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Development of an extended reset controller and its experimental demonstration

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Abstract: Reset control aims at enhanced performance that cannot be obtained by linear controllers. The conventional reset control is simple for implementation by resetting some of its controller states to zero when its input meets a threshold. However, it is found that in some cases the enhanced performance of conventional reset control is still limited such as with only partial reduction of the overshoot in a step reference response. Thus, the stability analysis and design of the reset control system are extended, where the reset time instances are prespecified and the controller states are reset to certain non-zero values, which are calculated online in terms of the system states for optimal performance. Experimental results on a piezoelectric positioning stage demonstrate that the extended reset control can further reduce the overshoot and thus achieve shorter settling time than the conventional reset control. Moreover, robustness tests against various step levels, disturbance and sensor noise are presented.

1 Introduction

Reset control was firstly proposed by Clegg [1] to overcome limitations of linear control. This reset controller, termed as Clegg integrator, consists of an integrator and a reset law which resets the amount of integration to zero when its input crosses zero. From the basic idea of reset control, one can see that reset control is capable of reducing windup caused by integration. Moreover, a Clegg integrator has a similar magnitude–frequency response as a pure integrator, but with 51.9° less phase lag. This favourable property helps to increase phase margin of a system. Krishman and Horowitz [2] developed a quantitative control design procedure of Clegg integrator. Horowitz and Rodenbaum [3] generalised the concept of reset control to higher-order systems. Relevant works can also be found in [4, 5].

A lot of works have shown the advantages of reset control over linear control. For instance, an example is presented in [6] showing that reset control can achieve some control specifications, which cannot be achieved by any ordinary linear control. Moreover, it is experimentally demonstrated in [7] that reset control can achieve better sensor noises suppression without degrading disturbance rejection or losing gain and phase margin. These advantages make reset control effective for performance improvement in a wide range of applications such as hard disc drive servos [8, 9] and vibration suppression [10, 11].

Typically, there are two steps in reset control design [12]: linear compensator design and reset element design. The linear compensator is firstly designed to meet all performance specifications except for the overshoot constraint; then the reset element is designed to reduce the overshoot. However, the reset controller can improve the closed-loop performance only when the reset law interacts well with the base linear system. In other words, if the reset controller is not appropriately designed, it may have little contribution to the performance improvement, or even cause system instability.

In reset control system design, there are three basic problems: stability analysis, base linear system design and reset law design. For stability analysis, there are lots of papers addressing this issue such as [13–16]. Most of these existing results require the base linear system to be stable. However, stability of a reset control system depends on both the base linear system and the reset actions. Either factor may destroy the stability of the overall system. Note that reset control systems are also known as impulsive systems. Many stability results have been obtained in the literature (see [17] for example).

For reset control design, more efforts are put on the design of base linear system in the literature. The reset law adopted is generally the conventional one, that is, resetting the controller states to zero when the controller input crosses zero [16]. The base linear system is then designed to interact well with the reset law. We refer to this kind of reset control as conventional reset control in this paper. The conventional reset control has been demonstrated to be able to achieve better performance than a pure linear controller. However, we find that in some cases the performance of conventional reset control is still limited, for example, the overshoot is only partially reduced based on the linear control system [7, 8]. Actually, reset control can be more generalised by designing the reset time instances and the reset values to push the performance improvement further. In [18], we have presented an extended reset control system, where the reset time instances are prespecified and the controller states are reset to certain non-zero values, which are calculated online in terms of the system states for optimal performance.

In this paper, we will describe the development of an extended reset controller and experimentally demonstrate its effectiveness on a positioning system. In Section 2, we firstly formulate the extended reset control system in a state space form. Then, the reset control system is reformulated as a new system, which is referred to as induced discrete system. We show that under some mild conditions the stability of a reset control system is equivalent to the stability of its induced discrete system. On the basis of this, we obtain some new results about the stability of reset control systems, which do not rely on the stability of base linear system. Lastly, we propose a reset law design approach, which aims at minimising a performance index related to the tracking error. Section 3 demonstrates the proposed reset controller on a piezoelectric (PZT) microactuator positioning stage. Experimental results showed its effectiveness in overshoot removal, disturbance and sensor noise suppression. Conclusions and future works are discussed in Section 4.

2 Extended reset control design

2.1 Formulation of reset control systems

A typical reset control system is depicted in Fig. 1, where r is the reference input, e the feedback error, u the reset controller output, d the output disturbance, n the sensor noise and y_p, y and y_m the plant, controlled and measurement outputs, respectively. The linear plant P is described by

$$\Sigma_P: \begin{cases} \dot{\boldsymbol{x}}_p = \boldsymbol{A}_p \boldsymbol{x}_p + \boldsymbol{B}_p u \\ y_p = \boldsymbol{C}_p \boldsymbol{x}_p \end{cases} \tag{1}$$

where $\boldsymbol{x}_p \in \mathbb{R}^{n_p}$, $u \in \mathbb{R}$ and $y_p \in \mathbb{R}$. The reset controller

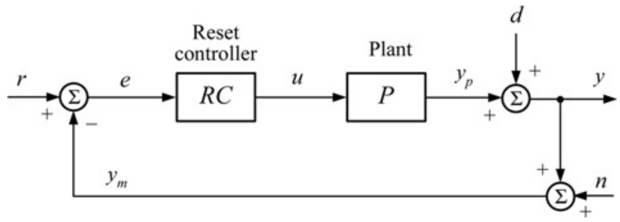

Figure 1 *Block diagram of a reset control system*

(RC) is described by impulsive differential equation

$$\Sigma_{RC}: \begin{cases} \dot{\boldsymbol{x}}_r = \boldsymbol{A}_r \boldsymbol{x}_r + \boldsymbol{B}_r e, & t \neq t_k \\ \boldsymbol{x}_r(t_k^+) = \boldsymbol{E}_k \boldsymbol{x}_p + \boldsymbol{F}_k \boldsymbol{x}_r + \boldsymbol{G}_k r, & t = t_k \\ u = \boldsymbol{C}_r \boldsymbol{x}_r + D_r e \end{cases} \tag{2}$$

where $\boldsymbol{x}_r \in \mathbb{R}^{n_r}$ is the reset controller state, t_k^+ the reset time instance and $e = r - d - n - y_p$. $\boldsymbol{A}_r$, $\boldsymbol{B}_r$, $\boldsymbol{E}_k$, $\boldsymbol{F}_k$, $\boldsymbol{G}_k$, $\boldsymbol{C}_r$ and D_r are appropriate dimensional constant matrices. The set of reset time instances $\{t_k\}$ is an unbounded time sequence increasing monotonously with respect to k, $k \in \mathbb{Z}^+$, that is, $t_k < t_{k+1}$ for any $k \in \mathbb{Z}^+$ and $\lim_{k\to\infty} t_k = +\infty$. In this paper, we assume that the reset time instances are prespecified and the reset actions are finite in any finite time interval to ensure the existence of solutions.

Combining (1) and (2) gives the closed-loop system as follows

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}w, & t \neq t_k \\ \boldsymbol{x}(t_k^+) = \boldsymbol{M}_k \boldsymbol{x} + \boldsymbol{N}_k r, & t = t_k \\ y = \boldsymbol{C}\boldsymbol{x} + d \end{cases} \tag{3}$$

where $\boldsymbol{x} = (\boldsymbol{x}_p^T, \boldsymbol{x}_r^T)^T$, $w = r - d - n$ and

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_p - \boldsymbol{B}_p D_r \boldsymbol{C}_p & \boldsymbol{B}_p \boldsymbol{C}_r \\ -\boldsymbol{B}_r \boldsymbol{C}_p & \boldsymbol{A}_r \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_p D_r \\ \boldsymbol{B}_r \end{bmatrix},$$

$$\boldsymbol{M}_k = \begin{bmatrix} \boldsymbol{I}_{n_p} & \boldsymbol{0} \\ \boldsymbol{E}_k & \boldsymbol{F}_k \end{bmatrix}, \quad \boldsymbol{N}_k = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{G}_k \end{bmatrix}, \quad \boldsymbol{C} = [\boldsymbol{C}_p \quad \boldsymbol{0}]$$

where I_{n_p} is the identity matrix of dimension n_p.

Remark 1: In conventional reset control, the reset element is generally defined as

$$\boldsymbol{x}_r(t_k^+) = \begin{bmatrix} \boldsymbol{I}_{n_{\bar{Q}}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0}_{n_Q} \end{bmatrix} \boldsymbol{x}_r, \quad t = t_k$$

$$\{t_k\} \triangleq \{t_k | e(t_k) = 0,\ t_k < t_{k+1}\}$$

where n_Q (with $n_{\bar{Q}} + n_Q = n_r$) is the amount of selected reset controller states [6]. It can bee seen that the reset actions are triggered by the feedback error and the controller states are selected to be always reset to zero. However, in our proposed reset control (2), the reset values have been generalized as a function of the system states and reference input for improved performance.

2.2 Stability analysis

In this subsection, we derive the stability condition of the reset control system (3) under the assumption of $r \equiv n \equiv d \equiv 0$ but subject to non-zero initial condition. Hence, the reset control system (3) can be rewritten as

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}, & t \neq t_k, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{x}(t_k^+) = \boldsymbol{M}_k\boldsymbol{x}, & t = t_k \end{cases} \tag{4}$$

Suppose that the solution to (4) is continuous from the left, we thus have $\boldsymbol{x}(t_k^+) = \lim_{t\to 0^+}\boldsymbol{x}(t_k) = \boldsymbol{M}_k\boldsymbol{x}(t_k)$ and

$$\boldsymbol{x}(t_{k+1}^+) = \boldsymbol{M}_{k+1}\mathrm{e}^{\boldsymbol{A}(t_{k+1}-t_k)}\boldsymbol{x}(t_k^+)$$

Defining $\boldsymbol{\eta}_k = \boldsymbol{x}(t_k^+)$, $\Delta t_k = t_k - t_{k-1}$, $\boldsymbol{L}_k = \boldsymbol{M}_k\mathrm{e}^{\boldsymbol{A}\Delta t_k}$ and $t_0 = 0$, we have

$$\boldsymbol{\eta}_{k+1} = \boldsymbol{L}_{k+1}\boldsymbol{\eta}_k, \quad k = 0, 1, \ldots, N \tag{5}$$

The system (5) is referred as the induced discrete system of the impulsive system (4).

Typically, the stability of system (4) is often analysed by finding a positive Lyapunov function $V(\boldsymbol{x})$ such that

$$\dot{V}(\boldsymbol{x}) = \left(\frac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)^{\mathrm{T}}\boldsymbol{A}\boldsymbol{x} \leq 0 \tag{6}$$

$$\Delta V(\boldsymbol{x}) = V(\boldsymbol{M}_k\boldsymbol{x}) - V(\boldsymbol{x}) \leq 0 \tag{7}$$

This indicates that the base linear system (without reset) is stable and the impulses always decrease the amount of the Lyapunov function. The following result reveals that under some mild conditions, the stability of an impulsive system is equivalent to the stability of its induced discrete system. Thus, the stability of the base linear system is not required any more, and the impulses are allowed to increase the value of Lyapunov function of the base linear system.

Proposition 1: If there exists a positive number $\Delta T > 0$ such that

$$\Delta t_k = t_k - t_{k-1} < \Delta T \tag{8}$$

for all $k \in \mathbb{Z}^+$, then the system (4) is (asymptotically) stable if and only if its induced discrete system (5) is (asymptotically) stable.

Proof: The necessity is obvious. We only prove the sufficiency. According to the fact that the solutions to the base linear system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ depend continuously upon initial conditions, we have for any $\varepsilon > 0$, there exists a positive number $\delta_1 > 0$ such that

$$\|\boldsymbol{x}_0\| < \delta_1 \Rightarrow \|\mathrm{e}^{\boldsymbol{A}s}\boldsymbol{x}_0\| < \varepsilon, \quad s \in [0, \Delta T] \tag{9}$$

Assume that the induced discrete system (5) is stable, then for δ_1 selected above, there exists a positive number $\delta > 0$ such that

$$\|\boldsymbol{\eta}_0\| < \delta \Rightarrow \|\boldsymbol{\eta}_k\| < \delta_1, \quad k \in \mathbb{Z}^+$$

Note that $\boldsymbol{\eta}_0 = \boldsymbol{x}_0$ and for any $t \in [0, +\infty)$, there is a non-negative integer k such that $t \in (t_k, t_{k+1}]$, so

$$\boldsymbol{x}(t) = \mathrm{e}^{\boldsymbol{A}(t-t_k)}\boldsymbol{x}(t_k^+) = \mathrm{e}^{\boldsymbol{A}(t-t_k)}\boldsymbol{\eta}_k$$

Since $t - t_k \in [0, \Delta T]$, we have

$$\|\boldsymbol{x}(t)\| = \|\mathrm{e}^{\boldsymbol{A}(t-t_k)}\boldsymbol{\eta}_k\| < \varepsilon, \quad t \in [0, +\infty)$$

So the system (4) is stable.

If the induced discrete system is asymptotically stable, then we have

$$\lim_{k\to\infty}\boldsymbol{\eta}_k = 0$$

For any $\varepsilon > 0$, choose δ_1 such that

$$\|\boldsymbol{x}_0\| < \delta_1 \Rightarrow \|\mathrm{e}^{\boldsymbol{A}s}\boldsymbol{x}_0\| < \varepsilon, \quad s \in [0, \Delta T] \tag{10}$$

Thus for any $\boldsymbol{\eta}_0(= \boldsymbol{x}_0)$, there exists a $K(\boldsymbol{x}_0) \in \mathbb{Z}^+$ for

$$\|\boldsymbol{\eta}_k\| < \delta_1, \quad k \geq K(\boldsymbol{x}_0)$$

Then for any $t > K(\boldsymbol{x}_0)$, there exists a $k \geq K(\boldsymbol{x}_0)$ such that $t \in (t_k, t_{k+1}]$ and

$$\boldsymbol{x}(t) = \mathrm{e}^{\boldsymbol{A}(t-t_k)}\boldsymbol{\eta}_k$$

so we have

$$\|\boldsymbol{x}(t)\| = \|\mathrm{e}^{\boldsymbol{A}(t-t_k)}\boldsymbol{\eta}_k\| < \varepsilon$$

according to $t - t_k \in [0, \Delta T]$. Therefore

$$\lim_{t\to\infty}\boldsymbol{x}(t) = 0$$

which implies that the system (4) is asymptotically stable. □

Corollary 1: If both $\Delta t_k = \delta$ is a constant and $\boldsymbol{M}_k \equiv \boldsymbol{M}$ is a constant matrix, then the reset control system (4) is (asymptotically) stable if and only if

$$|\lambda(\boldsymbol{M}\mathrm{e}^{\boldsymbol{A}\delta})| \leq 1, \quad (< 1) \tag{11}$$

where $\lambda(\cdot)$ denotes the eigenvalues of $(\cdot)$.

Remark 2: Proposition 1 indicates that in order to check the stability of an impulsive system, we only need to check the induced discrete system. For systems of which the base linear system is unstable and the induced system is stable, it is impossible to find a positive Lyapunov function $V(\boldsymbol{x})$ such that (6) and (7) hold. Hence, Proposition 1 which only requires the boundedness of Δt_k is more general.

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In practice, the base linear system is typically designed to be stable, thus the bounded constraint on $\{\Delta t_k\}$ can be relaxed. Thus we have the following.

Proposition 2: Assume the base linear system is stable, then the system (4) is (asymptotically) stable if and only if its induced discrete system (5) is (asymptotically) stable.

Proof: To complete the proof, just replace ΔT in the proof of Proposition 1 by $+\infty$ and simply follow the same lines of arguments. $\square$

Remark 3: Note that in Proposition 2, we only assume the stability (not asymptotical stability) of the base linear system. This is because the stability of the base linear system is adequate to assure the boundedness of the transition matrix $\mathrm{e}^{\boldsymbol{A}t}$.

2.3 Reset law design

Consider the reset control system (3) and assume that the base linear system has been appropriately designed for basic stability and performance, and the reset time instances are predefined. Our objective here is to find a set of reset values $\boldsymbol{x}_{\mathrm{r}}(t_k^+)$ of the controller states such that the system tracking error is minimised. More specifically, we suppose $n \equiv d \equiv 0$ and formulate our objective as solving $\boldsymbol{x}_{\mathrm{r}}(t_k^+)$ to minimise the following quadratic performance index

$$J_k = e^{\mathrm{T}}(t_{k+1})P_0 e(t_{k+1}) + \dot{e}^{\mathrm{T}}(t_{k+1})Q_0\dot{e}(t_{k+1}) + \int_{t_k}^{t_k+1} e(s)^{\mathrm{T}} P_1 e(s)\mathrm{d}s \tag{12}$$

where $e = r - y$, and P_0, P_1 and Q_0 are positive semi-definite matrices. To solve the problem, we use the following assumption.

Assumption 1: Assume that the reference r is constant, and for any $r \in \mathbb{R}$, there exists $\boldsymbol{x}_{\mathrm{ss}} = (\boldsymbol{x}_{\mathrm{pss}}^{\mathrm{T}}, \boldsymbol{x}_{\mathrm{rss}}^{\mathrm{T}})^{\mathrm{T}}$ such that

$$\begin{aligned} \boldsymbol{A}\boldsymbol{x}_{\mathrm{ss}} + \boldsymbol{B}r &= 0 \\ \boldsymbol{C}\boldsymbol{x}_{\mathrm{ss}} - r &= 0 \end{aligned} \tag{13}$$

Define

$$\begin{aligned} \boldsymbol{\xi}_{\mathrm{p}} &= \boldsymbol{x}_{\mathrm{p}} - \boldsymbol{x}_{\mathrm{pss}} \\ \boldsymbol{\xi}_{\mathrm{r}} &= \boldsymbol{x}_{\mathrm{r}} - \boldsymbol{x}_{\mathrm{rss}} \end{aligned}$$

we have

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= \boldsymbol{A}\boldsymbol{\xi}, \quad t \neq t_k \\ \boldsymbol{\xi}_{\mathrm{r}}(t_k^+) &= \boldsymbol{\rho}_k(\boldsymbol{\xi}, r), \quad t = t_k \end{aligned} \tag{14}$$

where $\boldsymbol{\xi} = (\boldsymbol{\xi}_{\mathrm{p}}^{\mathrm{T}}, \boldsymbol{\xi}_{\mathrm{r}}^{\mathrm{T}})^{\mathrm{T}}$. Hence we have

$$\begin{aligned} e &= -\boldsymbol{C}\boldsymbol{\xi} \\ \dot{e} &= -\boldsymbol{C}\boldsymbol{A}\boldsymbol{\xi} \end{aligned} \tag{15}$$

and the performance index J_k can thus be rewritten as

$$J_k = \boldsymbol{\xi}^{\mathrm{T}}(t_{k+1})\bar{\boldsymbol{P}}\boldsymbol{\xi}(t_{k+1}) + \int_{t_k}^{t_{k+1}} \boldsymbol{\xi}^{\mathrm{T}}(s)\bar{\boldsymbol{Q}}\boldsymbol{\xi}(s)\mathrm{d}s \tag{16}$$

where

$$\bar{\boldsymbol{P}} = \boldsymbol{C}^{\mathrm{T}}P_0\boldsymbol{C} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}Q_0\boldsymbol{C}\boldsymbol{A} \tag{17}$$

$$\bar{\boldsymbol{Q}} = \boldsymbol{C}^{\mathrm{T}}P_1\boldsymbol{C} \tag{18}$$

If $\boldsymbol{\xi}_{\mathrm{p}}(t_k)$, r and t_k, t_{k+1} are fixed, J_k is in fact a function of $\boldsymbol{\xi}_{\mathrm{r}}(t_k^+)$. In order to choose $\boldsymbol{\xi}_{\mathrm{r}}(t_k^+)$ such that J_k is minimised, we need to calculate $\partial J_k/\partial\boldsymbol{\xi}_{\mathrm{r}}(t_k^+)$ in the following. Note that

$$\boldsymbol{\xi}(t) = \mathrm{e}^{\boldsymbol{A}t}\boldsymbol{\xi}(t_k^+), \quad t \in (t_k, t_{k+1}]$$

We then have

$$\begin{aligned} \frac{\partial J_k}{\partial\boldsymbol{\xi}(t_k^+)} &= \frac{\partial\boldsymbol{\xi}(t_{k+1})}{\partial\boldsymbol{\xi}(t_k^+)}\frac{\partial}{\partial\boldsymbol{\xi}(t_{k+1})}\Big(\boldsymbol{\xi}^{\mathrm{T}}(t_{k+1})\bar{\boldsymbol{P}}\boldsymbol{\xi}(t_{k+1})\Big) \\ &\quad + \int_{t_k}^{t_{k+1}} \frac{\partial\boldsymbol{\xi}(s)}{\partial\boldsymbol{\xi}(t_k^+)}\frac{\partial}{\partial\boldsymbol{\xi}(s)}\Big(\boldsymbol{\xi}^{\mathrm{T}}(s)\bar{\boldsymbol{Q}}\xi(s)\Big)\mathrm{d}s \\ &= 2\mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}\Delta t_k}\bar{\boldsymbol{P}}\mathrm{e}^{\boldsymbol{A}\Delta t_k}\boldsymbol{\xi}(t_k^+) \\ &\quad + 2\int_{t_k}^{t_{k+1}} \mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}(s-t_k)}\bar{\boldsymbol{Q}}\mathrm{e}^{\boldsymbol{A}(s-t_k)}\boldsymbol{\xi}(t_k^+)\mathrm{d}s \\ &= 2\Big(\mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}\Delta t_k}\bar{\boldsymbol{P}}\mathrm{e}^{\boldsymbol{A}\Delta t_k} \\ &\quad + \int_{t_k}^{t_{k+1}} \mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}(s-t_k)}\bar{\boldsymbol{Q}}\mathrm{e}^{\boldsymbol{A}(s-t_k)}\mathrm{d}s\Big)\boldsymbol{\xi}(t_k^+) \\ &= 2\Big(\mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}\Delta t_k}\bar{\boldsymbol{P}}\mathrm{e}^{\boldsymbol{A}\Delta t_k} + \int_0^{\Delta t_k} \mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}s}\bar{\boldsymbol{Q}}\mathrm{e}^{\boldsymbol{A}s}\mathrm{d}s\Big)\boldsymbol{\xi}(t_k^+) \\ &= 2\boldsymbol{\Gamma}_k\boldsymbol{\xi}(t_k^+) \end{aligned} \tag{19}$$

where

$$\boldsymbol{\Gamma}_k = \mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}\Delta t_k}\bar{\boldsymbol{P}}\mathrm{e}^{\boldsymbol{A}\Delta t_k} + \int_0^{\Delta t_k} \mathrm{e}^{\boldsymbol{A}^{\mathrm{T}}s}\bar{\boldsymbol{Q}}\mathrm{e}^{\boldsymbol{A}s}\mathrm{d}s \tag{20}$$

Partition $\boldsymbol{\Gamma}_k$ as

$$\boldsymbol{\Gamma}_k = \begin{pmatrix} \boldsymbol{\Gamma}_k^{11} & \boldsymbol{\Gamma}_k^{12} \\ \boldsymbol{\Gamma}_k^{21} & \boldsymbol{\Gamma}_k^{22} \end{pmatrix}$$

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with $\boldsymbol{\Gamma}_k^{12} = (\boldsymbol{\Gamma}_k^{21})^{\mathrm{T}}$. Thus we have

$$\frac{\partial J_k}{\partial \boldsymbol{\xi}_{\mathrm{r}}(t_k^+)} = \frac{\partial \boldsymbol{\xi}(t_k^+)}{\partial \boldsymbol{\xi}_{\mathrm{r}}(t_k^+)} \frac{\partial J_k}{\partial \boldsymbol{\xi}(t_k^+)}$$

$$= 2(\mathbf{0} \quad \boldsymbol{I})\begin{pmatrix} \boldsymbol{\Gamma}_k^{11} & \boldsymbol{\Gamma}_k^{12} \\ \boldsymbol{\Gamma}_k^{21} & \boldsymbol{\Gamma}_k^{22} \end{pmatrix} \boldsymbol{\xi}(t_k^+)$$

$$= 2(\boldsymbol{\Gamma}_k^{21} \boldsymbol{\xi}_{\mathrm{p}}(t_k) + \boldsymbol{\Gamma}_k^{22} \boldsymbol{\xi}_{\mathrm{r}}(t_k^+)) \tag{21}$$

If $\boldsymbol{\Gamma}_k^{22}$ is positive definite, letting $\partial J_k/(\partial \boldsymbol{\xi}_{\mathrm{r}}(t_k^+)) = 0$ derives

$$\boldsymbol{\xi}_{\mathrm{r}}(t_k^+) = -(\boldsymbol{\Gamma}_k^{22})^{-1} \boldsymbol{\Gamma}_k^{21} \boldsymbol{\xi}_{\mathrm{p}}(t_k) \tag{22}$$

Thus the reset law which minimises J_k is given by

$$\boldsymbol{x}_{\mathrm{r}}(t_k^+) = -(\boldsymbol{\Gamma}_k^{22})^{-1} \boldsymbol{\Gamma}_k^{21} (\boldsymbol{x}_{\mathrm{p}} - \boldsymbol{x}_{\mathrm{pss}}) + \boldsymbol{x}_{\mathrm{rss}} \tag{23}$$

Proposition 3: Assume that Assumption 1 holds and $\boldsymbol{\Gamma}_k^{22} > 0$, then the reset law which minimises J_k of (12) is given by

$$\boldsymbol{x}_{\mathrm{r}}(t_k^+) = -(\boldsymbol{\Gamma}_k^{22})^{-1} \boldsymbol{\Gamma}_k^{21} (\boldsymbol{x}_{\mathrm{p}} - \boldsymbol{x}_{\mathrm{pss}}) + \boldsymbol{x}_{\mathrm{rss}} \tag{24}$$

Furthermore, if we consider equidistant reset control (i.e. $\Delta t_k \equiv \delta$ is a constant), then $\boldsymbol{\Gamma}_k \equiv \boldsymbol{\Gamma}$ is a constant matrix independent on k. According to the analysis above and by Corollary 1, we have the following.

Proposition 4: Suppose that $\Delta t_k = \delta$ is a constant and $\boldsymbol{\Gamma}^{22} > 0$, then the reset law which minimises J_k (12) is independent on k and is given by

$$\boldsymbol{x}_{\mathrm{r}}(t_k^+) = -(\boldsymbol{\Gamma}^{22})^{-1} \boldsymbol{\Gamma}^{21} (\boldsymbol{x}_{\mathrm{p}} - \boldsymbol{x}_{\mathrm{pss}}) + \boldsymbol{x}_{\mathrm{rss}} \tag{25}$$

In addition, the corresponding closed-loop system (4) under this reset law with $r = 0$ is asymptotically stable if and only if

$$\left| \lambda \left(\begin{pmatrix} \boldsymbol{I}_{n_{\mathrm{p}}} & \mathbf{0} \\ -(\boldsymbol{\Gamma}^{22})^{-1} \boldsymbol{\Gamma}^{21} & \mathbf{0} \end{pmatrix} \mathrm{e}^{\boldsymbol{A}\delta} \right) \right| < 1 \tag{26}$$

2.4 Trade-off between stability and performance

In the design of reset law, we have not taken account of stability. If the stability condition (26) cannot be satisfied, we have to retune the base controller or the reset law design parameters. Generally, the base linear system is designed to be asymptotically stable, then we can alternatively use the following reset law

$$\boldsymbol{x}_{\mathrm{r}}(t_k^+) = -\mu\Big[(\boldsymbol{\Gamma}_k^{22})^{-1} \boldsymbol{\Gamma}_k^{21} (\boldsymbol{x}_{\mathrm{p}} - \boldsymbol{x}_{\mathrm{pss}}) - \boldsymbol{x}_{\mathrm{rss}}\Big] + (1-\mu)\boldsymbol{x}_{\mathrm{r}}, \quad \mu \in [0, 1] \tag{27}$$

to compromise between the stability and the performance characterised by J_k. It is clear that when μ varies from 1 to 0, the reset control system reduces to the base linear system without reset that is assumed to be asymptotically stable.

On the other hand, according to (21), we have

$$\frac{\partial J_k}{\partial \mu} = \frac{\partial \boldsymbol{\xi}_{\mathrm{r}}(t_k^+)}{\partial \mu} \frac{\partial J_k}{\partial \boldsymbol{\xi}_{\mathrm{r}}(t_k^+)}$$

$$= -2(1-\mu)\Big[(\boldsymbol{\Gamma}_k^{22})^{-1} \boldsymbol{\Gamma}_k^{21} \boldsymbol{\xi}_{\mathrm{p}}(t_k) + \boldsymbol{\xi}_{\mathrm{r}}(t_k)\Big]^{\mathrm{T}} \times \boldsymbol{\Gamma}_k^{22}\Big[(\boldsymbol{\Gamma}_k^{22})^{-1} \boldsymbol{\Gamma}_k^{21} \boldsymbol{\xi}_{\mathrm{p}}(t_k) + \boldsymbol{\xi}_{\mathrm{r}}(t_k)\Big]$$

$$\leq 0, \quad |\mu| \leq 1$$

Denote $\boldsymbol{\rho}(\boldsymbol{\xi}, r, \mu) = \boldsymbol{x}_{\mathrm{r}}(t_k^+)$, thus $J_k(\boldsymbol{\rho}(\boldsymbol{\xi}, r, \mu))$ is monotonously decreasing when μ varies from 0 to 1. Thus we can always choose $\mu \in [0, 1]$ such that the closed-loop system is asymptotically stable and at the same time,

$$J_k(\boldsymbol{\rho}(\boldsymbol{\xi}, r, 1)) < J_k(\boldsymbol{\rho}(\boldsymbol{\xi}, r, \mu)) < J_k(\boldsymbol{\rho}(\boldsymbol{\xi}, r, 0))$$

The above inequality indicates that the performance index of the resulting reset control system is always less than that of the base linear system, though the minimal index cannot be achieved.

By Corollary 1, for equidistant reset control where $\Delta t_k = \delta$ and $\boldsymbol{\Gamma}_k = \boldsymbol{\Gamma}$ are constant, the overall system under the reset law (27) is asymptotically stable if and only if

$$\left| \lambda \left(\begin{pmatrix} \boldsymbol{I}_{n_{\mathrm{p}}} & \mathbf{0} \\ -\mu(\boldsymbol{\Gamma}^{22})^{-1} \boldsymbol{\Gamma}^{21} & (1-\mu)\boldsymbol{I} \end{pmatrix} \mathrm{e}^{\boldsymbol{A}\delta} \right) \right| < 1 \tag{28}$$

3 Experimental results

In this section, we verify the extended reset controller on a PZT microactuator-positioning stage (P-752 PZT Flexure Stage System, Polytec PI, Germany) as shown in Fig. 2*a* and compare the experimental results with a conventional reset controller.

3.1 Modelling of the PZT-positioning stage

The PZT-positioning stage consists of a PZT microactuator, a moving stage connected with the base via the flexures in four corners, a PZT power amplifier and an integrated capacitive position feedback sensor with 0.2 nm resolution to measure the displacement of the moving stage. The PZT microactuator is of high stiffness and has a maximum travel range of ± 15 μm. The mechanical resonance caused by the flexures is actively damped by the integrated control electronics. Thus, the dynamics of the PZT-positioning stage can be simply depicted by a mass-damper-spring

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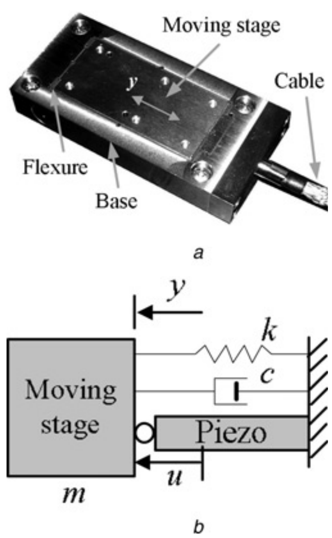

Figure 2 *PZT microactuator-positioning stage*

system as shown in Fig. 2*b*, which can be then described by a state space form as follows

$$\Sigma_{\mathrm{P}}:\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -a_1x_1 - a_2x_2 + bu\\ y = x_1\end{cases}\tag{29}$$

where x_1 and x_2 are the position and velocity of the moving stage, respectively, and u is the control input to the PZT amplifier.

The modal parameters in (29) are identified from experimental frequency response data. A dynamic signal analyser (HP 35670A, Hewlett Packard Company, Washington) is used to generate the swept-sinusoidal excitation signals and collect the frequency response data from the excitation signals u to the position output y. The dashed lines in Fig. 3 show the measured frequency responses of the

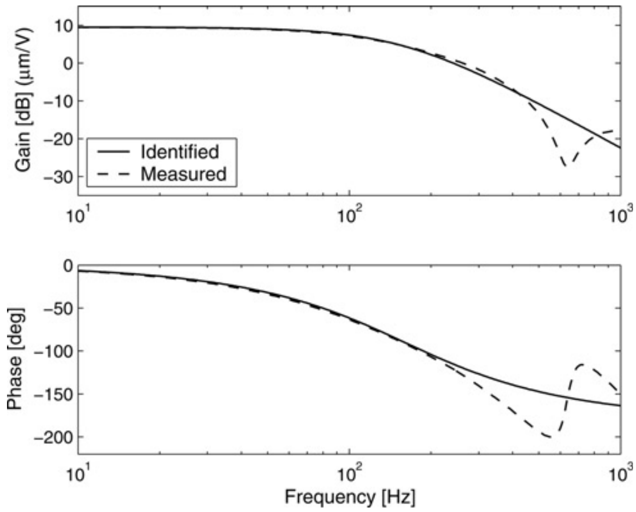

Figure 3 *Frequency responses of the PZT microactuator-positioning stage*

PZT-positioning stage. The PZT dynamics is of high stiffness that exhibits a flat gain in the low-frequency range. Using the least square estimation method [19], we obtain the modal parameters as follows

$$a_1 = 10^6, \quad a_2 = 1810, \quad b = 3\times 10^6$$

The solid lines in Fig. 3 show that the identified model match the measured model well in the frequency range of interest.

3.2 Reset control law

Our objective is to design a feedback controller for robust tracking of a step reference input with zero steady-state error and fast settling, for which high open-loop gains in low-frequency range and high bandwidth with sufficient stability margin are typically required. Thus, we employ the classical proportional–integral (PI) control structure as the base controller

$$\frac{u(s)}{e(s)} = k_{\mathrm{p}} + \frac{k_{\mathrm{i}}}{s}\tag{30}$$

where $k_{\mathrm{p}} = 0.08$ and $k_{\mathrm{i}} = 300$. The resultant base linear system has a closed-loop bandwidth 178 Hz, and gain/phase margin 11 dB/33°. Experimental result (see the dashed lines in Fig. 4) shows that the integrator increases the low-frequency gains and both fast rise time and zero steady-state error are achieved. However, the overshoot induced by the integrator is also significant (40% of the step level), which results in tedious settling time. Therefore it is expected that the overshoot could be reduced by resetting the integrator state to proper values.

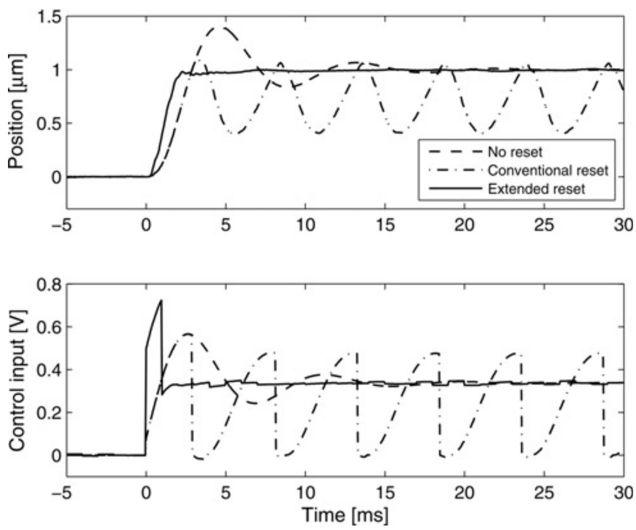

Figure 4 *Step responses ($r = 1\ \mu m$) of the reset control system*

The extended reset control exhibits little overshoot and the resultant settling time is 3 ms, which is identical to the rise time of the base linear system (no reset)

The conventional reset control still exhibits 15% overshoot in the first peak and causes limit cycles in steady state

To design the reset values, we set the reset time interval as a constant

$$\Delta t_k = 1\ \text{ms}$$

and select the tuning parameters of J_k in (12) as

$$P_0 = 2.1, \quad Q_0 = 1.0 \times 10^{-6}, \quad P_1 = 0 \qquad (31)$$

Thus, according to (13) and (20) we can easily obtain $\boldsymbol{x}_{\text{ss}}$ and $\boldsymbol{\Gamma}$ as

$$\boldsymbol{x}_{\text{ss}} = [1\ 0\ 0.0011]^{\text{T}}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1.16 & 0.0005 & 65.33 \\ 0.0005 & 0 & 0.16 \\ 65.33 & 0.16 & 230042 \end{bmatrix}$$

The resulting reset controller is then described in state space as follows

$$\begin{cases} \dot{x}_{\text{r}} = e, & t \neq t_k \\ x_{\text{r}}(t_k^+) = E_1 x_1 + E_2 x_2 + Gr, & t = t_k \\ u = k_{\text{i}} x_{\text{r}} + k_{\text{p}} e \end{cases} \qquad (32)$$

where $E_1 = -2.8 \times 10^{-4}$, $E_2 = -6.8 \times 10^{-7}$, $G = 0.0014$, $k_{\text{i}} = 300$ and $k_{\text{p}} = 0.08$. Moreover, it is easy to verify that the resulting reset control system satisfies the stability condition in (26), which implies that the unforced closed-loop system is asymptotically stable. For comparison, we also design a conventional reset controller as follows

$$\begin{cases} \dot{x}_{\text{r}} = e, & e(t) \neq 0 \\ x_{\text{r}}(t^+) = 0, & e(t) = 0 \\ u = 300 x_{\text{r}} + 0.08 e \end{cases} \qquad (33)$$

In fact, the conventional reset controller (33) can be seen as a special case of the extended reset controller. Thus, the stability of the closed-loop system under (33) can be easily verified through (11).

3.3 Results and discussion

The reset controllers were implemented by a real-time DSP system (dSPACE-DS1103) with the sampling time of $T_{\text{s}} = 50\ \mu\text{s}$. The position of the moving stage x_1 or equivalently the output y can be directly obtained through the sensor output. We estimate the velocity x_2 by backward differentiation of the position signal $x_1(t)$, that is

$$\hat{x}_2 = \frac{z-1}{T_{\text{s}} z} x_1$$

Fig. 4 shows the experimental results for 1 µm step response. It can be seen that the extended reset control nearly removes the overshoot and thus reduces the settling time from 15 ms (no reset) to 3 ms. Moreover, we observe that the extended reset control has a faster transient response compared with that under no reset because the integrator state was reset to minimise J_k from the beginning at $t = 0$ leading to a larger control input at the initial stage and thus faster response. When the position output approaches the target, the integrator state is reset to a smaller value (see the control input at $t = 1$ ms in Fig. 4) to reduce the overshoot and keep the moving stage at the desired position. In this case, the conventional reset control works badly, which can only partially reduces the first overshoot peak and results in limit cycles. This is because resetting the integrator state to zero tends to resetting the control input to zero, which will cause the moving stage going to its initial position due to the high stiffness of the PZT actuator. Thus, the conventional reset control needs an intentional interplay between the reset mechanism and an appropriately designed base linear controller [7].

Next, we test the step responses of the extended reset control system to various step levels. The results are shown in Fig. 5, which indicates that the overshoots in all cases are nearly removed and the settling time are still maintained to be 3 ms. In this paper, we have not theoretically considered how to select the reset time interval, which is in fact related to the overall system performance. This is however evaluated through experiments. To do this, we use the same design parameters (31) but select various reset time intervals. Following the same design procedure, we obtain a set of reset controller for implementation. The results in terms of net overshoot and settling time are summarised and compared in Fig. 6. It is interesting to see that a smaller reset time interval tends to exhibit less overshoot but results in longer settling time (i.e. slower system dynamics). Under the range of 0.4–2 ms, the settling time and overshoot simultaneously reach a low level. Thus, tuning the reset time interval as 20–60% of the rise time (3 ms in this case) by

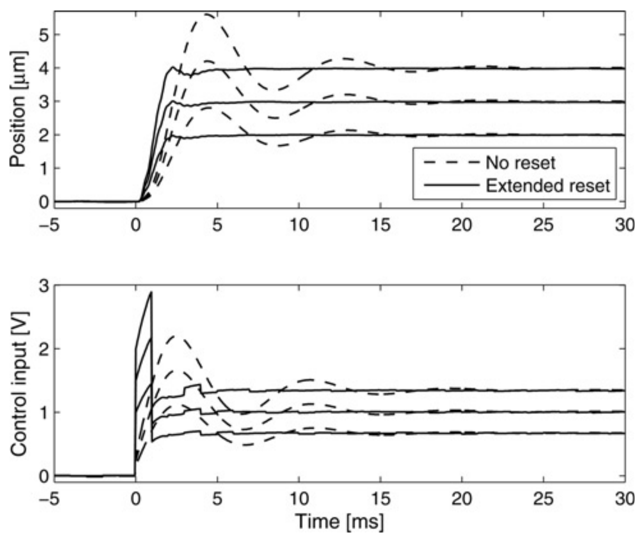

Figure 5 *Time responses of the reset control system to various step levels (r = 2, 3, 4 µm)*

The overshoots in all cases are nearly removed and the settling times are maintained to be 3 ms

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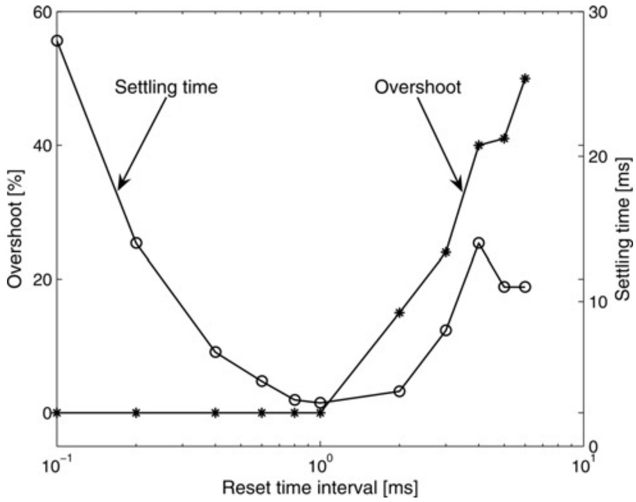

Figure 6 *Relationship between reset time interval, overshoot and settling time of the extended reset control system*

the base linear system might achieve relatively good performance.

Finally, we test the robustness of the extended reset control system against input disturbance and sensor noise. Figs. 7 and 8, respectively, show the time responses to a single-frequency (100 Hz) sinusoidal input disturbance u_d and sensor noise n, which are artifically introduced to the control system. We can see that the extended reset control simultaneously provides an improvement of 65% in both disturbance and noise suppression based on the base linear system (no reset). Further, we experimentally analyse the properties of disturbance and noise suppression in a wide frequency range according to the describing function approach. We used a dynamic signal analyser (DSA) to generate swept-sinusoidal excitation signals, which are then injected to the control input or the sensor output, respectively. The tests of disturbance and sensor noise responses are performed individually. The DSA is also used to collect the frequency response data from the position output to the excitation signals. Since the reset control system is essentially nonlinear, its frequency response may depend on the excitation levels. We thus vary the excitation level from 0.1 to 3 evenly spaced by 0.1, and it is interesting to find that the Bode plots of the frequency responses in all cases are very close to that in Fig. 9. In the plot, we can see that improved disturbance/noise suppression occurs simultaneously around 100 Hz, which matches the results in Figs. 7 and 8. However, we also find that the extended reset control adversely increases the low-frequency disturbance (<60 Hz) and high-frequency noise (>200 Hz) reduction ratio. This may be because the reset values in the steady state are calculated based on optimal

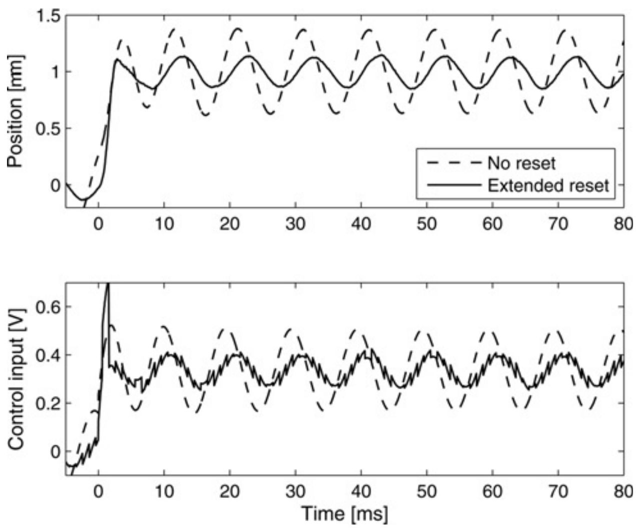

Figure 7 *Time responses of the reset control system to step input $r = 1$ μm and sinusoidal input disturbance $u_d = 0.1\sin(2\pi 100t)$ V*

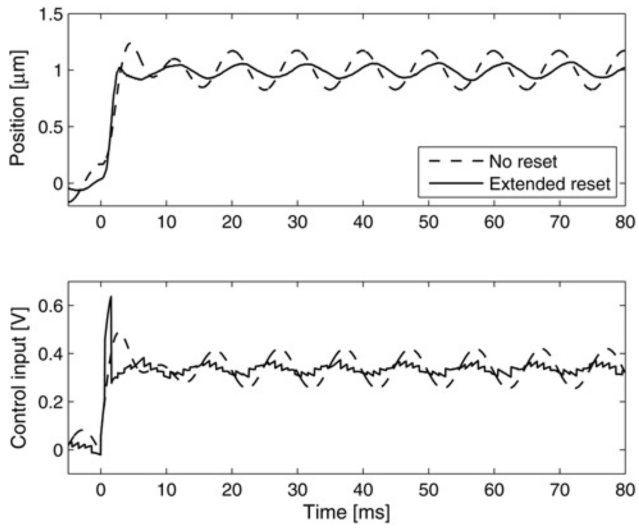

Figure 8 *Time responses of the reset control system to step input $r = 1$ μm and sinusoidal sensor noise $n = 0.1\sin(2\pi 100t)$ μm*

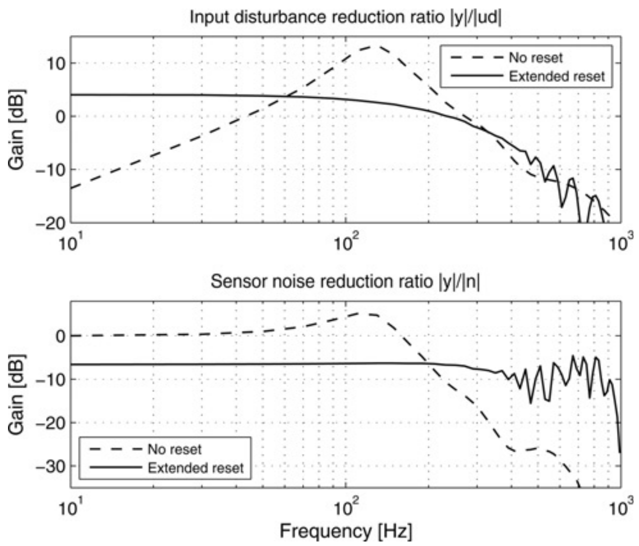

Figure 9 *Bode plot for describing functions of input disturbance/sensor noise suppression; the excitation level of u_d is 0.1 V and n is 0.1 μm*

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step-tracking control design instead of disturbance/noise rejection. Since the reset control system has nonlinear features, it cannot guarantee low-frequency disturbance rejection and high-frequency noise suppression as linear control does. One possible way to improve this is to include the disturbance and noise model in the synthesis of the reset law design, which is left in our future works.

4 Conclusions and future works

This paper has demonstrated an extended reset control system by resetting its controller states to certain non-zero values, which is more general than the conventional reset control which can only reset its controller state to zero. Some new results of stability were given, which do not rely on the stability of the base linear system. We also provide a reset law design method based on optimisation technique. Experimental results on a PZT-positioning stage showed that the extended reset control can achieve faster settling time than the conventional reset control by nearly overshoot removal. Moreover, it can simultaneously achieve sinusoidal disturbance and sensor noise suppression at some frequencies, though this is not guaranteed in other frequencies.

We still have a lot of challenges in the future work. The reset time interval is selected via experience by far. We need a theoretical design of the reset law and reset time interval (constant or time-varying) to minimise some performance index with finite or infinite horizon. In addition, we ultimately attempt to obtain conditions under which a reset system has disturbance and noise attenuation simultaneously in a wide frequency range. The proposed reset control can also be easily extended to nonlinear systems. Hence, our future works include the experimental demonstration on a nonlinear system platform and seeking other applications of the proposed reset control technique.

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