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New Results on Node Localizable Conditions for Sensor Networks

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***Abstract*— This paper studies the problem of localizable conditions for sensor nodes in a two-dimensional sensor network. A new group of conditions are proposed which judge each node's localizability through analyzing its connections with not only a set of already localized nodes but also its other neighbors. Our new conditions allow up to four nodes to be localized at a time. This algorithm offers a substantial improvement in localizability over the well-known *trilateration* method and *convex hull* method. We show that a newly developed *WHEEL extension* condition is a special case of our conditions. We also demonstrate that our result can be used as a guideline to modify topology of a network for better localizability.**

I. INTRODUCTION

Sensor node localization is a fundamental problem for applications of sensor networks, including wildlife tracking, ocean monitoring, intelligent manufacturing, information encryption and carbon pollution reduction.

There are two common approaches to localization. The first one is based on distance (or range) information, i.e., relative distance measurements between sensor nodes, whereas the second one employs other information including angular measurements, hop-count and other positioning measurements. In this paper, we only consider distance-based two-dimensional (2D) localization problems.

Known localization schemes can only compute the locations of nodes that are "easily localizable". There are two representative schemes. First is the well-known *trilateration* scheme which requires each "new" node to have distance measurements with at least three anchor nodes [3]. This approach localizes nodes fitting this condition one by one. The other scheme is called the convex hull method (also known as the DILOC algorithm) which requires each node to lie inside a *convex hull* by at least three anchor nodes [1]. The location of each node can be computed iteratively by these three anchors. This approach can localize all nodes concurrently, whereas the trialteration approach is sequential. But the main drawback of the DILOC algorithm is that nodes outside the convex hull of the anchor nodes are unable to be localized.

The 3-connected condition is used for localization in the trilaiteration method. This condition is limited because typically only a small part of the nodes in a network satisfies the 3-connected condition. Often the time, several "new" nodes may be jointly localized by anchor nodes. Therefore, we want to relax the 3-connected condition and develop a way to localize multiple nodes at a time.

Besides the 3-connected condition, many existing works on localization focus on localizing a whole network (see, e.g., [4]). The drawback of the localization conditions for a whole network is that, once the network is not fully localizable, they cannot be used to find localizable nodes in the network. Practically, a randomly deployed sensor network is hardly fully localizable [5], and the task of localization is to find the set of localizable nodes.

In this paper, we will seek for ways to relax the localizable condition to allow several nodes to be jointly localized at a time. Localizable conditions will be provided by analyzing the rigidity properties of the combined set of nodes.

II. PROBLEM STATEMENT AND RELATED WORK

The localization problem is often characterized through rigidity conditions of the sensor network graph. A sensor network graph is formed by the set of sensor nodes V and the available distance measurements between nodes (i.e., the edges E), and the graph is denoted by $G(V,E)$. We assume that $G(V,E)$ is a connected graph. A graph is rigid if it cannot be continuously deformed without changing the distances. A graph is called redundantly rigid if it is still rigid after removing any one edge. A graph is called 3-connected if it is still connected after removal of any two nodes. A graph is called globally rigid if there is a single realization with the given distance constraints. A globally rigid graph is called minimally globally rigid if it is no longer globally rigid after removing any edge. It is well known that a graph is globally rigid if and only if it is 3-connected and redundantly rigid [6]. Throughout the paper, we consider the so-called *generic* properties of graphs [2], which mean none of any three nodes in the graph is co-linear. This graph is generically globally rigid because the co-linear event is a zero-measure event. For notational simplicity, we will not mention the term "generic" explicitly.

It is obvious that a 2D graph is localizable if and only if the graph is globally rigid and there are at least three anchor nodes. Given this, the sequential localization problem we study in this paper is the following: *Given an anchor graph* $\mathbf{G_1} = (V_1, E_1)$ *(with known locations for all nodes in it) and a connected graph* $\mathbf{G_2} = (V_2, E_2)$*, we want to know the conditions on the connections (i.e. the set of connecting edges* E_3*) between the two graphs such that the concatenated graph* $\mathbf{G} = (V_1 \cup V_2, E_1 \cup E_2 \cup E_3)$ *is globally rigid.* In particular, we

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want to know how many connecting edges are needed and how they are connected.

It is clear that not all localizable nodes can be detected by using the above sequential method. The main advantage of the above method is that the computational load is relatively low. In practice, if a sensor network cannot be made localizable using the above method, it can be done so by adding a number of distance measurements (edges), and this number is typically small, we will show this fact via an example later.

Obviously, our sequential algorithm differs from the known trilateration method in the sense that several nodes are jointly localized each time. To demonstrate the need for this, we can have a look at the graph in Fig. 1(b) where nodes in $\mathbf{G_1}$ are anchors and nodes in $\mathbf{G_2}$ need to be localized. Nodes in $\mathbf{G_2}$ have an internal edge and they also have 4 connections with the anchor nodes. It is clear the new nodes are not 3-connected to nodes in $\mathbf{G_1}$ but they are still localizable jointly (see Theorem 1 later).

Our work can also be viewed as checking the localizability of two merging graphs. In [8], *Yu et al.* studied how to construct connecting edges between two set of nodes in order to get a globally rigid graph after merging, which is important for formation control in multi-agent systems. But they require both sets to be globally rigid. In this paper, we consider one graph of anchor nodes and one arbitrary graph to be merged, with no guarantee on global rigidity or even rigidity. Also, no anchor node appears in the graph to be merged.

III. LOCALIZABILITY CONDITIONS

This section provides the localizability conditions for a 2D concatenated graph formed by an anchor graph $\mathbf{G_1}$ and a merging graph $\mathbf{G_2}$. Since all the nodes in $\mathbf{G_1}$ are anchored, we can assume that $\mathbf{G_1}$ is globally rigid. The merging graph $\mathbf{G_2}$ is assumed to be a connected graph. We want to determine the number of necessary connecting edges and the connection patterns such that the concatenated graph $\mathbf{G}$ is also an anchored graph.

Since localization is closely related to global rigidity, we first give a necessary condition on the number of edges for a 2D graph to be globally rigid.

Lemma 1: For a graph $\mathbf{G}$ with n nodes to be globally rigid, it is necessary that $\mathbf{G}$ has at least $2n-2$ edges.

Proof: This follows directly from the definition of a global rigidity. Since removing any edge, the remaining graph $\mathbf{G}^-$ is still rigid, it implies that $\mathbf{G}^-$ has at least $2n-3$ edges due to the facts that each node has two degrees of freedom, that a rigid graph is still free to do rotation as well as vertical and horizontal translations which account for three degrees of freedom, and that each edge constraints one degree of freedom. That is, $\mathbf{G}^-$ has at least $2n-3$ edges. Hence, $\mathbf{G}$ has at least $2n-2$ edges. ■

This leads to the next result which states a necessary condition on the number of connecting edges for a concatenated graph to be globally rigid.

Corollary 1: Given two graphs $\mathbf{G_1}$, containing m nodes and p edges inside, and $\mathbf{G_2}$, containing n nodes and q edges, if their concatenated graph $\mathbf{G}$ is globally rigid, then there must be at least $2(m+n)-(p+q)-2$ edges connecting $\mathbf{G_1}$ and $\mathbf{G_2}$.

The following result characterizes the localizability conditions for cases of $|V_2| \leq 4$ (where for any finite set X, $|X|$ denotes its cardinal number).

Theorem 1: Given an anchor graph $\mathbf{G_1} = (V_1, E_1)$ with $|V_1| \geq 3$ and a connected merging graph $\mathbf{G_2} = (V_2, E_2)$ for $|V_2| \leq 4$, $\mathbf{G_2}$ is localized via a set of connecting edges E_3 with $\mathbf{G_1}$ with a minimum $|E_3|$ if and only if E_3 is such that the concatenated graph $\mathbf{G} = (V_1 \cup V_2, E_1 \cup E_2 \cup E_3)$ is 3-connected and the following condition is satisfied:

1) If $|V_2| = 1$, then $\min |E_3| = 3$;
2) If $|V_2| = 2$ and $|E_2| = 1$, then $\min |E_3| = 4$;
3) If $|V_2| = 3$ and $|E_2| = 2$, then $\min |E_3| = 5$;
4) If $|V_2| = 3$ and $|E_2| = 3$, then $\min |E_3| = 4$;
5) If $|V_2| = 4$ and $|E_2| = 3$, then $\min |E_3| = 6$;
6) If $|V_2| = 4$ and $|E_2| = 4$, then $\min |E_3| = 5$;
7) If $|V_2| = 4$ and $|E_2| = 5$ or 6, then $\min |E_3| = 4$.

The connection patterns satisfying the above conditions are all listed in Fig. 1.

Proof: Since $\mathbf{G_1}$ is an anchor graph, $\mathbf{G_2}$ is localized via E_3 if and only if $\mathbf{G}$ is globally rigid. Hence, it is necessary for $\mathbf{G}$ to be 3-connected. It remains to show that the additional condition on $|E_3|$ as itemized above is the minimum for guaranteeing that $\mathbf{G}$ is redundantly rigid, and that the connection patterns in Fig. 1 are all the patterns satisfying these conditions.

(Necessity) Above conditions on $|E_3|$ is easily to be checked as the minimum number to guarantee the redundantly rigidity of $\mathbf{G}$ and thus omitted here. To see the detail of proof, please check a full version of this work [11].

(Sufficiency) For each case of $|V_2|$, $|E_2|$ and $|E_3|$ as itemized in the theorem, we need to show that there exists at least one connection pattern satisfying the 3-connected condition and redundant rigidity condition, and then list all such patterns.

When $|V_2| = 1$ and $|E_3 = 3|$, the connection pattern in Fig. 1(a) is obvious.

When $|V_2| = 2$, $|E_2| = 1$ and $|E_3| = 4$, each node must have two connecting edges. The connection pattern is in Fig. 1(b) with either 3 or 4 connecting nodes in $\mathbf{G_1}$. To show the 3-connectedness for $\mathbf{G}$, we note that if both nodes in $\mathbf{G_2}$ are removed, the remaining graph, which is $\mathbf{G_1}$, is connected; and if one node in $\mathbf{G_2}$ and one node in $\mathbf{G_1}$ are removed, the remaining node in $\mathbf{G_2}$ is still connected to $\mathbf{G_1}$. To see redundant rigidity, if one connecting edge is removed, the node in $\mathbf{G_2}$ not connected to this edge is rigid (because of two connections to $\mathbf{G_1}$), and once this node is rigid, the other node is also rigid because of its connection to the first node and to $\mathbf{G_1}$.

When $|V_2| = 3$, $|E_2| = 2$ and $|E_3| = 5$, $\mathbf{G_2}$ is a chain, so each of the end nodes must have 2 connecting edges and the middle node must have one connecting edge. The connection pattern is in Fig. 1(c) with 3, 4 or 5 connecting nodes in $\mathbf{G_1}$.

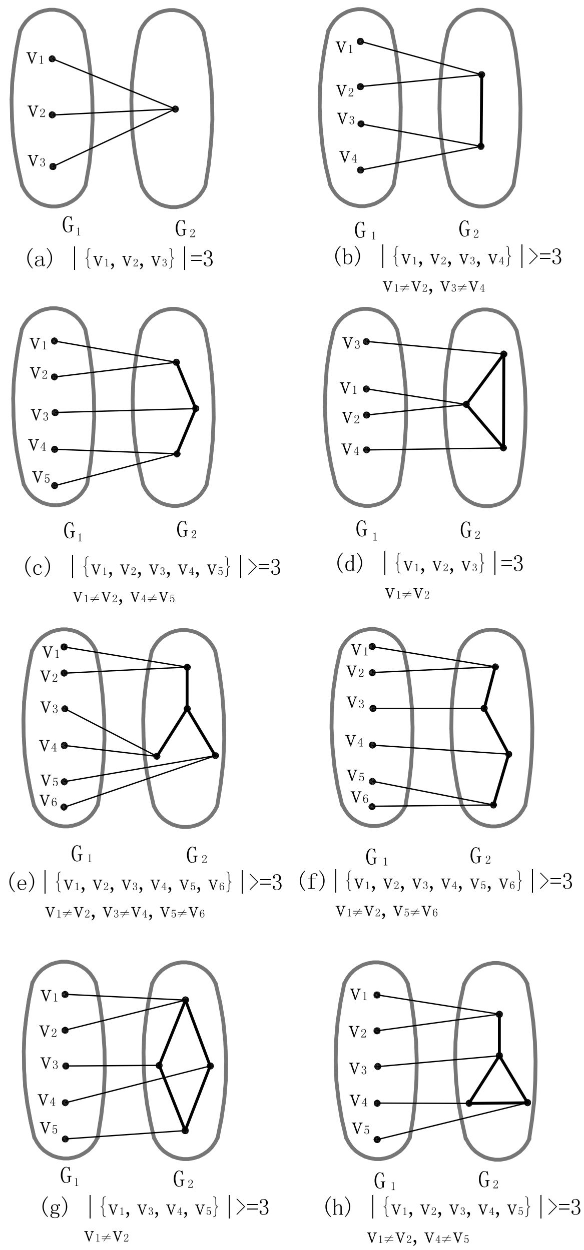

Fig. 1. Feasible Connection Patterns for Localization

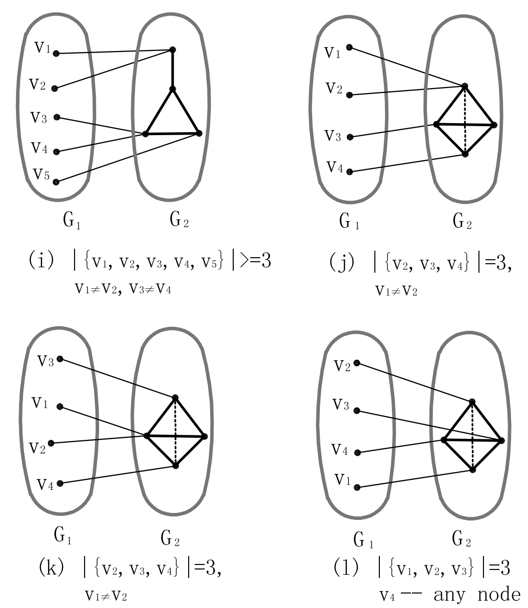

Fig. 1. Feasible Connection Patterns for Localization (con't)

The 3-connectedness and redundant rigidity of **G** are similar as above case and thus omitted here.

When $|V_2| = 3$, $|E_2| = 3$ and $|E_3| = 4$, $\mathbf{G_2}$ forms a triangle. It is thus necessary that each node in $\mathbf{G_2}$ has a connecting edge and one node has an extra connecting edge. There are either 3 or 4 connecting nodes in $\mathbf{G_1}$. To ensure 3-connectedness, it is necessary to avoid the situation that one node (say $node_a$) in $\mathbf{G_2}$ is connected to two nodes in $\mathbf{G_1}$ and the other two nodes in $\mathbf{G_2}$ is connected to a single node (say $node_b$) in $\mathbf{G_1}$ because, in this case, by removing $node_a$ and $node_b$ would disconnect **G**. This leaves the only possible connection pattern shown in Fig. 1(d). For this pattern, the 3-connectedness condition can be visually inspected.

When $|V_2| = 4$, $|E_2| = 3$ and $|E_3| = 6$, $\mathbf{G_2}$ is either a chain or a tree, as shown in Fig. 1(e)-(f). For the chain, each of the end nodes must have 2 connecting edges each and the middle nodes each one connecting edge each. For the tree, each of the end nodes must have 2 connecting edges each. The possible connection patterns are shown in Fig. 1(e)-(f), with the requirement that there are at least 3 connecting nodes in $\mathbf{G_1}$. The 3-connectedness and redundant rigidity conditions are easily checked as before and thus omitted.

When $|V_2| = 4$, $|E_2| = 4$ and $|E_3| = 5$, $\mathbf{G_2}$ is either a loop (Case A) or a triangle connected to an end node (Case B). For case A as shown in Fig. 1(g), each node in $\mathbf{G_2}$ must have a connecting edge, and the fifth connecting edge is connected to any node, e.g., the top node in Fig. 1(g). To guarantee the 3-connectedness condition, we must have $|v_1, v_3, v_4, v_5| \geq 3$. For Case B as shown in Fig. 1(h)-(i), the end node need 2 connecting edges and two nodes with only two edges need one connecting edge each, and the remaining connecting edge needs to be connected to any node in the triangle, as the connecting edge with v_3. To guarantee the 3-connectedness condition in Fig. 1(h), we must have $|v_1, v_2, v_3, v_4, v_5| \geq 3$, $v_1 \neq v_2$ and $v_4 \neq v_5$. To guarantee the 3-connectedness condition in Fig. 1(i), we must have $|v_1, v_2, v_3, v_4, v_5| \geq 3$, $v_1 \neq v_2$ and $v_3 \neq v_4$. In both cases, at least 3 connecting edges in $\mathbf{G_1}$ are necessary. The redundant rigidity condition is easily checked as before and thus omitted.

When $|V_2| = 4$, $|E_2| = 5$ and $|E_3| = 4$, $\mathbf{G_2}$ is formed by joining two triangles, as shown in Fig. 1(j) (without the dash line). It is clear that the two nodes with two edges each (the top and bottom nodes in Fig. 1(j)) need at least one connecting edge each. To ensure 3-connectedness, the remaining two connecting edges in E_3 cannot be allocated to the top and bottom nodes only (Otherwise, removing these two nodes would disconnect **G**). The only possible

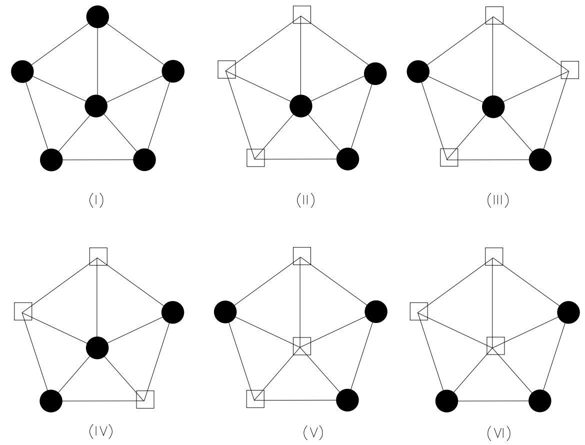

Fig. 2. (I) is a normal WHEEL structure. (II)-(VI) correspond to different anchor nodes deployment.

connection patterns are shown in Fig. 1(j)-(l), where the top node, bottom node and a third node (which is either of the remaining nodes) each has an edge connected to a different node in $\mathbf{G_1}$, plus another edge connecting any node in $\mathbf{G_1}$ to any node in $\mathbf{G_2}$. The 3-connectedness and redundant rigidity conditions are easily checked as before and thus omitted.

When $|V_2| = 4$, $|E_2| = 6$ and $|E_3| = 4$, $\mathbf{G_2}$ is shown as Fig. 1(j) (with the dash line). The feasible connection patterns are shown in Fig. 1(j)-(l) (with the dash line) and are identical to those for the $|E_2 = 5|$ because the extra internal edge does not affect the argument we had earlier for the allocation of the connecting edges. ■

IV. BEYOND TRILATERATION AND WHEEL EXTENSION

It is clear that the results above include the trilatertion condition as a special case. In this section, we compare our results with a method known as WHEEL extension [7].

In the work of [7], the authors explore the localizable nodes through detecting a WHEEL structure in the network. A node is claimed to be localizable if it is included in a WHEEL graph containing at least three anchor nodes. A common WHEEL graph is as shown in Fig. 2(I). Since there are six nodes in a WHEEL graph, one can detect at most three nodes jointly localizable according to this condition. In this way, the trilateration condition can be treated as a special case of WHEEL graph. In fact, according to different positions of anchor nodes, there are at most five possible WHEEL structures as shown in Fig. 2(II)-(VI). In a WHEEL graph, each rectangle indicates one anchor node and the solid circle indicates a node to be localized. It turns out that each graph in Fig. 2(II)-(VI) is a special case in our conditions shown in Fig. 1. The detailed relationship is shown in Table. I. We can see that the WHEEL extension is included in our conditions with no more than three nodes in $\mathbf{G_2}$. Since we also consider the case of four nodes in $\mathbf{G_2}$, our conditions can detect more localizable nodes compared with WHEEL extension.

V. SIMULATION

A. Comparison With Convex Hull Situation

To demonstrate the effectiveness of our proposed method, we first run a simulation to compare the number of localizable nodes detected by using the concept of convex hull in [1], trilateration and our given conditions. Given a 80-node network $\mathbf{G}$, when using trilateration and our localizable conditions, all nodes are randomly deployed in a 100×100 units area. Each anchor node is identical with other normal nodes except they know their positions of themselves. Every node has direct connections with their neighbors who living inside a communication radius, say 20 units in this simulation. But when using the convex hull method, the anchors are set manually and assumed to have a communication radius that guarantee all nodes lying inside the convex hull can be directly connected with the three anchors. Here, we use the maximum value of the distance measurements between three anchor nodes as the communication radius.

As shown in Fig. 3, each sensor node is indicated by a circle and has an edge connected with their neighbors. Three anchors are connected by thick solid line and the localizable nodes are marked by angles. Note that, besides all nodes lying inside the convex hull, some nodes lying outside the convex hull are also localizable. This is caused by the fact that these nodes are also covered inside the communication area of three anchors, i.e., they might have direct connections with three anchors. The topology here may different from that simulated in the DILOC method algorithm in [1]. In [1], they need to guarantee each node will be lying inside a triangle by its three neighbors. So, each node should have to find at least three neighbors to contain it inside. Since we only concern the number of localizable nodes here, we simplify the topology and only connect the anchor nodes with nodes that have less than three neighbors within radius of 20 unit. This will not affect the number of detected localizable nodes.

The localizable effect of our given condition is shown in Fig. 4. Different from the convex hull method, the anchors here are randomly chosen and assumed to have identical communication radius with other nodes. We notice that, after using our given conditions, a lot more localizable nodes than convex hull method can be detected for the same network. Actually, the group of nodes deployed on top-right, which cannot be detected as localizable by our conditions, can be easily checked to be definitely un-localizable since they are even not rigid.

TABLE I
CORRESPONDING RELATIONSHIP

WHEEL Graph in Fig. 2	Graph Described in Fig. 1
Fig. 2 (II)	Fig. 1(d)
Fig. 2 (III)	Fig. 1(c)
Fig. 2 (IV)	Fig. 1(c)
Fig. 2 (V)	Fig. 1(a)+(b)
Fig. 2 (VI)	Fig. 1(c)

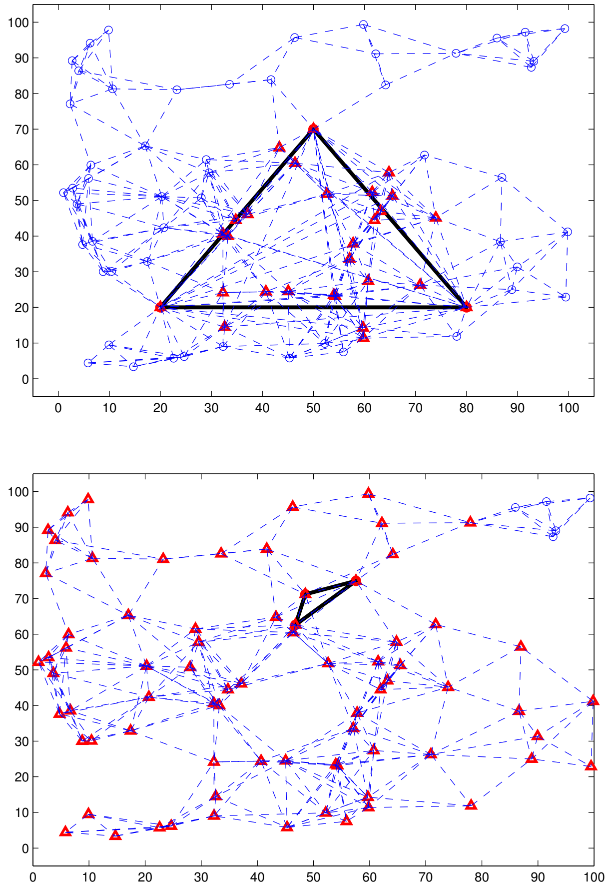

Fig. 3. Compare With DILOC

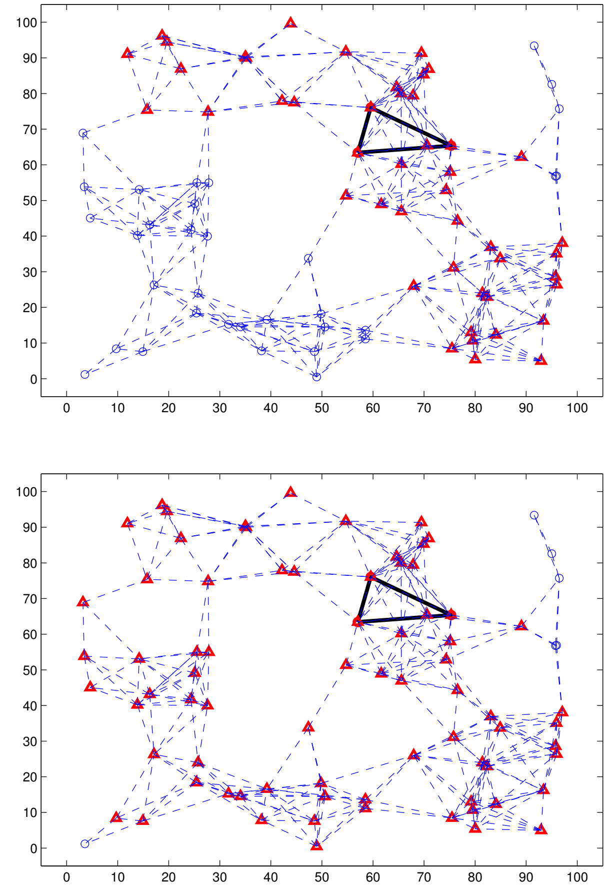

Fig. 4. Compare With WHEEL Extension

B. Comparison With Trilateration and WHEEL Extension

The network shown in Fig. 4 is one realization of a 80-node network **G**. Each node is also randomly deployed. The top figure in Fig. 4 shows the localizability results using WHEEL extension, whereas the bottom figure shows the results using our proposed conditions. In this simulation, our conditions show obvious improvement on localizable effectiveness.

We also run a 100-round Monte-Carlo simulation to show the effectiveness of our proposed conditions compared with both trilateration method and WHEEL extension in more general cases. In each round, a 80-node network **G** is randomly deployed in a 100×100 unit area. The anchor nodes are randomly selected. All nodes have the same communication radius. In each round, we compare four cases: (1) Fig. 1(a) only; (2) Fig. 1(a) and (b); (3) Fig. 1(a)-(d); (4) Fig. 1(a)-(l). As we mentioned in above section, the trilateration and WHEEL extension have a corresponding relationship with cases (1) and (3), respectively.

The result is shown in Fig. 5. Four groups of columns respect to the four cases we considered earlier. We also show in Fig. 5 the effect of communication radius on localizability. In each group, columns from left to right indicate three situations with communication radius 17, 20 and 25, respectively. The first group corresponds to localizable ratio of Fig. 1(a), i.e., the trilateration case. It is clear from Fig. 5 that improvement of our method is significant in comparison with the trilateration method, when the communication radius is not too large, especially when the average degree is close to the common suggestion of 6 or 8 in practical network deployment [9][10]. Note that the trilateration scheme in each round will not stop until no more localizable nodes can be found. So, the result shown in the first group of columns of Fig. 5 is indeed the localizable ratio detected by a sequential scheme of trilateration. It finds out all nodes satisfying the trilateration condition. Our condition also shows better localizable effectiveness than the WHEEL extension method.

C. Modification of Network Topology

The proposed localization conditions can also be used to eliminate the redundant connections in a given network and to improve localizability of the network through adding a small number of extra connections on specific nodes.

To eliminate the redundant connections, we can simply remove edges that are not necessary for each localizable pattern according to our conditions. Through this way, the average degree gets reduced and the communication and computational loads are reduced at the same time. Adding extra connections for un-localizable nodes need extra work

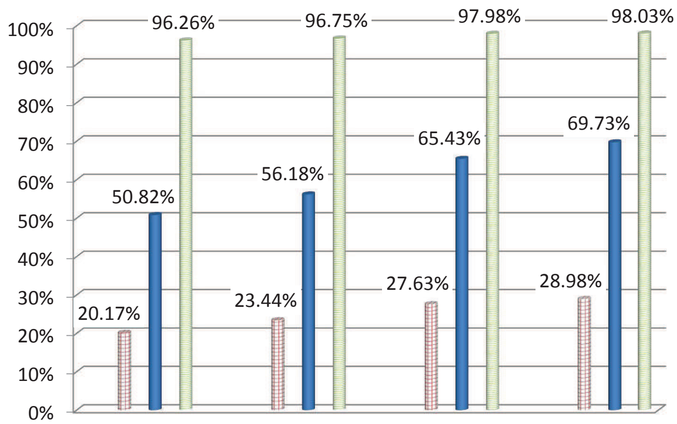

Fig. 5. Percentage of Localizable Nodes vs. Communication Radius

on finding proper un-localizable nodes and modifying communication radius. But this is easily to be done when the isolated nodes are not too many. Our localization conditions can be as a guideline to decide where to add connections.

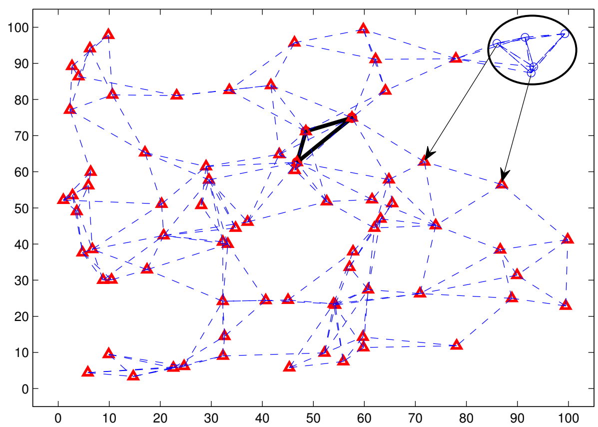

Fig. 6. A Modified Network Topology of Bottom Figure in Fig. 3

We show an example of such modification in Fig. 6 , which is modified from Fig. 3. The nodes inside the thick circles are those not localizable previously, and the required extra connections to make them localizable are marked by arrowed lines. We can see that the total number of connections is significantly reduced and only two new connections are added to make the whole network localizable.

To demonstrate the reduction effect according to our conditions in more general cases, we run a 100-round Monte Carlo simulation to compute the reduction ratio of the connecting edges. The results are shown in Table II, where Edges#1 and Degree#1 denote the number of connecting edges and average degree before the modification, respectively, whereas Edges#2 and Degree#2 denote those after the modification. We see that the reduction of the connecting edges is significant.

TABLE II
REDUCTION OF CONNECTING EDGES VS. RADIUS

Radius	Nodes	Edges#1	Edges#2	Degree#1	Degree#2
17	26.63	249.28	61.05	18.72	4.58
20	59	336.30	142.93	11.40	4.85
25	77.46	503.55	195.87	13.00	5.06

VI. CONCLUSION

In this paper, we have provided a group of localizable conditions for sensor networks. Our conditions can detect one to four localizable nodes in each step. In comparison with methods available in the literature, we have significantly generalized the well-known trilateration method and the convex hull method. The so-called WHEEL extension condition can also be treated as a subset of our localizable patterns. Simulation results show that our method is effective for detecting localizable nodes and offers significant improvements over known methods. We have also offered a simple modification to trim redundant connecting edges and at the same time to improve the localizability of the whole network.

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