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Optimal Control of Networked Systems with Limited Communication: a Combined Heuristic and Convex Optimization Approach

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***Abstract*—This paper discusses the optimal H_∞ control problem for networked systems with limited communication constraint. The limited communication constraint in control networks is taken into consideration in controller design by employing the notion of communication sequence. Our objective is to find an optimal communication sequence and the corresponding optimal controller for the given plant and communication resource under the H_∞ performance index. For a given communication sequence, the problem is formulated into a periodic control problem for which a direct LMI design method is developed. We also propose a heuristic search method for seeking a sub-optimal communication sequence, which in conjunction with the convex optimization gives a solution to the optimal limited communication control problem. As compared with the exhaustive search of communication sequence, our approach greatly reduces the computational cost. Examples are given to illustrate the design method. It clearly indicates that the solution of the heuristic search converges to the optimal communication sequence.**

I. Introduction

Since networks may greatly decrease the hardwiring, the cost of installation and implementation, it is popular to use networks in many complicated systems such as manufacturing plants, platoon vehicles and robotic systems. In addition, the more modular and more flexible structure of networked systems makes it much easier to remove, exchange, and add parts. However there are also drawbacks in employing serial communication network to exchange information between different system components. The main drawbacks are network-induced delay and the limitation on bandwidth both of which can affect system performance. Time-delay may be induced by networks when exchanging data among devices connected to the shared communication medium. The characteristics of this kind of time delay in different control networks can be found in [11]. On the other hand, the effects of limited bandwidth on control system performance has attracted a lot of interest recently, [1]-[8].

Control networks are different from data networks in that in the former data are continuously transmitted at relatively constant rates while in the latter, large data packets are sent out occasionally at high data rates. Furthermore, control networks need to meet time-critical requirement, that is to say, message should be sent out successfully within a prespecified time. The primary objective of control networks is to efficiently use the finite communication resources while maintaining good system performance. Thus some standard protocols for control networks have been developed such as Controlnet, Ethernet, Devicenet (see [12]), BACNet and Lonworks, which have been used in many practical applications.

In classic models, it may simplify problems and still work well to separate the communication aspects from the dynamics of a system. However when system performance is limited and degraded because of propagation delay and limited bandwidth, it is not proper without considering the effects brought by networks. Conventional control theories such as synchronized control and non delayed sensing and actuation must be reevaluated prior to application to networked control systems [1], [2]. This brings forth a lot of efforts to investigate the effects of time-delay and finite communication rates in control problems.

The problem of stabilizing an LTI system when only some elements of the outputs and/or some of the control actions can be transmitted at one time has been investigated in [1]-[3]. The LQ control with communication constraints is studied in [4] which indicates that an optimal communication sequence is typically such that the sampling resources should be focused on where they are needed most. In [5] and [6], stabilization of infinite-dimensional time-varying ARMA model under limited data rates is considered. In [7] and [8] coding and state estimation in limited communication channels are taken into account. [10] has shown that information exchange between local controllers through a network can enlarge the class of plants to be stabilized. An explicit stability condition dependent on the maximum time delay induced by networks is given in [13] and [15]. Recently, [16] shows that by using the estimated values instead of true values of system states at other nodes, a significant saving in bandwidth is achieved to allow more resources to utilize the network.

On the other hand, the standard H_∞ control problems are tackled by assuming that all the outputs are available to the controller at any time instant. Obviously, this is impossible due to the serial communication in control networks. In this paper, we want to find an optimal communication sequence and a controller to obtain the minimum H_∞ cost for networked control systems with limited bandwidth. The problem is first formulated as a periodic control problem by employing the notion of "communication sequence".

It is shown that under a given communication sequence, the design of an optimal periodic controller can be converted to a convex optimization. We then propose a heuristic search approach for a suboptimal communication sequence, which together with the convex optimization of controller, gives a solution to the joint optimization problem. The approach is known to be convergent. As compared to the exhaustive search in [4], our approach greatly reduces the computational cost. Several examples are included to demonstrate that the heuristic search in fact converges to a global optimal solution although no theoretical proof is given. It is worth noting that stabilization problem under limited communication constraint has been considered in [1], [2] and [10] using the lifting technique which will deal with much higher input and output dimensions. Our proposed direct approach has the advantage that it can avoid this and can be extended easily to deal with uncertain systems.

II. Problem Formulation

Consider a networked control system (NCS) shown in Fig 1. The plant, the sensors and the controller are spatially distributed and connected together through a control network, whereas conventional point-to-point link may be used. Now suppose that the spatially distributed plant is a linear time-invariant system described by

$$\begin{aligned} x(k+1) &= Ax(k) + B_1 w(k) + B_2 u(k) \quad (1)\\ z(k) &= C_1 x(k) + D_{11} w(k) + D_{12} u(k) \quad (2)\\ y(k) &= C_2 x(k) + D_{21} w(k) \quad (3) \end{aligned}$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $w(k) \in \mathcal{R}^p$ the disturbance input, $u(k) \in \mathcal{R}^m$ the control input, $y(k) \in \mathcal{R}^r$ the output, and $z(k) \in \mathcal{R}^q$ the controlled output. All the system matrices are with appropriate dimensions. The initial state x_0 is considered to be known and without loss of generality it is set to be zero. The system is assumed to be interconnected with spatially distributed subsystems. The output of each subsystem can only be sent to the controller through the network at a given time. Here we consider a simple case in which the control u is not transmitted by the network but in a way that it is transmitted to the plant directly.

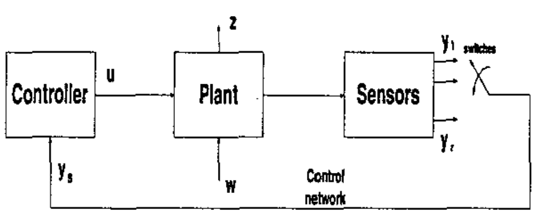

Fig. 1. Networked Control System

We assume that the control network adopts scheduled releasing policy to transmit data. Under this policy the start-sending time is scheduled to occur for each node and the signal is transmitted periodically. Hence, as pointed out in [12], time delay is little and the possibility of message collision is much lower. However, it is clear that under this policy the controller can't have simultaneous access to all outputs of the plant, but in a way that the multiple outputs are sequentially multiplexed from the sensors to the controller at every step periodically. The way of multiplexing can be described by the r switches

$$\begin{aligned} \sigma_1 &= [1 \;\; 0 \;\; \cdots \;\; 0],\\ \sigma_2 &= [0 \;\; 1 \;\; \cdots \;\; 0],\\ &\vdots\\ \sigma_r &= [0 \;\; 0 \;\; \cdots \;\; 1]. \end{aligned}$$

The switch σ_i, $i = 1, \cdots, r$, is a $1 \times r$ matrix with the i^{th} element being 1 and all other entries being zero. It determines the controller to communicate with which element of the outputs, because $\sigma_i y(k) = y_i(k)$, where $y_i(k)$ is the i^{th} element of the outputs.

We employ the idea of "communication sequence" which was originally introduced in [9] to jointly formulate the control and communication problem. Here the k^{th} element of a communication sequence, s_k, is defined as an arbitrary element of the above switches. Hence a communication sequence leads the controller to read which of the output signals at each time instant. It is reasonable to assume that the controller communicates with the plant following a periodic pattern, which can be specified by an N-periodic communication sequence s_k, where $s_{k+N} = s_k, \forall k \in \mathcal{Z}$.

Definition 2.1: An N-periodic communication sequence s_k, $k = 0, \cdots, N-1$, where $s_k \in \{\sigma_1, \sigma_2, \cdots, \sigma_r\}$, is admissible if the following condition is satisfied

$$rank \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-1} \end{bmatrix} = r. \quad (4)$$

The above definition has a more direct expression and is more flexible in converting a NCS to a periodic system compared with the similar definition given in [1]. This condition requires that no more than one of the outputs be measured by the controller at each time instant and the controller communicate with each of the plant output at least once within a period [2]. In this way, the bandwidth limitation in NCS is modeled in a manner that the controller can communicate with only one of the sensors at a discrete-time constant according to the communication sequence.

Remark 2.1: If more than one, say l, output measurements of the sensors can be transmitted in one-packet, then $s_k \in \{\bar{\sigma}_0, \bar{\sigma}_1, \cdots, \bar{\sigma}_{N-1}\}$, where

$$\bar{\sigma}_i = \begin{bmatrix} \tilde{\sigma}_1 \\ \tilde{\sigma}_2 \\ \vdots \\ \tilde{\sigma}_l \end{bmatrix}, \quad \tilde{\sigma}_i \in \{\sigma_1, \sigma_2, \cdots, \sigma_r\}, \quad i = 0, 1, \cdots, N-1$$

Note that s_k needs to satisfy (4). Here we mainly consider the case in which no element of the outputs is lumped into one packet. But the results can be extended to the general case easily.

Since for a given periodic communication sequence, the NCS is in fact a periodic system, we introduce a periodic controller of the form of (5)-(6) whose period is equal to that of the communication sequence

$$\hat{x}(k+1) = \hat{A}_k\hat{x}(k) + \hat{B}_k y_s(k) \quad (5)$$
$$u(k) = \hat{C}_k\hat{x}(k) + \hat{D}_k y_s(k) \quad (6)$$

where $\hat{x}(k) \in \mathcal{R}^n$ is the state of the controller, $\hat{A}_k \in \mathcal{R}^{n\times n}, \hat{B}_k \in \mathcal{R}^{n\times 1}, \hat{C}_k \in \mathcal{R}^{m\times n}, \hat{D}_k \in \mathcal{R}^{m\times 1}$ are the controller matrices which are N-periodic, i.e.,

$$\hat{A}_{k+N} = \hat{A}_k,\ \hat{B}_{k+N} = \hat{B}_k,\ \hat{C}_{k+N} = \hat{C}_k,\ \hat{D}_{k+N} = \hat{D}_k,\ \forall k \in \mathcal{Z}.$$

For convenience, we gather all the controller parameters into the following compact form

$$\Theta_k = \begin{bmatrix} \hat{A}_k & \hat{B}_k \\ \hat{C}_k & \hat{D}_k \end{bmatrix}. \quad (7)$$

And $y_s(k)$ is the information which is transmitted from the plant and fed into the controller. Notice that in general $y_s(k)$ is not the same as $y(k)$ because not all elements of $y(k)$ are communicated to the controller at time instant k. If the transmission delay of the data from the plant to the controller is negligible, then $y_s(k)$ can be expressed in the following way:

$$y_s(k) = s_k y(k) \quad (8)$$

where s_k is the communication sequence, $y(k)$ is the output of the system.

By defining the augmented state vector $\xi(k) = [x_k^T\ \ \hat{x}_k^T]^T$, then from (1)-(8) we have the following closed-loop system $(\mathcal{S}_c)$:

$$\xi(k+1) = \bar{A}_{s_k}\xi(k) + \bar{B}_{s_k}w(k) \quad (9)$$
$$z(k) = \bar{C}_{s_k}\xi(k) + \bar{D}_{s_k}w(k) \quad (10)$$

where

$$\begin{aligned} \bar{A}_{s_k} &= \begin{bmatrix} A + B_2\hat{D}_k s_k C_2 & B_2\hat{C}_k \\ \hat{B}_k s_k C_2 & \hat{A}_k \end{bmatrix}, \quad (11) \\ \bar{B}_{s_k} &= \begin{bmatrix} B_1 + B_2\hat{D}_k s_k D_{21} \\ \hat{B}_k s_k D_{21} \end{bmatrix}, \\ \bar{C}_{s_k} &= [\,C_1 + D_{12}\hat{D}_k s_k C_2 \quad D_{12}\hat{C}_k\,], \\ \bar{D}_{s_k} &= D_{11} + D_{12}\hat{D}_k s_k D_{21}. \end{aligned}$$

It is clear that the closed-loop system (9)-(10) is periodic in k.

The H_∞ control problem can be stated as follows: find a communication sequence s_k and a periodic controller Θ_k of the form of (5)-(6) such that the closed-loop system (9)-(10) is asymptotically stable and has an optimal H_∞ performance under scheduled releasing policy.

III. Main Results

With the introduction of communication sequence, the optimal H_∞ control problem under scheduled releasing policy will be formulated as that of periodic systems. Thus we first give the direct approach for periodic systems analysis and design which is in comparison with the traditional lifting technique. Now consider a linear discrete-time N-periodic system $(\mathcal{S}_k)$ described by the following state space model:

$$x(k+1) = A_k x(k) + B_k w(k) \quad (12)$$
$$z(k) = C_k x(k) + D_k w(k) \quad (13)$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $w(k) \in \mathcal{R}^p$ is the disturbance input, $z(k) \in \mathcal{R}^q$ is the output of the system, and $A_k \in \mathcal{R}^{n\times n}$, $B_k \in \mathcal{R}^{n\times p}$, $C_k \in \mathcal{R}^{q\times n}$, $D_k \in \mathcal{R}^{q\times p}$ are N-periodic matrices satisfying

$$A_{k+N} = A_k,\ B_{k+N} = B_k,\ C_{k+N} = C_k,\ D_{k+N} = D_k,\ \forall k \in \mathcal{Z}.$$

The transition matrix of the system (12)-(13) is defined as

$$\Phi(k,l) = \begin{cases} A_{k-1}A_{k-2}\cdots A_l, & k > l \\ I, & k = l. \end{cases} \quad (14)$$

The transition matrix $\Phi(k,l)$ is N-periodic in k and l, i.e., $\Phi(k+N, l+N) = \Phi(k,l),\ \forall k,l \in \mathcal{Z}$. The eigenvalues of $\Phi(k+N,k)$ which are independent of k are referred to as characteristic multipliers. Moreover the periodic system is stable if and only if all the characteristic multipliers of A_k are inside the unit circle of the complex plane [18].

Definition 3.1: Given a scalar $\gamma > 0$, the periodic system (12)-(13) is said to have an H_∞ performance γ if it is asymptotically stable and satisfies

$$\sup_{0\neq w\in\ell_2} \frac{\|z\|_2}{\|w\|_2} < \gamma \quad (15)$$

under zero initial state condition.

In the following we present the Periodic Bounded Real Lemma which will play a key role for the results in this section.

Lemma 3.1: [21] The N-periodic system (12)-(13) has an H_∞ performance γ if and only if there exists an N-periodic positive definite matrix X_k satisfying the $LMIs$

$$\begin{bmatrix} -X_{k+1} & X_{k+1}A_k & X_{k+1}B_k & 0 \\ A_k^T X_{k+1} & -X_k & 0 & C_k^T \\ B_k^T X_{k+1} & 0 & -\gamma I & D_k^T \\ 0 & C_k & D_k & -\gamma I \end{bmatrix} < 0, \quad (16)$$

for $k = 0, 1, \cdots N-1$, simultaneously.

By Lemma 3.1, the closed-loop system (9)-(10) has an H_∞ performance γ under a given communication s_k if and only if there exists an N-periodic positive definite matrix X_k such that

$$\begin{bmatrix} -X_{k+1} & X_{k+1}\bar{A}_{s_k} & X_{k+1}\bar{B}_{s_k} & 0 \\ \bar{A}_{s_k}^T X_{k+1} & -X_k & 0 & \bar{C}_{s_k}^T \\ \bar{B}_{s_k}^T X_{k+1} & 0 & -\gamma I & \bar{D}_{s_k}^T \\ 0 & \bar{C}_{s_k} & \bar{D}_{s_k} & -\gamma I \end{bmatrix} < 0, \quad (17)$$

for $k = 0, 1, \cdots N-1$. Next we shall use the approach of change of variables as proposed in [20] to derive an explicit expression for the controller parameters that solve the H_∞ control problem.

Denote

$$X_k = \begin{bmatrix} R_k & M_k \\ M_k^T & U_k \end{bmatrix}, \quad X_k^{-1} = \begin{bmatrix} Y_k & N_k \\ N_k^T & V_k \end{bmatrix} \tag{18}$$

and

$$\Phi_{2k} = \begin{bmatrix} I & Y_k \\ 0 & N_k^T \end{bmatrix}, \quad \Phi_{1k} = \begin{bmatrix} R_k & I \\ M_k^T & 0 \end{bmatrix}. \tag{19}$$

Now introduce the new controller variables as below

$$\mathcal{C}_k = \hat{D}_k s_k C_2 Y_k + \hat{C}_k N_k^T \tag{20}$$

$$\mathcal{B}_k = R_{k+1} B_2 \hat{D}_k + M_{k+1} \hat{B}_k \tag{21}$$

$$\begin{aligned} \mathcal{A}_k = {} & R_{k+1}(A + B_2 \hat{D}_k s_k C_2) Y_k + R_{k+1} B_2 \hat{C}_k N_k^T \\ & + M_{k+1} \hat{B}_k s_k C_2 Y_k + M_{k+1} \hat{A}_k N_k^T. \end{aligned} \tag{22}$$

Observe that given matrices $\mathcal{A}_k, \mathcal{B}_k, \mathcal{C}_k$ and $\hat{D}_k$ and non-singular matrices M_k and N_k, the controller parameters $\hat{A}_k, \hat{B}_k, \hat{C}_k$ and $\hat{D}_k$ are uniquely determined by (20)-(22).

Theorem 3.1: Given a scalar $\gamma > 0$, the H_∞ control problem under a given communication sequence s_k for the system (1)-(3) is solvable if and only if there exist N-periodic matrices $R_k > 0$, $Y_k > 0$, $\mathcal{A}_k$, $\mathcal{B}_k$, $\mathcal{C}_k$ and $\hat{D}_k$ satisfying the following set of LMIs

$$\begin{bmatrix} -R_{k+1} & -I & R_{k+1}A + \mathcal{B}_k s_k C_2 & \mathcal{A}_k & R_{k+1}B_1 + \mathcal{B}_k s_k D_{21} & 0 \\ * & -Y_{k+1} & A + B_2 \hat{D}_k s_k C_2 & AY_k + B_2 \mathcal{C}_k & B_1 + B_2 \hat{D}_k s_k D_{21} & 0 \\ * & * & -R_k & -I & 0 & (C_1 + D_{12}\hat{D}_k s_k C_2)^T \\ * & * & * & -Y_k & 0 & (C_1 Y_k + D_{12}\mathcal{C}_k)^T \\ * & * & * & * & -\gamma I & (D_{11} + D_{12}\hat{D}_k s_k D_{21})^T \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{23}$$

for $k = 0, 1, \cdots, N-1$, simultaneously, where $*$ denotes the entries which can be known from the symmetry of the matrix.

Remark 3.1: It is clear that under different communication sequences, the optimal achievable H_∞ performance will be different. Note that the optimal H_∞ performance relies on both the communication sequence and the periodic controller Θ_k. Let $\gamma_o(s_k, \Theta_k)$ be the optimal H_∞ performance under a given communication sequence s_k, then the optimal H_∞ performance under scheduled releasing policy can be obtained by the following optimization

$$\begin{array}{ll} \min\limits_{s_k, \Theta_k} & \gamma_o(s_k, \Theta_k) \\ \textit{subject to} & (23). \end{array}$$

Remark 3.2: It should be noted that the above problems are in fact non-convex optimization ones with integer and rank constraints, and are very difficult to settle directly. It may be solved by combining an exhaustive search [4] with the LMI optimization (23). However when the number of the measurements increases, the size of the search tree will grow quickly. To avoid combinatoric explosion, in the following, we shall propose a heuristic search method for a communication sequence which, in conjunction with a convex optimization approach for controller parameters, gives a simple solution to the H_∞ control problems.

Heuristic Search Method

- Form $s_0^o = \{\sigma_1, \sigma_2, \cdots, \sigma_r\}$. If the optimal H_∞ performance γ_0^o under this communication sequence satisfies $\gamma_0^o - \gamma^{opt} < \epsilon$, where ϵ is a pre-specified tolerance and γ^{opt} is the optimal H_∞ performance for the system without communication constraint, i.e., $y_s(k) = y(k)$, then s_0^o is the optimal communication sequence and the period is r. Otherwise, proceed to the next step.
- Step i ($1 \le i < N_m - r$, where N_m is the maximum period which the period of the desired sequence cannot exceed): Assume that the optimal communication sequence obtained in step $i-1$ is s_{i-1}^o and the optimal cost is γ_{i-1}^o. Add an additional sampling $\sigma_j, j = 1, 2, \cdots, r$, to s_{i-1}^o to form a set of r new switching sequences $s_{ij} = \{s_{i-1}^o, \sigma_j\}, j = 1, 2, \cdots, r$ and the period is increased by one. Calculate the H_∞ optimal performance under these r communication sequences by using Theorem 3.1. Assume that the optimal communication sequence among $s_{ij}, j = 1, 2, \cdots, r$, is s_i^o and the optimal cost is γ_i^o.
- Step $i+1$: If $\gamma_i^o - \gamma^{opt} < \epsilon$ or $|\gamma_i^o - \gamma_{i-1}^o| < \epsilon_1$ or $i = N_m - 1 - r$, where ϵ_1 is the pre-specified tolerance, then stop and record the optimal sequence s_i^o and the optimal controller Θ_k^o. Otherwise, let $i = i + 1$ and go back to step i.

Remark 3.3: From the heuristic search method, we can see that for each period we can just consider r switching sequences. This can avoid the combinatoric explosion and thus reduces the computation cost greatly compared to the exhaustive search method, especially when r and the period of the optimal sequence are large. The example in Section IV will show that the heuristic search method is convergent with respect to the period of the communication sequence.

IV. Illustrative Example

Consider a discrete-time linear system described by

$$x(k+1) = \left[\begin{array}{cc|cc} 1.45 & 0.2 & 0 & 0 \\ 0 & 0.4 & 0 & 0.2 \\ \hline 1 & 0.2 & 1.1 & 0.75 \\ 0 & -1 & 0 & 0.4 \end{array}\right] x(k)$$

$$+\left[\begin{array}{c} 0.6 \\ 0 \\ \hline 0.6 \\ 0 \end{array}\right] w(k) + \left[\begin{array}{c} 0 \\ 1 \\ \hline 0 \\ 1 \end{array}\right] u(k) \qquad (24)$$

$$z(k) = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \end{array}\right] x(k) + w(k) + u(k) \qquad (25)$$

$$y(k) = \left[\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \end{array}\right] x(k) + \left[\begin{array}{c} 1 \\ \hline 1 \end{array}\right] w(k) \qquad (26)$$

where $x(k), y(k), z(k), u(k), w(k)$ are the same as those in (1)-(3). It can be easily seen that the eigenvalues of the first subsystem are $\{1.45, 0.4\}$ and the eigenvalues of the second subsystem are $\{1.1, 0.4\}$. The feedback from the sensor to the controller is connected by a network with scheduled releasing policy, which can transmit only one measurement within one sampling period. From the previous section, we know that the switches for this system are

$$\sigma_1 = [1 \quad 0], \quad \sigma_2 = [0 \quad 1].$$

First it can be known that the optimal H_∞ performance without communication limitations is 3.9368. We start from the sequence $\{\sigma_1, \sigma_2\}$ and obtain the optimal H_∞ performance 4.8842. Then by adding the switch σ_1 and the switch σ_2 to the sequence $\{\sigma_1, \sigma_2\}$ respectively, we arrive at the sequences $\{\sigma_1, \sigma_2, \sigma_1\}$ and $\{\sigma_1, \sigma_2, \sigma_2\}$. By Theorem 3.1, we can obtain the H_∞ performance 4.3947 under the sequence $\{\sigma_1, \sigma_2, \sigma_1\}$ which is smaller than that under the sequence $\{\sigma_1, \sigma_2, \sigma_2\}$ as shown in Table I. So the sequence $\{\sigma_1, \sigma_2, \sigma_1\}$ is the resulted sequence at this step. Proceeding the heuristic search we have the following results shown in Table I, in which the sequence in boldface stands for the resulted sequence at each step.

step	*period*	*sequence*	H_∞ *norm*
$i=1$	$N=3$	$\boldsymbol{\{\sigma_1, \sigma_2, \sigma_1\}}$	**4.3947**
$i=2$	$N=4$	$\boldsymbol{\{\sigma_1, \sigma_2, \sigma_1, \sigma_1\}}$	**4.1390**
$i=3$	$N=5$	$\boldsymbol{\{\sigma_1, \sigma_2, \sigma_1, \sigma_1, \sigma_1\}}$	**3.9647**
$i=4$	$N=6$	$\boldsymbol{\{\sigma_1, \sigma_2, \sigma_1, \sigma_1, \sigma_1, \sigma_1\}}$	**3.9368**

step	*period*	*sequence*	H_∞ *norm*
$i=1$	$N=3$	$\{\sigma_1, \sigma_2, \sigma_2\}$	5.6995
$i=2$	$N=4$	$\{\sigma_1, \sigma_2, \sigma_1, \sigma_2\}$	5.1950
$i=3$	$N=5$	$\{\sigma_1, \sigma_2, \sigma_1, \sigma_1, \sigma_2\}$	4.8717
$i=4$	$N=6$	$\{\sigma_1, \sigma_2, \sigma_1, \sigma_1, \sigma_1, \sigma_2\}$	4.6338

TABLE I

THE RESULTS OF USING THE HEURISTIC SEARCH METHOD

From the results shown above, we can see that under different communication sequences with the same period the optimal H_∞ performance are different and the more sampling we allocate for y_1 a better performance is achieved. This may be explained by the fact that the first subsystem is less stable than the second one. As a result, communicating more often with y_1 will lead to better performance. Clearly, when we choose the communication sequence $s^1_{opt} = \{\sigma_1, \sigma_2, \sigma_1, \sigma_1, \sigma_1, \sigma_1\}$, we can achieve the optimal H_∞ performance which are in fact the same as those without limited communication. However, it should be noted that even though the measurement of the second subsystem seems to play a less important role, it cannot be ignored. In fact, there does not exist any controller to stabilize the system by only using the measurement of the first subsystem.

We can also obtain the optimal sequence and the corresponding performance value by using the exhaustive search, which will need 651691 flops in H_∞ control problem. However, in the above heuristic search, only 355796 flops is needed to get the desired sequence. When the number of the outputs of the original system and the periods of the desired sequence become larger, more time will be saved by using the heuristic search method. The convergence of the heuristic search method with respect to the period of the communication sequence for this example is shown in Figure 2.

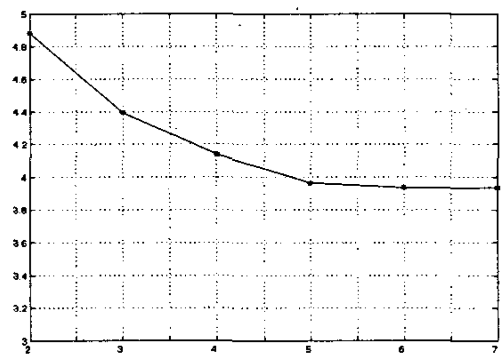

Fig. 2. Convergence of the heuristic search method

In the simulation, we also tried to get different systems by changing the value of the element $A(1,1)$ from 1.3 to 1.45 and use the heuristic search method to deal with these systems. Here we give four typical cases and the results are shown in Table II. It should be noted that the value of H_∞ norm for each case is exactly the same as that obtained when there is no communication constraint. It can be clearly seen from Table II that the period of the optimal communication sequence varies with the characteristic of the system.

$A(1,1)$	*optimal sequence*	H_∞ *norm*	$flops_e$	$flops_h$
1.43	$\{\sigma_1, \sigma_2, \sigma_1, \sigma_1, \sigma_1\}$	3.8559	381642	248268
1.4	$\{\sigma_1, \sigma_2, \sigma_1, \sigma_1\}$	3.7336	189656	151772
1.38	$\{\sigma_1, \sigma_2, \sigma_1\}$	3.6508	75254	75254
1.3	$\{\sigma_1, \sigma_2\}$	3.3106	13924	13924

TABLE II

THE OPTIMAL PERFORMANCE AND SEQUENCE FOR DIFFERENT SYSTEMS: $flops_e$ STANDS FOR THE CASE OF EXHAUSTIVE SEARCH AND $flops_h$ STANDS FOR THE CASE OF HEURISTIC SEARCH

V. CONCLUSION

This paper has investigated the optimal H_∞ control problem for networked control systems. Based on the notion of communication sequence and a direct controller design approach for periodic systems, a solution to the problem is given in terms of a set of LMIs. Given a communication sequence, an explicit expression for the controller is provided in terms of the solution of the LMIs. Then a heuristic search method is provided to obtain the desired sequence under which the H_∞ performance is better than that under other sequence. More efficient algorithm to obtain the optimal communication sequence will be investigated in the future. Moreover, we have assumed that the control u is transmitted to the plant directly. In practice, the controller may also transmit the control signals to the plant via a network. The approach of this paper can be extended to deal with this situation.

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