similarly, $\mathcal{N}_i^-$ represents the out-neighbour set of node i, i.e., $\mathcal{N}_i^- = \{o : (i, o) \in \mathcal{E}\}$. While in undirected graphs, we just use $\mathcal{N}_i$ to denote the set of node i's neighbours. An undirected graph is called connected if any pair of two nodes can be connected by a path. A digraph is strongly connected if every two nodes are mutually reachable. Diameter $\mathfrak{D}$ of a graph is the longest path among the shortest ones between any pair of nodes.

For an undirected graph, the ijth entry of its Laplacian matrix L is defined as $L(i, j) = -\xi_{ij}, i \neq j, j \in \mathcal{N}_i$, $L(i, j) = 0, i \neq j, j \notin \mathcal{N}_i$ and $L(i, j) = \sum_{j\in\mathcal{N}_i} \xi_{ij}, i = j$. By replacing $\mathcal{N}_i$ with $\mathcal{N}_i^+$, we can also get the Laplacian matrix of a directed graph. It is significant to point out that we ensure $0 < \xi_{ij} < 1$ and $\sum_{j\in\mathcal{N}_i} \xi_{ij} < 1$ when choosing the weights for all Laplacian matrices throughout this paper.

Lemma 1 [7] Zero is an eigenvalue of the Laplacian matrix L with $\mathbf{1}$ as a corresponding right eigenvector and all nonzero eigenvalues have positive real parts. For an undirected graph, zero is a simple eigenvalue of L if and only if the graph is connected.

Definition 1 [8] Semi-simple eigenvalue is a matrix's eigenvalue whose algebraic multiplicity is equal to its geometric multiplicity.

Lemma 2 [8] Suppose an n-dimension matrix $A(\gamma)$ depends on a small real parameter $\gamma \geq 0$ smoothly. Let $\lambda_1 = \cdots = \lambda_h, h \in (1, 2, \cdots, n)$, be a semi-simple eigenvalue of $A(0)$, with right eigenvectors $p_1, \cdots, p_h$ and left eigenvectors $q_1, \cdots, q_h$ such that

$$(q_1 \cdots q_h)^H (p_1 \cdots p_h) = I \tag{1}$$

$\lambda_i(\gamma)$ is denoted as the eigenvalue of $A(\gamma)$ with respect to $\lambda_i, i \in (1, 2, \cdots, h)$. Then the differential $d\lambda_i(\gamma)/d\gamma|_{\gamma=0}$ exists and is the eigenvalue of the h-dimension matrix as follows:

$$\begin{pmatrix} q_1^T \dot{A} p_1 & \cdots & q_1^T \dot{A} p_h \\ \vdots & \vdots & \vdots \\ q_h^T \dot{A} p_1 & \cdots & q_h^T \dot{A} p_h \end{pmatrix}, \qquad \dot{A} := dA(\gamma)/d\gamma|_{\gamma=0} \tag{2}$$

3 Clock Models and Synchronization Achieving Protocol

3.1 Models and Protocol

Above all, each local clock's physical time changes according to the following principle

$$\tau_i(t + T) = \tau_i(t) + \Delta_i(t, T), \tag{3}$$

where t denotes the global physical time and $\tau_i(t)$ represents the actual local time of node i corresponding with t from the global view. And $\Delta_i(t, T) > 0$ denotes the time variation of local clock i in sampling period T, where t implies that this variation might be slightly different between different neighboring sample time. It is also assumed that the value of $\Delta_i(t, T) > 0$ is bounded. Since the physical clock rate cannot be actually adjusted, every node needs to generate a virtual local clock whose time can be adjusted by a control input $v_i(t)$ directly. And activities asking for clock consensus all refer to the virtual local time. In this way, we just need to achieve the virtual clock synchronization of all nodes. Then the model of virtual clock with control effect can be rewritten as

$$\begin{aligned} \overline{\tau}_i(t + T) =& \overline{\tau}_i(t) + [\tau_i(t + T) - \tau_i(t)] + v_i(t) \\ =& \overline{\tau}_i(t) + \Delta_i(t, T) + v_i(t) \end{aligned} \tag{4}$$

where $\overline{\tau}_i(t)$ denotes the virtual time of local clock i and we let $\overline{\tau}_i(0) = \tau_i(0)$. Then an achieving protocol whose output is $v_i(t)$ can be designed to assist all the local clocks in obtaining the consensus of their virtual time.

Then for each node, the synchronization achieving protocol can be designed as

$$\begin{cases} \omega_i(t + T) = \varepsilon \sum_{j\in\mathcal{N}_i} d_{ij}(\overline{\tau}_j(t) - \overline{\tau}_i(t)) \\ v_i(t + T) = \sum_{j\in\mathcal{N}_i} d_{ij}(\overline{\tau}_i(t) - \overline{\tau}_j(t)) + v_i(t) \\ \qquad\qquad + \omega_i(t + T) - \alpha\omega_i(t) \end{cases}$$

where d_{ij} is the weight associated to the edge (j, i). How to choose the parameters ε and α will be shown in the next subsection to guarantee the stability of the system. The above equations can be expressed in a vector form as

$$\begin{cases} \boldsymbol{\omega}(t + T) = -\varepsilon D\overline{\boldsymbol{\tau}}(t) \\ \boldsymbol{v}(t + T) = D\overline{\boldsymbol{\tau}}(t) + \boldsymbol{v}(t) + \boldsymbol{\omega}(t + T) - \alpha\boldsymbol{\omega}(t) \end{cases} \tag{5}$$

where D consisting of d_{ij} is a Laplacian matrix. Here we have to assume that the underlying graph of the message channel topology is connected undirected as other distributed synchronization methods [5] do. Under this assumption, $d_{ij} = d_{ji}$ establishes and D is symmetric with zero as its simple eigenvalue.

Remark 1: Another form of expression can be derived from the above equations

$$\boldsymbol{v}(t + T) = \boldsymbol{v}(t) + (1 - \varepsilon)D\overline{\boldsymbol{\tau}}(t) + \alpha\varepsilon D\overline{\boldsymbol{\tau}}(t - T).$$

This shows that $\boldsymbol{v}(t + T)$ can be treated as the update of an estimator. In other words, errors from the input $\tau_j(t) - \tau_i(t)$ do not affect $v_i(t + T)$ directly. Hence, the protocol does not need additional filtering algorithms against noisy inputs like [5], which decreases the amount of energy consumed for data transmission and computation. It can be seen from the protocol that only estimation of virtual time and control input instead of physical clock correction is executed. This ensures more energy conservation.

3.2 Stability and Consensus Analysis

Since D is symmetric, there should exist a unitary matrix U satisfying that $U^H DU = \Lambda = \text{diag}\{\lambda_1, \cdots, \lambda_n\}$, where λ_i represents one of the eigenvalues of D. With the underlying graph being connected undirected, we can have the relation that $0 = \lambda_1 < \lambda_2 \cdots \leq \lambda_n$. According to the model (3) and the synchronization protocol (5), the state-space expression of this discrete-time system can be summarized in the following way (to fit the column width, we use $\boldsymbol{\Delta}$ instead of $\boldsymbol{\Delta}(t, T)$ in this subsection when necessary):

$$\begin{pmatrix} \overline{\boldsymbol{\tau}}(t+T) \\ \boldsymbol{v}(t+T) \\ \boldsymbol{\omega}(t+T) \end{pmatrix} = \begin{pmatrix} I & I & \mathbf{0} \\ (1-\varepsilon)D & I & -\alpha I \\ -\varepsilon D & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \overline{\boldsymbol{\tau}}(t) \\ \boldsymbol{v}(t) \\ \boldsymbol{\omega}(t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{\Delta} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}.$$

Then, we utilize the unitary transformation to make the system decoupled by letting

$$\begin{cases} \hat{\boldsymbol{\omega}}(t) = U^H \boldsymbol{\omega}(t), & \hat{\boldsymbol{v}}(t) = U^H \boldsymbol{v}(t) \\ \hat{\boldsymbol{\tau}}(t) = U^H \overline{\boldsymbol{\tau}}(t), & \hat{\boldsymbol{\Delta}}(t,T) = U^H \boldsymbol{\Delta}(t,T) \end{cases}. \tag{6}$$

The component-wise expression of the system after the transformation (6) can be obtained as

$$\begin{pmatrix} \hat{\tau}_i(t+T) \\ \hat{v}_i(t+T) \\ \hat{\omega}_i(t+T) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ (1-\varepsilon)\lambda_i & 1 & -\alpha \\ -\varepsilon\lambda_i & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{\tau}_i(t) \\ \hat{v}_i(t) \\ \hat{\omega}_i(t) \end{pmatrix} + \begin{pmatrix} \hat{\Delta}_i \\ 0 \\ 0 \end{pmatrix}.$$

To ensure the stability of the whole discrete-time system, there are certain extra constraints for the choice of the parameters in the designed protocol. As we know, the system will be stable if all eigenvalues of the system matrix are strictly included in a unit circle under the condition that values of $\Delta_i(t, T)$ are bounded. To utilize this principle, the eigenvalue expression has to be figured out as

$$f(\delta) = \delta^3 - 2\delta^2 + [1 + (\varepsilon - 1)\lambda_i]\delta - \alpha\varepsilon\lambda_i.$$

Let $f(\delta) = 0$, then the three eigenvalues δ_1, δ_2 and δ_3 are also the roots of the cubic eigenvalue equation. Here we utilize Jury Stability criterion to calculate the value ranges of those parameters with constraints that $|\delta_i| < 1$, for $i \in (1, 2, 3)$. The test chart is given as Table 1,

Table 1: Chart of Jury stability criterion

Linage	δ^0	δ^1	δ^2	δ^3
1	$-\alpha\varepsilon\lambda_i$	$1+(\varepsilon-1)\lambda_i$	-2	1
2	1	-2	$1+(\varepsilon-1)\lambda_i$	$-\alpha\varepsilon\lambda_i$
3	b_0	b_1	b_2	

where

$$b_0 = \begin{vmatrix} -\alpha\varepsilon\lambda_i & 1 \\ 1 & -\alpha\varepsilon\lambda_i \end{vmatrix}, \quad b_1 = \begin{vmatrix} -\alpha\varepsilon\lambda_i & -2 \\ 1 & 1+(\varepsilon-1)\lambda_i \end{vmatrix},$$
$$b_2 = \begin{vmatrix} -\alpha\varepsilon\lambda_i & 1+(\varepsilon-1)\lambda_i \\ 1 & -2 \end{vmatrix}.$$

Due to the criterion, we can get the following equations as constraints,

$$\begin{cases} f(1) > 0, f(-1) < 0 \\ |-\alpha\varepsilon\lambda_i| < 1, |b_0| > |b_2| \end{cases}.$$

Therefore, the value ranges of the parameters can be figured out as $1 < \varepsilon < 1 + \frac{1}{\lambda_i}$ and $0 < \alpha < 1 - \frac{1}{\varepsilon}$. The two exact value ranges can be obtained by letting $\rho(D)$ take the place of λ_i, where $\rho(D)$ is the spectral radius of D.

Next we consider the consensus attribute under the protocol. Let $e_i(t) = \overline{\tau}_i(t) - \frac{1}{n}\sum_{j=1}^{n} \overline{\tau}_j(t)$, then we have

$$\boldsymbol{e}(t) = (I - \frac{1}{n}\mathbf{1}\mathbf{1}^T)\overline{\boldsymbol{\tau}}(t). \tag{7}$$

From the expression of $e_i(t)$, we can see that $\overline{\boldsymbol{\tau}}(t)$ can achieve average consensus if $e_i(t)$ converges to zero. And it is not difficult to find that $I - \frac{1}{n}\mathbf{1}\mathbf{1}^T$ is also a symmetric Laplacian matrix with a special form. So it also satisfies that $U^H(I - \frac{1}{n}\mathbf{1}\mathbf{1}^T)U = \text{diag}\{0, I_{n-1}\}$. Similarly, by the transformation $\hat{\boldsymbol{e}}(t) = U^H\boldsymbol{e}(t)$, the formula (7) can be transformed as $U^H\boldsymbol{e}(t) = U^H(I - \frac{1}{n}\mathbf{1}\mathbf{1}^T)UU^H\overline{\boldsymbol{\tau}}(t)$, which infers

$$\hat{\boldsymbol{e}}(t) = \text{diag}\{0, I_{n-1}\}\hat{\boldsymbol{\tau}}(t). \tag{8}$$

From the above expression, we can see that $\hat{e}_1(t) \equiv 0$, while $\hat{e}_i(t) = \hat{\tau}_i(t)$ when $i \geq 2$. Since the component-wise form of the state-space expression still establishes when $t \to \infty$, we have

$$\begin{cases} \hat{v}_i(t+T) = (1-\varepsilon)\lambda_i\hat{\tau}_i(t) + \hat{v}_i(t) - \alpha\hat{\omega}_i(t), t \to \infty \\ \hat{\omega}_i(t+T) = -\varepsilon\lambda_i\hat{\tau}_i(t), t \to \infty \end{cases}.$$

From the equations, we can figure out that $(\alpha\varepsilon + 1 - \varepsilon)\lambda_i\hat{\tau}_i(t) = 0, t \to \infty$. Then the facts that $0 = \lambda_1 < \lambda_2 \cdots \leq \lambda_n$ and $\alpha\varepsilon + 1 - \varepsilon \neq 0$ help to conclude that $\hat{\tau}_i(t) \equiv 0, t \to \infty$ when $i \geq 2$. Combining the inference with the relation inferred from the formula (8), we can get that $\hat{\boldsymbol{e}}(t) \equiv 0, t \to \infty$. Since the fact that zero is not one eigenvalue of U^H, we can conclude that $\boldsymbol{e}(t) \equiv 0, t \to \infty$ due to the relationship $\hat{\boldsymbol{e}}(t) = U^H\boldsymbol{e}(t)$. Then till now, it has been proved that the whole system under protocol (5) can achieve asymptotic consensus.

To make the virtual reference time more precise, substituting $\lambda_1 = 0$ into the component-form state-space expression of the system yields that

$$\begin{cases} \hat{\tau}_1(t+T) = \hat{\tau}_1(t) + \hat{v}_1(t) + \hat{\Delta}_1(t,T) \\ \hat{v}_1(t+T) = \hat{v}_1(t) - \alpha\hat{\omega}_1(t) \\ \hat{\omega}_1(t+T) = \hat{\omega}_1(t) = 0 \end{cases}.$$

From the above expressions, we can obtain the recursion expression of $\hat{\tau}_1(t)$ as

$$\hat{\tau}_1(t) = \sum_{j=0}^{t-T} \hat{\Delta}_1(j,T) + \hat{\tau}_1(0) + (t/T)\hat{v}_1(0).$$

It is easy to figure out that $\hat{v}_1(0)$ in the last term only acts as a constant factor of t/T. Then to make the later analysis easier, we can remove the uncertainty by letting the initial value of the transformed control input $\hat{v}_1(0) = 0$ when the protocol is run. In this way, we get a more compact expression

$$\hat{\tau}_1(t) = \sum_{j=0}^{t-T} \hat{\Delta}_1(j,T) + \hat{\tau}_1(0).$$

Now, we can find out what the value of the virtual reference clock time is. According to the above statements, we can get the following process of inference as

$$\begin{aligned} &\lim_{t\to\infty} \overline{\boldsymbol{\tau}}(t) = \lim_{t\to\infty} U\hat{\boldsymbol{\tau}}(t) \\ &= \lim_{t\to\infty} U \begin{pmatrix} \hat{\tau}_1(t) & \cdots & \hat{\tau}_n(t) \end{pmatrix}^T \\ &= U \begin{pmatrix} \sum_{j=0}^{t-T} \hat{\Delta}_1(j,T) + \hat{\tau}_1(0) & 0 & \cdots & 0 \end{pmatrix}^T \\ &= U\{\text{diag}\{1, \mathbf{0}_{n-1}\}U^H[\sum_{j=0}^{t-T} \boldsymbol{\Delta}(j,T) + \overline{\boldsymbol{\tau}}(0)]\} \end{aligned}$$

$$= [I - (I - \frac{1}{n}\mathbf{1}\mathbf{1}^T)][\sum_{j=0}^{t-T} \boldsymbol{\Delta}(j,T) + \overline{\boldsymbol{\tau}}(0)]$$

$$= \frac{1}{n}\mathbf{1}\mathbf{1}^T[\sum_{j=0}^{t-T} \boldsymbol{\Delta}(j,T) + \overline{\boldsymbol{\tau}}(0)].$$

With $\overline{\tau}_i(0) = \tau_i(0)$, the virtual reference clock time can be denoted as

$$\tau_c(t) = \frac{1}{n}\sum_{i=1}^{n}(\sum_{j=0}^{t-T} \Delta_i(j,T) + \tau_i(0)). \quad (9)$$

Remark 2: From the result, we can see that the time variation of virtual reference clock has achieved a consensus value $\frac{1}{n}\sum_{i=1}^{n}\sum_{j=0}^{t-T} \Delta_i(j,T)$. Simultaneously, the initial time error of reference clock also gains the common value $\frac{1}{n}\sum_{i=1}^{n}\tau_i(0)$. Achieving the consensus of virtual time variations under the achieving protocol and the maintaining one introduced later means that local clocks in the whole network will not need re-synchronizing in a long period of global physical time for the fact that all virtual local time can change with the same pace. Therefore, compared to methods like [9] which executes synchronization frequently, this protocol saves more power.

4 Time Variation Model and Synchronization Maintaining Protocol

4.1 Model and Protocol

After the clock synchronization has been achieved with required accuracy identified by the stopping criterion stated in next section, there are almost no virtual time errors between these local clocks corresponding to the same global time instant. Then, the protocol can be degraded into another form with better power conservation. Since the maintaining protocol is run on the basis of completion of the previous protocol, only the consensus precision of virtual time variations needs maintaining or even enhancing to keep all virtual time changing at the same pace. Above all, the model of node i's virtual time variation with the control effect $u_i(\bar{t})$ can be represented as

$$\overline{\Delta}_i(\bar{t}+T,T) = \overline{\Delta}_i(\bar{t},T) + u_i(\bar{t}) \quad (10)$$

where $\overline{\Delta}_i(t_0,T) = \overline{\tau}_i(t_0+T) - \overline{\tau}_i(t_0)$. t_0 is assumed to be the time instant when the stopping margin is satisfied. After one sampling period from t_0, the achieving protocol stops. $\bar{t}$, distinct from t, denotes the global time instant from t_0 on. Then the maintaining protocol can be presented as follows:

$$\begin{cases} u_i(\bar{t}) = \sum_{j\in\mathcal{N}_i^+} l_{ij}(\overline{\Delta}_j(\bar{t},T) - \overline{\Delta}_i(\bar{t},T)) + \gamma\hbar_i(\bar{t}) \\ \hbar_i(\bar{t}+T) = (1 - \sum_{o\in\mathcal{N}_i^-} l_{oi})\hbar_i(\bar{t}) + \sum_{j\in\mathcal{N}_i^+} l_{ij}\hbar_j(\bar{t}) - u_i(\bar{t}) \end{cases}$$

where $\hbar_i(t_0) = 0$ and the choice of parameter $\gamma \geq 0$ is constrained to help the whole system achieve stability. Besides, $l_{oi} = d_{oi}$ when $i \neq o, o \in \mathcal{N}_i^-$ and $l_{oi} = 0$ when $i \neq o, o \notin \mathcal{N}_i^-$. In this way, the weights of communication channels of the network do not need to be redesigned when switching the protocols.

Substituting the relationship that $\sum_{o\in\mathcal{N}_i^-} l_{oi} = \sum_{o=1}^{n} l_{oi}$ into the above equations, we can obtain the compact vector form as follows:

$$\begin{cases} \boldsymbol{u}(\bar{t}) = -L\overline{\boldsymbol{\Delta}}(\bar{t},T) + \gamma\boldsymbol{\hbar}(\bar{t}) \\ \boldsymbol{\hbar}(\bar{t}+T) = (I - \underline{L})\boldsymbol{\hbar}(\bar{t}) - \boldsymbol{u}(\bar{t}) \end{cases}. \quad (11)$$

where L has the same form with the Laplacian matrix mentioned in the part of graph theory and $\underline{L}$'s oith entry is $L(o,i) = -l_{oi}, i \neq o, o \in \mathcal{N}_i^-$, $L(o,i) = 0, i \neq o, o \notin \mathcal{N}_i^-$ and $L(o,i) = \sum_{o\in\mathcal{N}_i^-} l_{oi}, i = o$. It is easy to figure out that $\underline{L}$'s column sum is zero and $\underline{L}^H$ is also a Laplacian matrix. Then combining the equations (10) and (11) gets the state-space expression of the whole system as

$$\begin{pmatrix} \overline{\boldsymbol{\Delta}}(\bar{t}+T,T) \\ \boldsymbol{\hbar}(\bar{t}+T) \end{pmatrix} = \begin{pmatrix} I-L & \gamma I \\ L & I-\underline{L}-\gamma I \end{pmatrix} \begin{pmatrix} \overline{\boldsymbol{\Delta}}(\bar{t},T) \\ \boldsymbol{\hbar}(\bar{t}) \end{pmatrix}.$$

4.2 Stability and Consensus Analysis

First, the stability of the whole system is analyzed in this part. According to the matrix perturbation theory, the system matrix A in above expression can be treated as the sum of an unperturbed matrix A_0 and a perturbing matrix γB. In symbol words, it is

$$A = \begin{pmatrix} I-L & \gamma I \\ L & I-\underline{L}-\gamma I \end{pmatrix} = \begin{pmatrix} I-L & \mathbf{0} \\ L & I-\underline{L} \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \gamma I \\ \mathbf{0} & -\gamma I \end{pmatrix}$$
$$= A_0 + \gamma B.$$

Due to the lower block triangular structure A_0 has, its spectrum $\sigma(A_0) = \sigma(I-L)\bigcup\sigma(I-\underline{L})$. Besides, the two block diagonal matrices of A_0 are row and column stochastic respectively. Therefore, the spectral radius $\rho(I-L) = \rho(I-\underline{L}) = 1$. Here, it has to be assumed that the underlying unbalanced graph of the new communication topology is strongly connected. Then these two matrices are irreducible. It can be inferred that 1 is a simple eigenvalue of both those two matrices according to the Perron-Frobenius Theorem [10]. A_0's eigenvalues can be presented in order as $1 = |\overline{\lambda}_1| = |\overline{\lambda}_2| > |\overline{\lambda}_3| \geq |\overline{\lambda}_4| \geq \cdots \geq |\overline{\lambda}_{2n}|$.

Then we let $\mathfrak{p}^T$, one n-dimension column vector satisfying $\mathfrak{p}\mathbf{1} = 1$, be the left eigenvector of the matrix $I-L$ corresponding with 1. Also let $\mathfrak{q}$ stand for $\rho(I-\underline{L})$'s right eigenvector of $I-\underline{L}$, satisfying $\mathbf{1}^T\mathfrak{q} = 1$. The Perron-Frobenius Theorem also ensures that such two positive eigenvectors can be found. Then two left eigenvectors of A_0 corresponding to the eigenvalue 1 can be designed as $\boldsymbol{q}_1^T = (\mathbf{1}^T,\mathbf{1}^T)^T$ and $\boldsymbol{q}_2^T = (\mathfrak{p},\mathbf{0})^T$, where $\mathbf{0}$ is a row vector. Similarly, the two right ones can be chosen as $\boldsymbol{p}_1 = (\mathbf{0},\mathfrak{q}^T)^T$ and $\boldsymbol{p}_2 = (\frac{1}{n}\mathbf{1}^T, -\mathfrak{q}^T)^T$. Then $(\boldsymbol{q}_1^T,\boldsymbol{q}_2^T)^T(\boldsymbol{p}_1,\boldsymbol{p}_2) = I$, which satisfies the condition (1) required in ***Lemma 2***. Also due to that lemma, it can be obtained that $\dot{A} = B$. Therefore the matrix in expressions (2) can be specified as follows:

$$\begin{pmatrix} q_1Bp_1 & q_1Bp_2 \\ q_2Bp_1 & q_2Bp_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \mathfrak{p}\mathfrak{q} & -\mathfrak{p}\mathfrak{q} \end{pmatrix}.$$

Due to ***Lemma 2***, if γ is small enough, $d\overline{\lambda}_1(\gamma)/d\gamma$ and $d\overline{\lambda}_2(\gamma)/d\gamma$ are eigenvalues of the above matrix on the right side, which equate with 0 and $-\mathfrak{p}\mathfrak{q}$, respectively. $\overline{\lambda}_i(\gamma), i \in (1,2,\cdots,2n)$ are eigenvalues of A corresponding with $\overline{\lambda}_i$ of A_0. Then it can be inferred that $\overline{\lambda}_1(\gamma)$ equates with $\overline{\lambda}_1$ and will not change as the value of γ becomes larger from zero, while $\overline{\lambda}_2(\gamma)$'s value will get smaller from $\overline{\lambda}_2$ with larger values of γ. According to [11], eigenvalues are continuous functions of matrix entries. There must exist an appropriate value $\overline{\gamma}$ which ensures $|\overline{\lambda}_i(\overline{\gamma})| < 1$ for all $i \in (2, 3, \cdots, 2n)$. Until now, it has been proved that the whole system can achieve the stability. As for the method of figuring out the upper bound value of $\overline{\gamma}$, one can refer to [12] in which a common form of that upper bound value is proposed with specific proof.

To analyze the consensus attribute, we can get the following equations from the state-space expression above all.

$$\begin{cases} \overline{\boldsymbol{\Delta}}(\bar{t}+T, T) = (I-L)\overline{\boldsymbol{\Delta}}(\bar{t}, T) + \gamma \boldsymbol{\hbar}(\bar{t}) \\ \boldsymbol{\hbar}(\bar{t}+T) = L\overline{\boldsymbol{\Delta}}(\bar{t}, T) + (I - \underline{L} - \gamma I)\boldsymbol{\hbar}(\bar{t}). \end{cases} \tag{12}$$

Then the sum of above equations can be induced as:

$$\overline{\boldsymbol{\Delta}}(\bar{t}+T, T) + \boldsymbol{\hbar}(\bar{t}+T) = \overline{\boldsymbol{\Delta}}(\bar{t}, T) + (I - \underline{L})\boldsymbol{\hbar}(\bar{t}). \tag{13}$$

Combining formula (13) and the first one in equations (12) yields the following expressions when $\bar{t}$ tends to infinity:

$$L\overline{\boldsymbol{\Delta}}(\infty, T) = \gamma \boldsymbol{\hbar}(\infty), \qquad \underline{L}\boldsymbol{\hbar}(\infty) = 0. \tag{14}$$

According to the above two expressions, it is easy to obtain that $\underline{L}L\overline{\boldsymbol{\Delta}}(\infty, T) = 0$. Since $\underline{L}L$ is a special Laplacian matrix whose row sum and column sum are both zero, it can be asserted that $\overline{\boldsymbol{\Delta}}(\infty, T) = \overline{\Delta}_c \mathbf{1}$. This also means that virtual time variations of all nodes can achieve asymptotic consensus. Then substituting this formula back into the left side equation in (14) yields that $\boldsymbol{\hbar}(\infty) = 0$.

On the other hand, the equation (13) enables us to get one further expression:

$$\begin{aligned} &\mathbf{1}^T[\overline{\boldsymbol{\Delta}}(\bar{t}+T, T) + \boldsymbol{\hbar}(\bar{t}+T)] = \mathbf{1}^T[\overline{\boldsymbol{\Delta}}(\bar{t}, T) + (I - \underline{L})\boldsymbol{\hbar}(\bar{t})] \\ &= \mathbf{1}^T[\overline{\boldsymbol{\Delta}}(\bar{t}, T) + \boldsymbol{\hbar}(\bar{t})] \\ &= \mathbf{1}^T[\overline{\boldsymbol{\Delta}}(t_0, T) + \boldsymbol{\hbar}(t_0)] = \mathbf{1}^T[\overline{\boldsymbol{\Delta}}(t_0, T)]. \end{aligned}$$

When $\bar{t}$ tends to infinity, the recursive expression above can be simplified to $\mathbf{1}^T[\overline{\boldsymbol{\Delta}}(\infty, T) + \boldsymbol{\hbar}(\infty)] = \mathbf{1}^T[\overline{\boldsymbol{\Delta}}(t_0, T)]$. Then substituting the above two formulas induced from expressions (14) into it, we can obtain the consensus value $\overline{\Delta}_c = \frac{1}{n}\sum_{i=1}^{n} \overline{\Delta}_i(t_0, T)$.

Remark 3: Since the underlying graph of the topology only needs being strongly connected instead of being connected balanced, the number and complexity of messages for transmission and storage decrease, which will be specifically shown in the simulation part. Besides, the protocol has a more succinct form with less burden of computation. Both of these two aspects conserve energy while maintaining the synchronization of clocks.

5 Stopping Criterion

Before introducing the criterion, it is worth reminding that the virtual time and virtual time variations of all nodes achieve consensus respectively at the same time. Thus, we can design the criterion due to the virtual time variations. Then, two more protocols are needed. It has been proven in [13] that the minimum and maximum consensus protocols can be applied to assisting average consensus protocols in finding the stopping margin as long as the underlying graph is undirected or strongly connected balanced (see [13] for details). The maximum and minimum consensus protocols can be presented as follows:

$$\begin{aligned} y_i(t+T) &= \max_{j \in \mathcal{N}_i} y_j(t) \\ z_i(t+T) &= \min_{j \in \mathcal{N}_i} z_j(t) \end{aligned} \tag{15}$$

where we set $\boldsymbol{y}(0) = \overline{\boldsymbol{\Delta}}(0, T), \boldsymbol{z}(0) = \overline{\boldsymbol{\Delta}}(0, T)$ and $\overline{\Delta}_j(t, T) = \overline{\tau}_j(t+T) - \overline{\tau}_j(t)$. From [13], we know that after every $\mathfrak{D}$ time steps the maximum/minimum value of the virtual local time decreases/increases strictly, where $\mathfrak{D}$ represents the diameter of the underlying graph of the topology. And $\mathfrak{D}$ can be obtained by all nodes in a distributed way according to the method mentioned in [14]. Then we have to define $\mathfrak{t}(k) = (k-1)\mathfrak{D}T$, $k = 1, 2, \cdots$ as the time instants when the maximum and minimum protocols are reset. In other words, the values $y_i(\mathfrak{t}(k)), z_i(\mathfrak{t}(k))$ of the two protocols at these instants are reset to be equal with the current virtual time variation value $\overline{\Delta}_i(\mathfrak{t}(k), T)$ of node i. Then we also have to define $\overline{x}_k, \underline{x}_k$ as the consensus values of the maximum and minimum protocols respectively. The synchronization achieving protocol in this paper can stop when the difference between the outputs $\mathfrak{x}_k = \overline{x}_k - \underline{x}_k < \rho$, where ρ is a preset constant parameter corresponding with the required accuracy.

6 Implementation

When applied to WSNs, this synchronization achieving method can take a reactive flooding scheme as its communication means with the help of MAC layer. MAC-layer technique does not only have the function of preventing transmission collisions, but also conserves energy[15]. *Reactive*, similar with *post-facto* in the method RBS [4], represents that sensors in the networks are wakened to find edges only when necessary instead of maintaining routing information proactively. This makes it possible that sensors stay in a state of low power without keeping the processor or radio used for synchronization on. After having listened to the signal for synchronization, the nodes can power their radios on to exchange the messages containing synchronization information. In this way, much less energy is consumed for maintaining the high-power state. Flooding means that a receiving node would be triggered by the incoming time-stamps to record its virtual local time of their arrival, then broadcast its synchronization message containing its virtual local time on to its neighbours at its periodic transmitting instants after updating the time-stamp. Since neighbours are nodes within the listening distance of each other, this way ensures reduction of energy consumption by letting messages transmit over multiple short distances instead of a single long path.

7 Numerical Simulation

In this section, we mainly present certain simulation results to show the performance of our method. We consider an example of a network consisting of six sensors, whose topologies for achieving and maintaining protocols are shown respectively in Fig. 1. In the figure, the left undirected one is the underlying graph for achieving protocol and it can be degraded to the middle one for the maintaining protocol. While if the purpose is to save energy as much as possible, the underlying graph of the topology can be simplified to the right one. Then we choose parameters as $\varepsilon = 1.30, \alpha = 0.23$ and $\gamma = 0.0010$. The simulation of

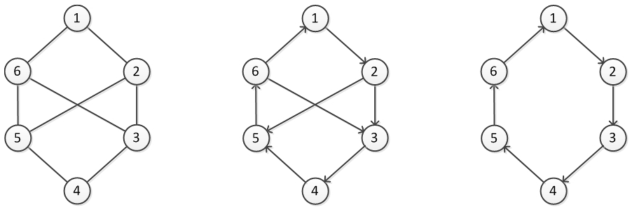

Fig. 1: Topologies of one WSN with six nodes

all local clocks' virtual time according to the physical time t with the achieving protocol is obtained in Fig. 2. Fig. 3, in which the blue circle represents the stopping margin, shows time variations of all local clocks in the achieving phase with the stopping criterion. Fig. 4 shows the the change of time variations with the maintaining protocol.

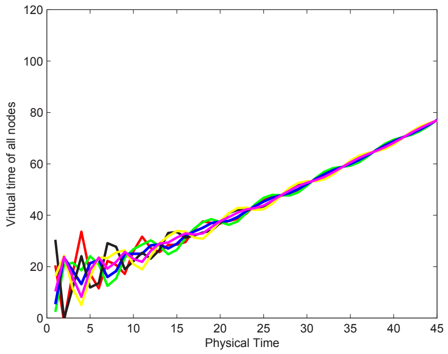

Fig. 2: Virtual time of all local clocks due to the physical time

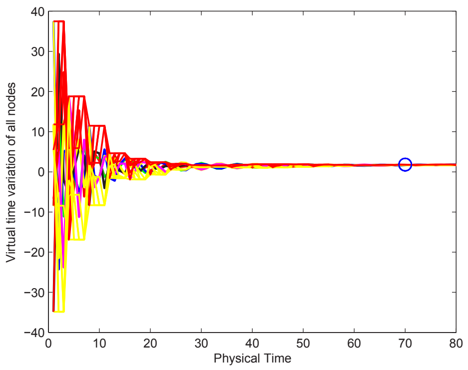

Fig. 3: Time variations with synchronization achieving protocol

8 Conclusion

By utilizing the communication scheme presented in this paper, the synchronization achieving protocol can realize synchronization of virtual time and time variations simultaneously with lower power consumption. Then the stopping criterion helps the protocol to be transformed to the maintaining one to save more energy while keeping the synchronization accuracy.

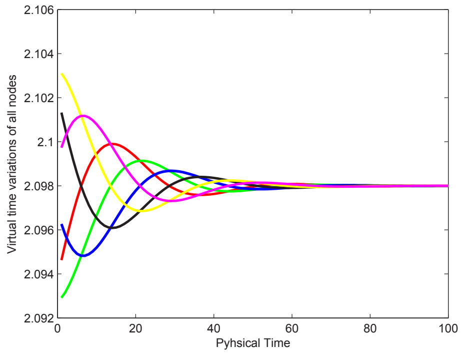

Fig. 4: Time variations with synchronization maintaining protocol

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