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A Fully Distributed Approach to Formation Maneuvering Control of Multi-Agent Systems

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***Abstract*— This paper studies the formation maneuvering control problem for a network of agents with the objective of achieving a desired group formation shape and a constant overall group maneuvering velocity. A fully distributed approach is developed to solve the problem. That is, a control law is proposed for each agent in the network, with its parameters capable of being designed in a distributed manner, and is implementable locally via relative sensing and communication with neighbors. Necessary and sufficient conditions regarding a critical control parameter are obtained to guarantee the globally asymptotic convergence of the overall system for both the single-integrator kinematics case and the double-integrator dynamics case.**

I. INTRODUCTION

In many applications, a network of multiple agents holds eminent promises to achieve a level of performance, capability, robustness, and efficiency beyond what a single agent can provide. However, to be advantageous, multiple agents have to work in an organized manner. A basic requirement is to achieve certain desired internal group formation shape and overall group maneuver at the same time.

The paper considers mobile agents in the plane and aims to provide a distributed algorithm to achieve a desired group formation shape and a constant overall group maneuvering velocity. This is referred as the formation maneuvering control problem in the paper. The basis for formation maneuvering control is the formation shape stabilization problem, which has been systematically investigated using various approaches in recent years.

Assuming the existence of a common directional sense for all the agents in a network, a displacement-based control approach is proposed regarding different agent models to achieve formation shape stabilization [1], [2]. However, unlike the displacement-based approach, most works using the distance-based approach are limited to multi-agent systems over undirected graphs. On the other hand, [3] and [4] adopt the idea of combining the alignment of local frames on different agents and the displacement-based control to achieve globally asymptotic stability, which can deal with directed networks. More recently, the work of [5], [6] introduces a new approach based on complex-valued Laplacian for formation shape stabilization, which leads to a linear control law and can address both undirected and directed networks with ensured global stability properties. However, the design of the control law requires a centralized computation using global information of the entire network.

These approaches for formation shape stabilization are also applied to solve the formation maneuvering control problem under a single-leader following network [7] or a co-leader following network [8]–[10]. The basic idea is to decouple the formation shape stabilization task and the velocity synchronization task. However, the deficiency of these approaches is also inherited.

This paper addresses the formation maneuvering control problem for a leaderless network. The main contribution of this paper is a simple, distributed control algorithm that achieves the desired formation shape and a constant group maneuvering velocity. In contrast to the work of [5], [6], [9], [10], the control algorithm in this paper is fully distributed. Not only the implementation but also the design of control are made by the agents themselves in a distributed manner without centralized computation. Also, in contrast to most gradient control laws, the control algorithm in this paper is linear, ensures global stability, and is suitable for directed networks. For a network of agents with either the single-integrator kinematic model or the double-integrator dynamic model, the control algorithm requires the same relative sensing information and communication information. Necessary and sufficient conditions for a critical control parameter in the algorithm are obtained, which guarantee the globally asymptotic convergence to the desired formation shape as well as a constant group maneuvering velocity.

Notation: $\mathbb{C}$ and $\mathbb{R}$ denote the set of complex and real numbers, respectively. $\iota = \sqrt{-1}$ denotes the imaginary unit. $\mathbf{1}_n$ represents the n-dimensional vector of ones and I_n denotes the identity matrix of order n. $\bar{w}_{ij}$ is the conjugate of w_{ij} and L^* is the conjugate transpose of the matrix L.

II. PRELIMINARIES AND PROBLEM SETUP

This section presents some basic notions from graph theory and several preliminary results, and then formulates the problem we study.

A. Basic notions in graphs

A *directed graph* (digraph for short) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a non-empty node set $\mathcal{V} = \{1, 2, \cdots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge of $\mathcal{G}$ is denoted by an ordered pair of nodes (j, i), which means that the edge has tail at node j

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and has head at node i. Alternatively, the edge (j,i) is called an *incoming edge* of node i and an *outgoing edge* of node j. In the paper, we let $\mathcal{N}_i^+$ denote the in-neighbor set of node i, i.e., $\mathcal{N}_i^+ = \{j : (j,i) \in \mathcal{E}\}$, and let $\mathcal{N}_i^-$ denote the out-neighbor set of node i, i.e., $\mathcal{N}_i^- = \{j : (i,j) \in \mathcal{E}\}$. For a directed graph, if both (i,j) and (j,i) are edges of the graph, we call it a *bidirectional graph*. For a bidirectional graph, the in-neighbor set is the same as the out-neighbor set, then we do not distinguish them and just use $\mathcal{N}_i$ to stand for the neighbor set of node i.

For a directed graph $\mathcal{G}$, a node v is said to be *reachable* from another node u if there exists a path from u to v. A directed graph $\mathcal{G}$ is said to be *rooted* if there exists a node, from which every other node is reachable. For a directed graph $\mathcal{G}$, a node v is said to be *2-reachable* from a non-singleton subset of nodes $\{u_1, \dots, u_k\}$ if there exists a path from a node in $\{u_1, \dots, u_k\}$ to v after removing any one node except v. A directed graph $\mathcal{G}$ is said to be *2-rooted* if there exists a subset of two nodes, from which every other node is 2-reachable. These two nodes are called the *roots* of the graph.

For a digraph $\mathcal{G}$, we associate to each edge (j,i) a weight $w_{ij} \neq 0$. Then the Laplacian L of $\mathcal{G}$ is defined as follows.

$$L(i,j) = \begin{cases} -w_{ij} & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i^+ \\ 0 & \text{if } i \neq j \text{ and } j \notin \mathcal{N}_i^+ \\ \sum\limits_{k \in \mathcal{N}_i^+} w_{ik} & \text{if } i = j \end{cases}$$

where $L(i,j)$ is the (i,j)th entry of L. The weights w_{ij}'s can be real or complex numbers, for which the Laplacian is called real-valued Laplacian and complex-valued Laplacian, respectively. Certainly, any Laplacian matrix L satisfies $L\mathbf{1}_n = 0$.

B. Preliminary results on consensus and formations

Consider a directed graph $\mathcal{G}$ of n nodes and suppose that each agent i updates its state $x_i \in \mathbb{R}$ by the following rule

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i^+} a_{ij}(x_j - x_i) \tag{1}$$

where a_{ij}'s are real and positive. Denote by $x = [x_1, \dots, x_n]^\intercal$ the aggregate state and denote by L the Laplacian of $\mathcal{G}$ with weights a_{ij}'s. Then the following is the well-known consensus result.

Lemma 2.1 ([11]): A group of agents governed by (1) asymptotically reaches state consensus if and only if $\mathcal{G}$ is rooted. Moreover,

$$\lim_{t \to \infty} x(t) = \left(\frac{c^\intercal x(0)}{c^\intercal \mathbf{1}_n} \right) \mathbf{1}_n$$

where $c^\intercal$ is an associated left-eigenvector of L for the zero eigenvalue.

Consider a configuration of n points in the plane, denoted by

$$\xi = [\xi_1, \xi_2, \cdots, \xi_n]^\intercal \tag{2}$$

where $\xi_i, \ i = 1, \dots, n$ are complex numbers. Throughout the paper, we use a complex number to represent a state in the plane for simplicity. Then the set

$$F_\xi = \{c_1\mathbf{1}_n + c_2\xi : c_1, c_2 \in \mathbb{C}\}$$

specifies all configurations in the plane, which have the same formation shape as ξ. In other words, every configuration in F_ξ can be transformed from ξ by a composition of rotations, translations and dilations. The next preliminary result from [12] shows that the formation shape similar to ξ can be encoded into a complex-valued Laplacian of a directed graph.

Consider a directed graph $\mathcal{G}$ and suppose that complex weights on edge (j,i)'s of $\mathcal{G}$, denoted by w_{ij}'s, satisfy

$$\sum_{j \in \mathcal{N}_i^+} w_{ij}(\xi_j - \xi_i) = 0 \text{ for } i = 1, \dots, n. \tag{3}$$

Let L be the complex-valued Laplacian of $\mathcal{G}$ with these complex weights.

Lemma 2.2 ([12]): $\ker(L) = F_\xi$ almost surely if and only if $\mathcal{G}$ is 2-rooted.

Remark 2.1: The linear constraint (3) also provides a distributed approach to calculate the weights w_{ij}'s that encode the information of the target formation shape. More specifically, for each agent i, it knows its own ξ_i and its in-neighbors ξ_j's, $j \in \mathcal{N}_i^+$ and thus it is able to compute w_{ij}'s from the linear equation (3). There are multiple solutions for w_{ij}'s, but agent i can just randomly pick one.

C. Problem setup

This paper considers the formation maneuvering control problem for a group of n agents in the plane. The agents are with either a single-integrator kinematic model or a double-integrator dynamic model. The position of each agent i, $i = 1, \dots, n$, is denoted by a complex variable z_i.

Suppose each agent i with a local reference frame can sense the relative position of their neighbors (namely, $z_j - z_i$). More practically, mutual sensing may not be possible. Thus, we use a directed graph $\mathcal{G}$ of n nodes to represent the *sensing graph*, for which each node represents an agent and an edge (j,i) indicates the availability of relative position measurement $(z_j - z_i)$ by agent i. We use $\mathcal{N}_i^+$ and $\mathcal{N}_i^-$ to denote the in-neighbor set and out-neighbor set of agent i in the sensing graph $\mathcal{G}$. To make our problem solvable, we make the following assumption for the sensing graph $\mathcal{G}$.

Assumption 2.1: The sensing graph $\mathcal{G}$ is 2-rooted.

Moreover, each agent is supposed to be able to communicate with its communication neighbors that may be different from its sensing neighbors. We use another directed graph $\mathcal{H}$ of n nodes to represent the *communication graph*, for which each node represents an agent and an edge (j,i) indicates that agent j can communicate to agent i. Usually, communication is bidirectional, meaning that if agent i can communicate to agent j, then agent j can also communicate to agent i. Moreover, the communication range is often greater than the sensing range in practice. Therefore, we make the following assumption for the communication graph.

Assumption 2.2: The communication graph $\mathcal{H}$ is bidirectional and rooted. Moreover, the communication graph $\mathcal{H}$ contains the sensing graph $\mathcal{G}$ as a subgraph.

In this paper we use $\mathcal{M}_i$ to denote the neighbor set of agent i in the communication graph $\mathcal{H}$ as a bidirectional graph has the same in-neighbor and out-neighbor set.

In many applications, a basic requirement for multi-agent systems is to achieve certain desired internal group formation shape and overall group maneuver with certain velocity at the same time. We call it the *formation maneuvering control* problem. In this paper, we use a configuration ξ as defined in (2) to represent a desired formation shape, for which $\xi_i \neq \xi_j$ for $i \neq j$. As discussed in Subsection II-B, a configuration in the set $F_\xi = \{c_1\mathbf{1}_n + c_2\xi : c_1, c_2 \in \mathbb{C}\}$ preserves the formation shape. Thus, the control objective of formation maneuvering can be formally stated as to make

$$\lim_{t\to\infty} z(t) = c_1\mathbf{1}_n + c_2\xi + v_s t\mathbf{1}_n \tag{4}$$

for some $c_1, c_2 \in \mathbb{C}$ and $v_s \in \mathbb{C}$, where v_s here represents the group maneuvering velocity. When (4) is satisfied, we say a network of agents achieves the desired formation shape ξ and a constant maneuvering velocity.

III. Main Results

This section develops distributed control strategies for a network of single-integrator kinematic agents or double-integrator dynamic agents to achieve the desired formation shape ξ and a constant maneuvering velocity. Rigorous analysis on the convergence will also be given.

A. Formation maneuvering of single-integrator kinematic agents

Consider a network of agents labeled from 1 to n. Suppose that each agent is governed by a single-integrator kinematic model

$$\dot{z}_i = u_i, \tag{5}$$

where $z_i \in \mathbb{C}$ represents the position of agent i in the plane and $u_i \in \mathbb{C}$ represents the velocity control input.

The following control law is proposed for a network of single-integrator agents to achieve the desired formation shape and a constant group maneuvering velocity.

$$\begin{cases} \dot{\zeta}_i = -a\zeta_i - \sum\limits_{j\in\mathcal{N}_i^+} w_{ij}(z_j - z_i), \\ \dot{\eta}_i = \sum\limits_{j\in\mathcal{M}_i} \alpha_{ij}(\eta_j - \eta_i), \\ u_i = \eta_i - \sum\limits_{j\in\mathcal{N}_i^+} \bar{w}_{ij}\zeta_i + \sum\limits_{j\in\mathcal{N}_i^-} \bar{w}_{ji}\zeta_j, \end{cases} \tag{6}$$

where $\zeta_i \in \mathbb{C}$ and $\eta_i \in \mathbb{C}$ are auxiliary states, w_{ij} is a complex weight associated to edge (j, i) in the sensing graph $\mathcal{G}$, which can be designed in a distributed manner for a given target shape as discussed in Remark 2.1, $\bar{w}_{ji}$ is the conjugate of w_{ji}, α_{ij} can be any positive real number picked by agent i itself, and $a \in \mathbb{R}$ is a parameter to be designed.

Remark 3.1: The control law (6) requires the following relative sensing information by agent i:

- $(z_j - z_i)$ of all in-neighbors in the sensing graph $\mathcal{G}$,

and it requires the following information via communications:

- the auxiliary state η_j from all communication neighbors,
- the auxiliary information $\bar{w}_{ji}\zeta_j$ from all out-neighbors in the sensing graph $\mathcal{G}$.

Note that the whole piece of information $\bar{w}_{ji}\zeta_j$ is known by agent j and also that the sensing graph $\mathcal{G}$ is contained in the communication graph $\mathcal{H}$ as assumed in Assumption 2.2, so both the required information can be available to i via communications. Therefore, the control law (6) can be implemented in a distributed manner.

Denote $z = [z_1, z_2, \dots, z_n]^\intercal$, $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_n]^\intercal$ and $\eta = [\eta_1, \eta_2, \dots, \eta_n]^\intercal$. With the distributed control law (6), the overall closed-loop system can be described as

$$\begin{bmatrix} \dot{z} \\ \dot{\zeta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & -L^* & I_n \\ L & -aI_n & 0 \\ 0 & 0 & -H \end{bmatrix} \begin{bmatrix} z \\ \zeta \\ \eta \end{bmatrix} \tag{7}$$

where L is the complex-valued Laplacian of $\mathcal{G}$ with weights w_{ij}'s, L^* is the conjugate transpose of L, and H is the real-valued Laplacian of $\mathcal{H}$ with weights α_{ij}'s.

Remark 3.2: The motivation of introducing the auxiliary state ζ is to make the stability of the system (7) irrelevant to the complex-valued Laplacian L, which encodes the target formation shape, though in general, the eigenvalues of L can be everywhere in the complex plane. On the other hand, the purpose of including the auxiliary state η is to synchronize the maneuvering velocity. The initial values of the auxiliary states ζ and η can be arbitrary.

Next, we present our main result for formation maneuvering control of single-integrator agents.

Theorem 3.1: A network of agents achieves the desired formation shape ξ and a constant maneuvering velocity under the local control law (6) if and only if $a > 0$.

Proof. Denote

$$A = \begin{bmatrix} 0 & -L^* \\ L & -aI_n \end{bmatrix} \text{ and } B = \begin{bmatrix} I_n \\ 0 \end{bmatrix}.$$

Then the overall closed-loop system can be treated as a cascade system as shown in Fig. 1. The subsystem $\mathcal{A}_s$ is

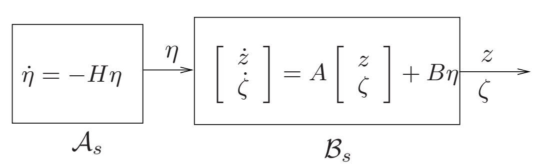

Fig. 1. A cascade system representation.

an autonomous system with a state η, while the subsystem $\mathcal{B}_s$ is a non-autonomous system with a state (z, ζ) and an external input η.

For subsystem $\mathcal{A}_s$, it is govern by a simple first-order consensus control law. Since the communication graph $\mathcal{H}$ is connected by Assumption 2.2, by Lemma 2.1, the state η of subsystem $\mathcal{A}_s$ asymptotically reaches consensus, that is,

$$\lim_{t\to\infty} \eta(t) = v_s\mathbf{1}_n$$

where $v_s = \frac{c^{\intercal}\eta(0)}{c^{\intercal}\mathbf{1}_n}$ with $c^{\intercal}$ being a left-eigenvector of H corresponding to the zero eigenvalue.

Define $y := z - v_s t\mathbf{1}_n$ and $\delta := \eta - v_s\mathbf{1}_n$. Then the cascade system given in Fig. 1 can be transformed to the one in Fig. 2, for which the subsystem $\mathcal{B}_s'$ has an external input δ. The external input δ vanishes asymptotically according to what we show above. So it remains to analyze asymptotic

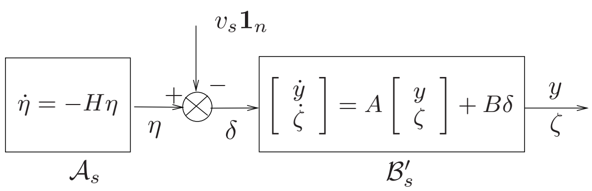

Fig. 2. A transformed cascade system representation.

behaviors of subsystem $\mathcal{B}_s'$ with zero external input, that is,

$$\begin{bmatrix} \dot{y} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} 0 & -L^* \\ L & -aI_n \end{bmatrix} \begin{bmatrix} y \\ \zeta \end{bmatrix}. \tag{8}$$

It can be simply verified that

$$\{(y,\zeta) : y = c_1\mathbf{1}_n + c_2\xi \text{ for } c_1, c_2 \in \mathbb{C} \text{ and } \zeta = 0\}$$

is the equilibrium subspace of system (8). Next, let's check the eigenvalues of the system matrix A of system (8).

Let λ be an eigenvalue of A and let $\begin{bmatrix} \omega \\ \varpi \end{bmatrix}$ be its associated eigenvector, where $\omega, \varpi \in \mathbb{C}^n$. Then we have

$$\left(\lambda I_{2n} - \begin{bmatrix} 0 & -L^* \\ L & -aI_n \end{bmatrix}\right) \begin{bmatrix} \omega \\ \varpi \end{bmatrix} = 0$$

or equivalently, we have

$$\left(\begin{bmatrix} \lambda I_n & 0 \end{bmatrix} - \begin{bmatrix} 0 & -L^* \end{bmatrix}\right) \begin{bmatrix} \omega \\ \varpi \end{bmatrix} = 0 \tag{9}$$

and

$$\left(\begin{bmatrix} 0 & \lambda I_n \end{bmatrix} - \begin{bmatrix} L & -aI_n \end{bmatrix}\right) \begin{bmatrix} \omega \\ \varpi \end{bmatrix} = 0. \tag{10}$$

By mathematical manipulations for eqs. (9)-(10), ϖ can be eliminated and the following is obtained $-L^*L\omega = \lambda(\lambda + a)\omega$, which means, $\lambda(\lambda+a)$ is an eigenvalue of $-L^*L$ with ω being its associated eigenvector. In other words, let σ_i be an eigenvalue of the matrix L^*L. Then the roots of the polynomial equation

$$\lambda^2 + a\lambda + \sigma_i = 0,\ i = 1, \ldots, n \tag{11}$$

are the eigenvalues of A. Note that the roots of the polynomial equation (11) have the following explicit formula.

$$\lambda = \frac{-a \pm \sqrt{a - 4\sigma_i}}{2},\ i = 1, \ldots, n. \tag{12}$$

The matrix L^*L is positive semi-definite with its rank the same as L. So it has two zero eigenvalues and all other eigenvalues are positive and real. Thus, from (12), if and only if $a > 0$, the eigenvalues of A lies in the open left complex plane except two zero eigenvalues. Therefore, system (8) is asymptotically stable with respect to its equilibrium subspace.

This means, $z(t)$ of system (7) asymptotically converges to $(c_1 + v_s t)\mathbf{1}_n + c_2\xi$, which is equivalently to say, a network of agents achieves the desired formation shape ξ and a constant maneuvering velocity v_s. ■

Remark 3.3: Theorem 3.1 shows that the distributed control law (6) solves the formation maneuvering control problem for single-integrator agents if all the agents adopt a same positive parameter a. As discussed immediately after the control law (6), all other control parameters in (6) can be picked by agent i itself except the parameter a. In order to design the parameter a in a distributed way, every agent can propose a positive value for a and then runs a consensus algorithm to reach a common value.

B. Formation maneuvering of double-integrator dynamic agents

Consider a network of agents labeled from 1 to n. Suppose each agent is governed by a double integrator dynamic model

$$\begin{cases} \dot{z}_i = v_i \\ \dot{v}_i = a_i \end{cases}, \tag{13}$$

where the position $z_i \in \mathbb{C}$ and the velocity $v_i \in \mathbb{C}$ are the states of the system and the acceleration $a_i \in \mathbb{C}$ is the control input.

The following control law is proposed for a network of double-integrator agents to achieve a desired formation shape and a constant group maneuvering velocity.

$$\begin{cases} \dot{\zeta}_i = -a\zeta_i - \sum_{j\in\mathcal{N}_i^+} w_{ij}(z_j - z_i), \\ \dot{\eta}_i = \sum_{j\in\mathcal{M}_i} \alpha_{ij}(\eta_j - \eta_i), \\ a_i = \eta_i - v_i - \sum_{j\in\mathcal{N}_i^+} \bar{w}_{ij}\zeta_i + \sum_{j\in\mathcal{N}_i^-} \bar{w}_{ji}\zeta_j, \end{cases} \tag{14}$$

where $\zeta_i \in \mathbb{C}$ and $\eta_i \in \mathbb{C}$ are auxiliary states, w_{ij} is a complex weight associated to edge (j,i) in the sensing graph $\mathcal{G}$, which can be designed in a distributed manner for a given target shape as discussed in Remark 2.1, $\bar{w}_{ji}$ is the conjugate of w_{ji}, α_{ij} can be any positive real number picked by agent i itself, and $a \in \mathbb{R}$ is a parameter to be designed.

Compared to the control law (6) for single-integrator agents, the control law (14) for double-integrator agents only has one extra damping term $-v_i$, which is available by agent i itself. Thus, the required sensing information and communication information are the same as the ones described in Remark 3.1.

Denote $z = [z_1, z_2, \ldots, z_n]^{\intercal}$, $v = [v_1, v_2, \ldots, v_n]^{\intercal}$, $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_n]^{\intercal}$ and $\eta = [\eta_1, \eta_2, \ldots, \eta_n]^{\intercal}$. Then the overall closed-loop system can be described as

$$\begin{bmatrix} \dot{z} \\ \dot{v} \\ \dot{\zeta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & I_n & 0 & 0 \\ 0 & -I_n & -L^* & I_n \\ L & 0 & -aI_n & 0 \\ 0 & 0 & 0 & -H \end{bmatrix} \begin{bmatrix} z \\ v \\ \zeta \\ \eta \end{bmatrix} \tag{15}$$

where L is the complex-valued Laplacian of $\mathcal{G}$ with weights w_{ij}'s, L^* is the conjugate transpose of L, and H is the real-valued Laplacian of $\mathcal{H}$ with weights α_{ij}'s.

Next, we present our main result for formation maneuvering control of double-integrator agents.

Theorem 3.2: A network of agents achieves the desired formation shape ξ and a constant maneuvering velocity under the local control law (14) if and only if

$$a > \frac{\sqrt{1+4\lambda_{max}(L^*L)}-1}{2}. \quad (16)$$

Proof: Similar to the single-integrator case, the overall closed-loop system (15) of double-integrator agents can also be considered as a cascade system and transformed into following form,

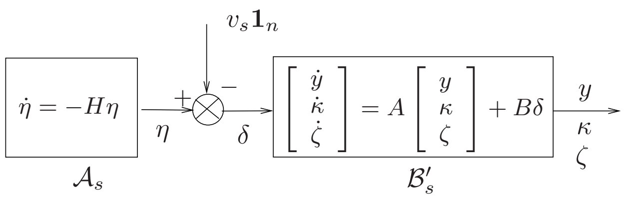

Fig. 3. A transformed cascade system representation.

where $y := z - v_s t\mathbf{1}_n$, $\kappa := v - v_s\mathbf{1}_n$ and $\delta := \eta - v_s\mathbf{1}_n$,

$$A = \begin{bmatrix} 0 & I_n & 0 \\ 0 & -I_n & -L^* \\ L & 0 & -aI_n \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ I_n \\ 0 \end{bmatrix}.$$

As the external input δ in Fig. 3 asymptotically vanishes, we only need to analyze asymptotic behaviors of subsystem $\mathcal{B}'_s$ with zero external input, that is,

$$\begin{bmatrix}\dot{y}\\ \dot{\kappa}\\ \dot{\zeta}\end{bmatrix} = \begin{bmatrix} 0 & I_n & 0 \\ 0 & -I_n & -L^* \\ L & 0 & -aI_n \end{bmatrix}\begin{bmatrix}y\\ \kappa\\ \zeta\end{bmatrix}. \quad (17)$$

It can be simply verified that

$$\{(y,\kappa,\zeta) : y = c_1\mathbf{1}_n + c_2\xi \text{ for } c_1, c_2 \in \mathbb{C}, \kappa = 0 \text{ and } \zeta = 0\}$$

is the equilibrium subspace of system (17).

With the similar mathematical manipulations, by the Hurwitz stability criteria, we can conclude that the system matrix A has at most two zero eigenvalues if and only if

$$a > \frac{\sqrt{1+4\lambda_{max}(L^*L)}-1}{2},$$

which means system (17) is asymptotically stable with respect to its equilibrium subspace

$$\{(y,\kappa,\zeta) : y = c_1\mathbf{1}_n + c_2\xi \text{ for } c_1, c_2 \in \mathbb{C}, \kappa = 0 \text{ and } \zeta = 0\}.$$

In other words, $z(t)$ of system (15) asymptotically converges to $(c_1+v_s t)\mathbf{1}_n + c_2\xi$, which is equivalently to say, a network of agents achieves the desired formation shape ξ and a constant maneuvering velocity v_s. ■

Remark 3.4: As stated in Theorem 3.2, in order to solve the formation maneuvering control problem for double-integrator agents, the parameter a needs to satisfy the condition (16). Note that L in the condition (16) is a piece of global information that is not known locally by every agent. However, since L^*L is symmetric, its maximum eigenvalue can be solved in a distributed way by a distributed power iteration algorithm as described in [13] or by some other distributed techniques for eigenvalue estimation such as that in [14] and [15]. By knowing $\lambda_{max}(L^*L)$ with a distributed algorithm, then a common parameter a can be found to satisfy (16) in a distributed way.

IV. SIMULATION

In this section, we present several simulations to validate our proposed control laws and theoretic results.

We consider a multi-agent system consisting of six agents. Suppose the sensing graph $\mathcal{G}$ is shown in Fig. 4(a). The sensing graph satisfies Assumption 2.1. In other words, the graph $\mathcal{G}$ is 2-rooted and it has two roots (e.g., 1 and 2). The communication graph $\mathcal{H}$ is shown in Fig. 4(b), which satisfies Assumption 2.2.

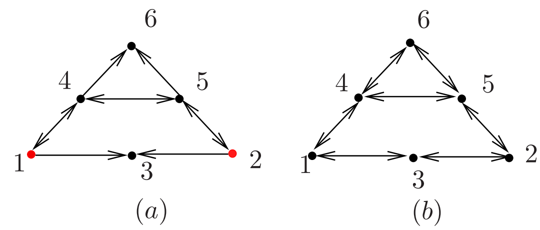

Fig. 4. Sensing graph $\mathcal{G}$ and Communication graph $\mathcal{H}$.

The desired formation shape is a triangle and $\xi = \begin{bmatrix} -2-1\iota & 2-1\iota & -1\iota & -1 & 1 & 1\iota \end{bmatrix}^{\mathsf{T}}$ is a representative configuration of the desired shape. The complex weights w_{ij}'s are solved from (3).

A. Simulation of single-integrator agents

For the single-integrator case, the distributed control law (6) is used with $a = 5 > 0$. A simulation result is shown in Fig. 5 with an arbitrarily initial state. Fig. 5(a) plots the simulation trajectories of the six agents from the initial state marked with small circles, from which it can be seen that they asymptotically converge to the desired formation shape and achieve a constant maneuvering velocity. Fig. 5(b) plots the evolution of the components of Lz. As we can see, all the components converge to zero, which also means the aggregate state of the six agents approaches the null space of L (or equivalently, they reach the desired formation shape).

B. Simulation of double-integrator agents

For the double-integrator case, the distributed control law (14) is used. For L given above, the right-hand side of eq. (16) (namely, the lower-bound of a) is 5.532. We choose $a = 10 > 5.532$ in the simulation in order to satisfy the condition in Theorem 3.2. With this parameter, a simulation result is shown in Fig. 6 with a randomly selected initial state. Fig. 6(a) plots the simulation trajectories of the six agents who asymptotically converge to the formation shape and achieve a constant maneuvering velocity. The same as Fig. 5(b), Fig. 6(b) plots the evolution of the components of Lz. From the simulation result, the same conclusion can be drawn as for the single-integrator case.

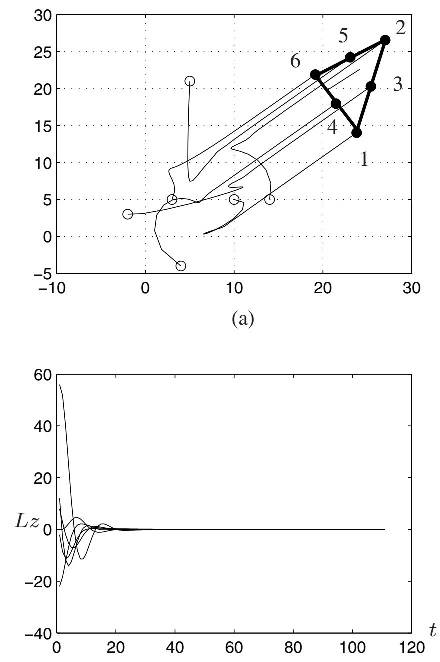

Fig. 5. Asymptotically achieves the desired triangle formation shape and a constant maneuvering velocity under the control law (6) with $a = 5$.

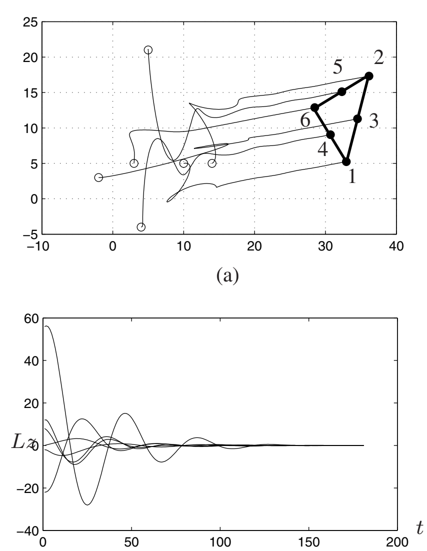

Fig. 6. Asymptotically achieves the desired triangle formation shape and a constant maneuvering velocity under the control law (14) with $a = 10$.

V. CONCLUSIONS

This paper presents a fully distributed approach to solve the formation maneuvering control problem for a multi-agent system. It is assumed that each agent in a network has two ways to access information: one by relative sensing and the other by communication. Several practical assumptions are made for the sensing graph and the communication graph, which might be of different topology. Then distributed control laws are proposed for formation maneuvering control of agents with either the single-integrator kinematic model or the double-integrator dynamic model. The proposed control laws not only are implemented in a distributed manner but also are designed in a distributed way. It is shown that a control parameter plays a key in guaranteeing globally asymptotical convergence to the desired formation shape while achieving a constant maneuvering velocity. Necessary and sufficient conditions are obtained regarding the unique control parameter for both the single-integrator case and the double-integrator case.

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