Comments on "Optimal Gain for Proportional-Integral-Derivative Feedback"

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Editor's Note: Several readers, including Dr. T. Hägglund of SattControl Instruments in Sweden, have written in to say that there were discrepancies in the numerical values listed in Tables 1, 2, and 3 in the paper "Optimal Gain for Proportional-Integral-Derivative Feedback" by Salem A. K. Al-Assadi and Lamya A. M. Al-Chalabi, which appeared in the December 1987 issue of the *IEEE Control Systems Magazine* [1]. The authors, Al-Assadi and Al-Chalabi, apologize for these errors. Part of the discrepancies may have been due to misinterpretation of terms, but there are some even more fundamental problems as explained in this comment on that paper.

ABSTRACT: This comment concerns the comparison of tuning parameters for proportional-integral-derivative (PID) controllers proposed in [1]. We focus on the time-delay example contained in [1] and the resulting Table 2; however, we first remark on some fundamental points regarding PID controllers that have been misunderstood in [1] as well as in Kinney [2], which serves as one of the principal references for [1].

Background and Preliminary Remarks

Recently, we have been working on problems of robust stability of time-delay systems. In looking for a "real" example, we focused on the one given in [1], where a model for a heat exchange process is given as a first-order system with time delay:

$$G(s) = \exp(-\tau_{DT}s)/(\tau_1 s + 1) \quad (1)$$

with τ_{DT} given to be 0.67 min and τ_1 given to be 0.33 min. A proportional-integral-derivative (PID) controller of the following form was proposed in [1]:

$$G_c(s) = K_p + K_I/s + K_D s \quad (2)$$

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Our initial idea was to investigate the stability of the closed-loop system with respect to parameter variations, using the proposed values in Table 2 of [1] as nominal values. This proved impossible since a Nyquist plot of the proposed closed-loop system showed it to be unstable. Further investigation of [1] has led to this comment.

The importance of the proper tuning of PID controllers has been recognized for some 50 years. Various methods to accomplish the desired tuning are contained in [2]–[4] and more recently in [5]—to cite but a few of the many possible references. Most of these methods are empirical and are "optimal" only in the sense of yielding "good" practical values in many typical cases. It is well known that classical tuning methods do not produce optimal settings in terms of minimizing the integral-squared-error (ISE). They are by no means optimal in any strict mathematical sense. In fact, Shinskey ([4], Table 4.3, p. 97) points out that for "noninteracting" PID controllers, the optimum settings require large K_D, making the closed-loop system more sensitive to process parameter variations. Furthermore, some authors (see [5], p. 231) consider the ISE criterion to be *unacceptable* due to the highly oscillatory nature of the resulting closed-loop system. In view of the preceding remarks, it is a basic misconception to suggest, as is done in both [1] and [2], that the methods used for comparison purposes in [1] yield optimal PID settings or that minimizing the ISE yields the best PID controller settings. Since the process model is only an approximation, we believe robustness with respect to model uncertainties is more important than optimality (methods used for comparison purposes in [1] at least provide good stability margins). Furthermore, comparing methods without mentioning the inherent assumptions and limitations can lead to erroneous conclusions: The various methods are "good" only under certain assumptions, for example, to quote ([5], p. 225), "these formulas [Ziegler-Nichols open loop] ... should not be extrapolated outside a range of τ_{DT}/τ_1 of around 0.1 to 1.0." For the time-delay example in [1], $\tau_{DT}/\tau_1 = 2.03$, again calling into question the validity of the comparisons made in [1].

Time-Delay Example

A PID representation that is equivalent to Eq. (2) is

$$G_c(s) = K[1 + 1/(T_i s) + T_d s] \quad (3)$$

Thus, comparing Eqs. (2) and (3), we have

$$K_p = K$$

$$K_I = K/T_i$$

$$K_D = KT_d \quad (4)$$

The columns in Table 2 of [1] are not labeled properly [implementing what the authors propose on a process described by Eq. (1)

Table 2 (revised)
Results for Time-Delay Example

Method	Parameters of PID Controller: K	T_d	T_i
Ziegler-Nichols (Open Loop)	0.591	0.335	1.34
Ziegler-Nichols (Closed Loop)	0.910	0.229	0.915
Cohen and Coon [2]	0.766	0.176	1.06
Shinskey (Noninteracting) [4]	0.755	0.146	0.622
Proposed	0.8416	0.3555	0.4359

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results in an unstable closed-loop system] and many of the values given in this table are incorrect. Using Eq. (3), we are convinced that K_p, K_D, and K_I in Table 2 of [1] must be replaced by K, T_d, and T_i, respectively. (*Editor's Note:* This assertion has been confirmed by the authors of [1].) If Table 2 of [1] is revised as given herein, then the values proposed by the authors do yield a stable system; the robustness of their proposed controller, however, is relatively poor. For example, if τ_1 is decreased by about 10 percent from 0.33 to 0.298, the closed-loop system becomes unstable, i.e., for $\tau_1 = 0.298$, $s = j42.22$ is a root of the characteristic equation. We now give detailed calculations to show how the values in the revised version of Table 2 are obtained.

Kinney [2] references Shinskey [4], who gives the PID representation (where P is in percent) as

$$G_c(s) = (100/P)[1 + 1/Is + Ds] \quad (5)$$

Thus, using the notation in Eqs. (2), (3), and (5), we have

$$K = 100/P = K_p$$
$$T_i = I = K_p/K_I$$
$$T_d = D = K_D/K_p \quad (6)$$

Using the Ziegler-Nichols open-loop method, we have from Eqs. (11a)-(11c) of [2], with $P = PB$ and $R_R = 3.7$ min in [2],

$$K = 1.2/(\tau_{DT}R_R) = 0.484$$
$$T_i = 2\tau_{DT} = 1.34$$
$$T_d = 0.5\tau_{DT} = 0.335$$

These are the values given in Table 2 of [1]. However, it should be pointed out that if we utilize Eq. (1) as the process model (the authors make this assumption in calculating their "proposed" values), R_R in [2] should be replaced by $1/\tau_1$, resulting in $K = 0.591$.

In Eqs. (12a)-(12c) of [2], the Ziegler-Nichols closed-loop relations are given (once again P equals PB) as

$$P = 1.66P^*$$
$$I = 0.5\tau_0$$
$$D = \tau_0/8$$

The variable τ_0 is the "natural period" found by using only proportional control and adjusting P to P^* (PB^* in [2]) so that the closed-loop process oscillates around the set point. Note that in [2], the value of τ_0 is measured to be 2.2 min, and P^* is chosen as 140 percent. Using these values yields the results given in Table 2 of [1], i.e.,

$$K = 0.43$$
$$T_d = 0.275$$
$$T_i = 1.1 \quad (7)$$

This supports our interpretation of Table 2. If, however, we assume the process model (1) to be exact, then τ_0 and P^* can be calculated explicitly as follows. Assuming a proportional controller, the closed-loop characteristic equation is as follows:

$$1 + \left(\frac{100}{P}\right)\frac{\exp(-\tau_{DT}s)}{(\tau_1 s + 1)} = 0 \quad (8)$$

Replacing s by $j\omega$ and solving for the values of P and ω (denoted by P^* and ω^*), which produce oscillations, is equivalent to solving

$$\left(\frac{100}{P}\right)\frac{\exp(-0.67j\omega^*)}{(0.33j\omega^* + 1)} = -1 \quad (9)$$

Setting the amplitude of Eq. (9) equal to 1 and the argument equal to π yields $\omega^* = 3.43 = 2\pi/\tau_0$, resulting in $\tau_0 = 1.83$ min and $P^* = 66.2$ percent. Thus, from Eqs. (12a)-(12c) of [2], Eqs. (4) and (6) here, we have

$$K = 100/1.66P^* = 0.910$$
$$T_d = \tau_0/8 = 0.229$$
$$T_i = I = 0.5\tau_0 = 0.915 \quad (10)$$

The difference between the values in Eq. (10) and those in Eq. (7) is that the theoretical model [Eq. (1)] does not match the real process considered in [2]. In fact, the Ziegler-Nichols closed-loop method really *does not assume any process model* and is based on process measurements. Thus a comparison between the authors' method and the Ziegler-Nichols closed-loop method *should use the values given in Eq. (10), if it is at all meaningful.*

We now turn to the Cohen and Coon method [2]. Since the original reference [6] was unavailable to us, we assume that [2] is correct. From Eqs. (13a)-(13c) of [2], we have, with $\mu = \tau_{DT}/\tau_1$,

$$K = \frac{100}{P} = 1.35\frac{(1 + \mu/5)}{\tau_{DT}R_R} = 0.766$$

$$T_d = D = 0.37\frac{\tau_{DT}}{(1 + \mu/5)} = 0.176$$

$$T_i = I = 2.5\tau_{DT}\left[\frac{(1 + \mu/5)}{(1 + 3\mu/5)}\right] = 1.06$$

These values, with the exception of T_d, do not agree with those in [1]!

For Shinskey's method, the values given in Table 2 of [1] agree with those that can be calculated from Eqs. (14a)-(14c) of [2], with $P^* = 140$ percent and $\tau_0 = 2.2$ min. We believe, however, that Eqs. (14a)-(14c) of [2] contain errors. Going directly to the procedure given in Shinskey ([4], p. 99), we find that his "noninteracting controller" parameters are as follows:

$$P = 2.0P^*$$
$$I = 0.34\tau_0 \ (\tau_R \text{ in [4]})$$
$$D = 0.08\tau_0 \quad (11)$$

Note that the coefficients in Eq. (11) are not those given in [2]. Using P^* and τ_0 calculated from Eq. (9) in Eq. (11) yields

$$K = 0.755$$
$$T_i = 0.622$$
$$T_d = 0.146$$

In summary, we believe the corrected values of the tuning parameters are given here in the revised version of Table 2.

References

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