Design of Serially Concatenated Continuous Phase Modulation with Symbol-Wise Interleaving

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Abstract—In this letter we consider serially concatenated continuous phase modulation (SCCPM) schemes with symbol-wise interleaving and an additional intra-symbol interleaving. We use extrinsic information transfer (EXIT) chart to optimize the outer code and intra-symbol interleaver. We show that, for various continuous phase modulation (CPM) parameters, the outer code with memory 2 may not always be the best choice. For CPM with modulation index $h = 1/2$, considerable improvements can be achieved by moderately increasing the outer code memory.

Index Terms—continuous phase modulation, serially concatenated CPM, symbol-wise interleaving, EXIT chart.

Fig. 1. The proposed symbol-wise interleaved SCCPM scheme.

I. INTRODUCTION

CONTINUOUS phase modulation (CPM) plays an important role in radio and satellite communications due to its constant envelope property and high spectral efficiency [1], [2]. The spectral efficiency of a CPM scheme depends on a number of parameters. These include the shape function $g(t)$ (rectangular, raised cosine or Gaussian), the duration L of the shape function, the symbol period T, the cardinality of the CPM symbols M, and the modulation index h. In this letter, the modulation index is restricted to the form of $h = 1/P$, where P is the number of phase states. A high spectral efficiency can be achieved by increasing the values of L, M and P. This comes with the tradeoff of decoding complexity and bit-error-rate (BER) performance. On the other hand, decoding complexity and BER performance can be improved by increasing the modulation index h. To improve the power efficiency by coding, a CPM system is typically cascaded with an outer code. Shortly after the invention of turbo codes, serially concatenated CPM (SCCPM) showed their advantages in power efficiency [3]–[12].

A typical SCCPM system consists of a binary outer code, a bit-wise interleaver and an inner code (CPM) which is connected to the channel directly. When $M > 2$, a mapper is also needed to bridge between the bit-wise interleaver and the non-binary CPM. Iterative methods are typically used for decoding. Recently, it is shown that SCCPM systems with symbol-wise interleaving can achieve substantial improvement in BER performance over their counterparts with bit-wise interleaving [11], [12]. The outer code in a symbol-wise interleaved SCCPM can still be a binary code, provided that the symbols for the outer code have a size of alphabet equal to M. For an SCCPM, nonrecursive convolutional codes (CCs)
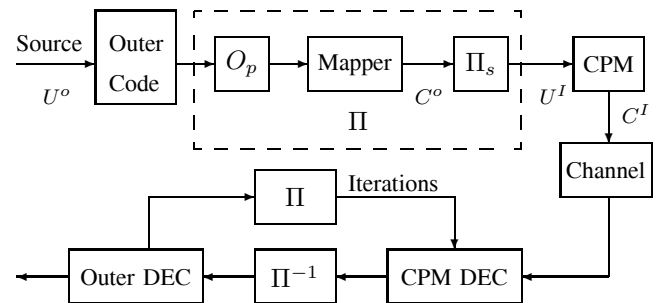
with very low memory, acting as outer codes, have shown their advantages over those with high memory. Typically, nonrecursive convolutional code with the generator matrix $G(D) = [1 + D + D^2, \ 1 + D^2]$, where D is a delay operator, or $(7, 5)$ in octal form is widely-used and believed to offer excellent performance in concatenation with a large class of CPMs. To the authors' knowledge, there is no literature showing the advantage of using outer code with memory higher than 2.

In this letter we consider symbol-wise interleaved SCCPM schemes. Motivated by the approach in [13], we add an intra-symbol interleaver in the schemes to further improve the BER performance. For the proposed schemes we use extrinsic information transfer (EXIT) chart technique to optimize the outer code and intra-symbol interleaver. For the SCCPM schemes with modulation index $h = 1/2$, we show that considerable improvements can be achieved by moderately increasing the outer code memory.

II. THE PROPOSED SCCPM

The proposed scheme shown in Fig. 1 is similar to the symbol-wise interleaved scheme in [11] except for the addition of an intra-symbol interleaver O_p. The motivation of adding an intra-symbol interleaver is based on the idea presented in [13] as well as the observation that the ordering of the output bits within a coded symbol from the outer encoder has some impact on the BER performance. The choices for O_p can be fixed or periodic. Take the outer encoder $(7, 5)$ as an example, intra-symbol interleaver $O_p = [2 \ 1]$ indicates that the order of output bits from the two branches of encoder $(7, 5)$ is fixed and the bit from branch 5 precedes the bit from branch 7 within the output symbol. Another example is $O_p = \{[1 \ 2], [2 \ 1]\}$ which means that O_p is chosen to be $[2 \ 1]$ and $[1 \ 2]$ periodically.

In Fig. 1 an information binary sequence U^o is fed into the outer encoder. For each input symbol which consists of
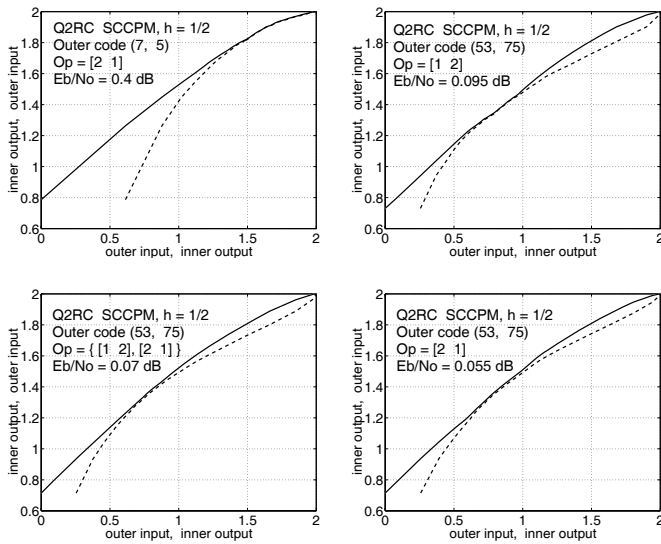
Fig. 2. EXIT charts of the symbol-wise interleaved SCCPMs.

Fig. 3. Simulation results of the coded quaternary 2RC CPMs with rate-$1/2$ binary outer code, $h = 1/2$, $N = 1024$ and 12 decoding iterations.

n_i bits at each time instant, the encoder generates a coded symbol consisting of n_o bits. The intra-symbol interleaver O_p permutates the order of the n_o bits within the coded symbol and the mapper converts each symbol of O_p into a symbol according to Natural Binary Code (NBC) or Gray mapping rule, assuming that $n_o = \log_2 M$. The symbol sequence C^o from the mapper is then interleaved by the symbol-wise interleaver Π_s into another symbol sequence U^I which is treated as the input of CPM. Through CPM, the sequence U^I is modulated to give the CPM symbol sequence C^I before it is transmitted over the communication channel. In this paper, symbol-wise interleavers are chosen to be S-random interleavers [14].

III. OUTER CODE DESIGN USING EXIT CHART

EXIT chart technique is a well-known numerical tool for analyzing the convergence behavior of iterative decoders and predicting the convergence threshold of turbo-like concatenated codes. The convergence threshold is the lowest signal-to-noise ratio at which the decoding can converge at a given BER. In this letter, we restrict ourselves to quaternary CPMs with raised cosine shape function of duration 2 and 3, denoted by Q2RC CPM and Q3RC CPM, respectively. For the purpose of optimization, we choose the outer codes to be different convolutional codes (CCs) with memory varying from 2 to 5. Also, we try different intra-symbol interleavers for the schemes. For comparison, we choose the modulation index h to be $1/2$ and $1/4$ and use both bit-wise interleaving and symbol-wise interleaving. EXIT charts are computed using the approach in [11] which is based on [15], [16].

From the EXIT charts, we have the following observations. Gray mapping is always preferred to NBC mapping for the bit-wise interleaved schemes, whereas NBC mapping performs better for the most symbol-wise interleaved schemes. For bit-wise interleaved schemes and symbol-wise interleaved schemes with $h = 1/4$, outer code $(7, 5)$ are superior to those with higher memory. Nevertheless, for symbol-wise interleaved schemes with $h = 1/2$, outer code $(7, 5)$ is
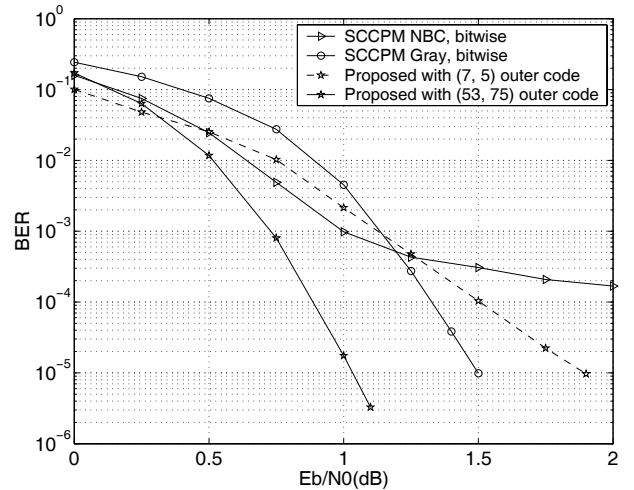
not the best choice and improvements can be achieved by increasing code memory moderately. In addition, a symbol-wise interleaved scheme with the optimized outer code, mapping and intra-symbol interleaver substantially outperforms its bit-wise interleaved counterparts. The best intra-symbol interleaver varies for different schemes.

Fig. 2 shows the EXIT charts of symbol-wise interleaved SCCPMs with Q2RC CPM and $h = 1/2$ as typical examples. The input information block has $20,000$ bits. The EXIT chart on the left-top of Fig. 2 shows the decoding trajectories and the convergence threshold in E_b/N_0 of the scheme using CC $(7, 5)$ and $O_p = [2\ 1]$. Other EXIT charts in the figure are from the schemes using CC $(53, 75)$, with difference in O_p. We observe that each pair of EXIT curves associated with CC $(53, 75)$ match much better than those associated with CC $(7, 5)$. In particular, the schemes with CC $(53, 75)$ and $O_p = [2\ 1]$ gives the lowest convergence threshold, although different intra-symbol interleavers do not cause much difference.

IV. SIMULATIONS AND COMPARISONS

In this section, we present and compare some simulation results when AWGN channels and coherent demodulations are assumed. The input block size for the outer codes is $N = 1024$ bits and the outer code rate is $1/2$. For each comparison, we keep the CPM parameters unchanged. Also for the convenience of comparison, we only show the simulation results of symbol-wise interleaved schemes with NBC mapping in the following figures.

Fig. 3 shows the simulated results for SCCPMs with Q2RC CPM and $h = 1/2$ using bit-wise or symbol-wise interleaving. When the outer code is CC $(7, 5)$, the result for the symbol-wise interleaved scheme is worse than that of its bit-wise counterpart. However, when the outer code is CC $(53, 75)$, a gain of about 0.5 dB at BER of 10^{-5} is observed for the symbol-wise interleaved scheme over the best bit-wise interleaved scheme.

To show the impact of an intra-symbol interleaving, Fig. 4 compares the simulated results of the symbol-wise interleaved
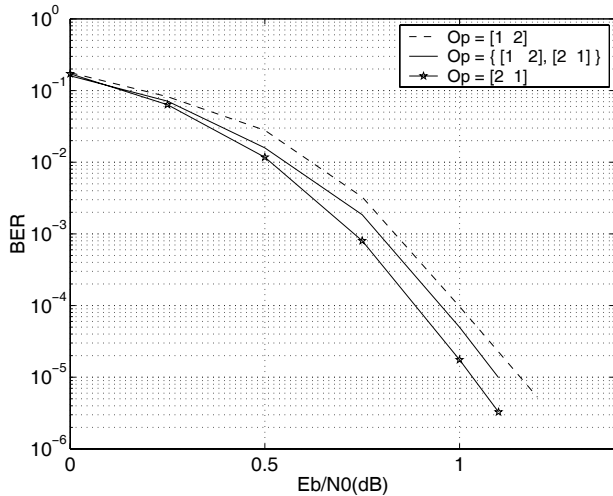
Fig. 4.   Simulation results of the coded quaternary 2RC CPMs with rate-$1/2$ binary outer code $(53, 75)$, various intra-symbol interleavers, $h = 1/2$, $N = 1024$ and 12 decoding iterations.
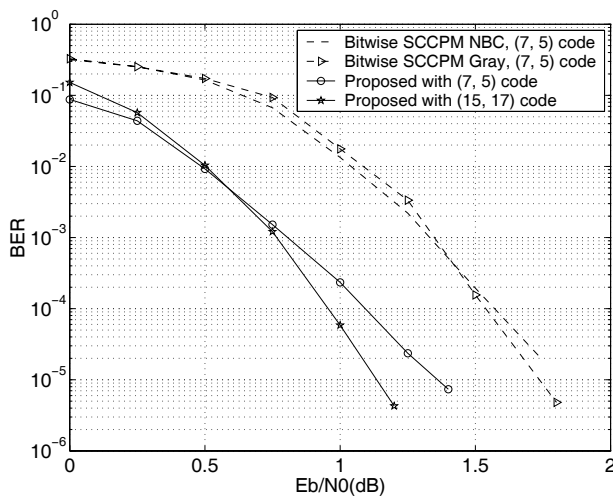

Fig. 5.   Simulation results of the coded quaternary 3RC CPMs with rate-$1/2$ binary outer code, $h = 1/2$, $N = 1024$ and 12 decoding iterations.

Q2RC CPM schemes with outer code $(53, 75)$ but various intra-symbol interleavers. It turns out that intra-symbol interleaver $O_p = [2\ 1]$ gives the best performance while $O_p = [1\ 2]$ performs worst, although the difference between them is only about 0.12 dB at BER of 10^{-5}.

For SCCPMs with Q3RC CPM, Fig. 5 shows the simulated results for SCCPMs with Q3RC CPM and $h = 1/2$ using bit-wise or symbol-wise interleaving. We observe that the symbol-wise interleaved schemes outperform the bit-wise interleaved schemes with $(7, 5)$ outer code. Using symbol-wise interleaving, the schemes with $(15, 17)$ outer code and $O_p = \{[1\ 2],\ [2\ 1]\}$ outperforms the schemes with $(7, 5)$ outer code and $O_p = [2\ 1]$ by about 0.25 dB.

From the simulation results of the SCCPM schemes with $h = 1/2$ and Q2RC CPM or Q3RC CPM, we observed that considerable improvements can be achieved by moder-

ately increasing the outer code memory. Note that for these examples, outer codes with memory higher than 2 do not give any improvement for bit-wise interleaved schemes. These simulated results are in line with our EXIT chart analysis.

V. Conclusion

In this letter we have considered symbol-wise interleaved SCCPM schemes and designed outer codes using EXIT chart. It turned out that, for the schemes with moderate modulation index, the outer code with memory 2 is superior, while for the schemes with high modulation index, the outer codes with moderate memory are preferred. We have also shown that the use of an additional intra-symbol interleaving can help the SCCPM systems achieve the best decoding performance. Finally, we have presented some simulation results to show the superior performance of the proposed schemes.

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