Proceedings of the 30th Conference
on Decision and Control
Brighton, England • December 1991

Dynamic Decoupling of MIMO Systems: Nonlinear Case

Peter W. Gibbens
Department of Electrical
Engineering and Computer Science
University of Newcastle, N.S.W.
2308 Australia

Carla A. Schwartz
Department of Computer Science
and Electrical Engineering
University of Vermont, Burlington,
VT. 05405-0156 U.S.A.

Minyue Fu
Department of Electrical
Engineering and Computer Science
University of Newcastle, N.S.W.
2308 Australia

Abstract. It is known that static state feedback compensation is insufficient to decouple nonlinear systems which do not have *vector relative degree.* Instead, a dynamic precompensation is first required to achieve vector relative degree. In such cases, a non-diagonal dynamic precompensation can ordinarily be used. However, a diagonal dynamic precompensator is usually preferred as an exact knowledge of system parameters is then unnecessary. In this paper, we determine conditions under which a diagonal dynamic precompensation is sufficient to achieve vector relative degree for multivariable nonlinear systems, and describe a simple algorithm which determines such compensation.

1 Introduction

There has been much effort applied in solving the problem of decoupling of multivariable nonlinear systems [1]-[3]. Decoupling is usually achieved by applying a precompensator to the plant, or equivalently, by applying a state feedback when the full state of the system is available, see for example [1] and the references therein.

It is known that the decoupling of a multivariable nonlinear system is closely related to the so-called vector relative degree of the system, which serves as the generalisation of relative degree for single-input-single-output systems (see [1]). For systems with a vector relative degree, decoupling can be simply realised by using *static* state feedback compensation. This case has been studied by many authors, see for example [1]. For systems which do not have vector relative degree, *dynamic* state feedback compensation is required. An excellent description of the general approach to static and dynamic decoupling of nonlinear systems is found in [3], and the references therein.

An alternative approach to decoupling is to study the following decouplability problem: given a multivariable nonlinear system which does not have vector relative degree, search for a diagonal precompensator such that the resulting system will achieve vector relative degree. Once this precompensation is found, the resulting system can be decoupled by using static state feedback compensation, as already mentioned. This approach has been used for linear systems [4], [5], [6] and for nonlinear systems [1]. In [1] the so-called Dynamic Extension Algorithm is applied for finding the diagonal precompensator. Diagonal dynamic precompensators are usually preferred as an exact knowledge of system parameters is then unnecessary for achieving decouplability.

In this paper, we provide a *necessary and sufficient* condition for the existence of diagonal precompensation which achieves vector relative degree. Based on this, a simple algorithm for finding the diagonal precompensator is given. The result is an extension of the result developed by the authors [6] and is similar to another by [7], for linear systems. We show that the decouplability question is essentially associated with the nonsingularity or (non)generic singularity of a certain matrix related to the system.

2 Problem Formulation and Preliminaries

Consider the following class of multivariable nonlinear systems:

$$\begin{aligned} T: \ \dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) + k(x(t))u(t) \end{aligned} \tag{1}$$

where the state $x(t) \in M$, with M being a submanifold of $\mathbf{R}^n$, the control $u(t) \in \mathbf{R}^m$, the output $y(t) \in \mathbf{R}^m$, and $f(\cdot), g(\cdot), h(\cdot)$ and $k(\cdot)$ are smooth functions of appropriate dimensions. The matrix $k(x)$ is called the direct transmission matrix. In the sequel, we denote the system (1) by $y = T(u)$ or simply T, the row vectors of f, g, h and k by f_i, g_i, h_i and k_i, the elements of g and k by g_{ij} and k_{ij}, and the differential operator d/dt by ρ.

Definition 2.1 *Given $x_o \in M$, the system T in (1) is called* bicausal *at x_o if there exists a neighbourhood N of x_o such that $k(x)$ is nonsingular for all $x \in N \subset M$. The system T is called* bicausal *if it is bicausal at every point $x \in M$.*

For simplicity of exposition, we will implicitly assume that T is restricted to a connected submanifold on which the properties of interest, such as bicausality, hold.

We now give a definition which is a prerequisite to the results established in Section 3. It concerns the concepts of generic and nongeneric singularity of transfer matrices as discussed by Singh and Narendra [7], however as we find the discussion and definition therein to be somewhat unclear, we outline these concepts from a different and more precise viewpoint.

Definition 2.2 *Given a set of m $n-$dimensional row vectors $\{v_i = (v_{i1}\ v_{i2}\ \ldots v_{in});\ i = 1, \ldots, m\}$, v_1 is said to be generically linearly dependent on $v_2, \ldots, v_m$ if for every possible vector $\tilde{v}_1 = (\tilde{v}_{11}\ \tilde{v}_{12}\ \ldots v_{1n})$ defined by*

$$\tilde{v}_{1j} = 0 \ \ if \ v_{1j} = 0 \tag{2}$$

$$\neq 0 \ \ if \ v_{1j} \neq 0, \ j = 1, \ldots, n, \tag{3}$$

the vectors $\tilde{v}_1, v_2, \ldots, v_m$ are linearly dependent, i.e., the linear dependency is independent of the specific values of the nonzero elements of v_1. Otherwise, v_1 is said to be nongenerically linearly dependent. By the same token, a set of linearly dependent row vectors are said to be generically linear dependent if at least one of them is generically linearly dependent on the others, otherwise they are said to be nongenerically linearly dependent.

These definitions also apply to column vectors by considering their transposes, and to matrices by considering their row/column vectors.

Remark 2.1: The definition 2.2 above indicates that the linear dependence of a set of nongenerically linear dependent vectors can be invalidated by varying the nonzero elements of the vectors. The number of vectors must exceed one in order to have nongeneric linear dependence. Furthermore, a set of row (resp. column) vectors are generically linearly dependent *if and only if* either of the following cases happens:

(i) there is a zero row (resp. column);

(ii) there exists a subset of vectors such that by forming them as a matrix, the number of nonzero columns (resp. rows) in the matrix is strictly less than the number of rows (resp. columns) in the matrix, i.e., the nonzero columns (resp. rows) form a "tall" (resp. "wide") submatrix.

3 Main Result

Given the system T in (1) and M, we wish to define conditions under which we can find a diagonal dynamic precompensator of the form

$$D(\rho^{-1}) = \text{diag}\{\rho^{-d_j}\}, \quad d_j \geq 0, \quad 1 \leq j \leq m \qquad (4)$$

which ensures that the composite system $T \circ D(\rho^{-1})$ has some vector relative degree $\{r_1, r_2, \cdots, r_m\}$ on M. i.e., the new system $K = R(\rho) \circ T \circ D(\rho^{-1})$ is bicausal, where

$$R(\rho) = \text{diag}\{\rho^{r_1}, \ldots, \rho^{r_m}\}. \qquad (5)$$

Defining $z(t) = R(\rho)y(t)$ and $u(t) = D(\rho^{-1})v(t)$, the system K can be written as follows:

$$K: \begin{array}{rcl} \dot{\bar{x}}(t) &=& \bar{f}(\bar{x}(t)) + \bar{g}(\bar{x}(t))v(t) \\ z(t) &=& \bar{h}(\bar{x}(t)) + \bar{k}(\bar{x}(t))v(t) \end{array} \qquad (6)$$

where $\bar{x}$ is the state of the new system which contains the state of T and new internal state variables introduced by $D(\rho^{-1})$.

Assumption 3.1 (well posedness): The nonlinear system (1) is called well-posed if the following conditions are satisfied:

(i) For every $1 \leq i \leq m$ there exists some $u \in R^n$ and integer $r_i \geq 0$ such that the direct transmission from u to $\rho^{r_i} z_i$ is nonzero for all $x \in M$.

(ii) For every $u \in R^m$, there exists $1 \leq i \leq m$ and $\hat{r}_i$ such that the direct transmission from u to $\rho^{\hat{r}_i} z_i$ is nonzero for all $x \in M$.

Remark 3.1: Condition (i) is equivalent to saying that there exists $\{r_1, \ldots, r_m\}, r_i \geq 0$, such that every row of the matrix $\bar{k}$ in (6) is nonzero for all $x \in M$ (but $\bar{k}$ is not necessarily nonsingular). That is, each z_i should be "influenced" by *some* u directly through $\rho^{r_i} z_i$, Condition (ii) implies that every u should have "influence" in some z_i directly through $\rho^{\hat{r}_i} z_i$. These two conditions are satisfied by most nonlinear engineering systems and all linear systems with nonsingular transfer matrices.

We now have the following result.

Theorem 3.1 *Given the system $y = T(u)$ in (1) satisfying Assumption 3.1, one of the following two cases must occur and they are mutually exclusive:*

(i) There exists a pair $D(\rho^{-1})$ and $R(\rho)$ in the form of (4) and (5) such that the direct transmission matrix $\bar{k}(\bar{x})$ of the resulting system K in (6) is nonsingular on M. In this case, $T \circ D(\rho^{-1})$ has vector relative degree described by $R(\rho)$ and therefore is decouplable by static state feedback.

(ii) There exists $D(\rho^{-1})$ and $R(\rho)$ in the form of (4) and (5) such that the direct transmission matrix $\bar{k}(\bar{x})$ is nongenerically singular for some $x \in M$. In this case, there exists no other diagonal precompensator $D(\rho^{-1})$ of the form (4) such that $T \circ D(\rho-1)$ has vector relative degree.

The theorem above indicates that we only need to find $R(\rho)$ and $D(\rho^{-1})$ which make $\bar{k}(\bar{x})$ either nonsingular or nongenerically singular in order to determine whether a suitable precompensator exists or not. The means for finding such a precompensator is given in the following algorithm. The proof for Theorem 3.1 and Algorithm 3.2 is given in [8].

Algorithm 3.2 Initially let $D(\rho^{-1}) = I$.

Step1. Find $R(\rho)$ such that every row of $\bar{k}(\bar{x})$ is nonzero. There are three possibilities:

1. $\bar{k}(\bar{x})$ is nonsingular on M: $D(\rho^{-1})$ is a diagonal precompensator for T and $R(\rho)$ gives the associated vector relative degree.
2. $\bar{k}(\bar{x})$ is nongenerically singular at some $x \in M$: No diagonal compensator exists which will give a vector relative degree.
3. $\bar{k}(\bar{x})$ is generically singular on M: Proceed to Step 2.

Step 2. Extract the maximum number of rows of $\bar{k}(\bar{x})$, each of which are generically linearly dependent of the others. Note that the nonzero columns in these rows form a tall matrix. Denote this set of rows by i, the set of nonzero columns by j, and the set of remaining columns by $j^{\perp}$. Then for each column index l in j, increment d_l by 1. Return to step 1.

The algorithm is complete when either of cases (i) or (ii) is achieved.

References

[1] Isidori, A. (1989) *Nonlinear Control Systems: An Introduction,* 2nd ed., Springer-Verlag, Berlin.

[2] Nijmeijer, H. and A. J. van der Schaft, (1990) Nonlinear Dynamical Control Systems, Springer, New York.

[3] Descusse, J. (1990) New results on the structure of non-linear systems: application to the (dynamic) block-decoupling problem, *International J. of Adaptive Control and Signal Processing,* vol. 4, 453-464.

[4] Falb, P. L. and W. A. Wolovich, (1967) Decoupling in the design and synthesis of multivariable control systems, *IEEE Transactions on Automatic Control,* 12, 651-659.

[5] Descusse, J. (1980) State feedback decoupling with stability of linear constant (A, B, C, D)-quadruples, *IEEE Transactions on Automatic Control,* 25, 739-743. Correction:*IEEE Transactions on Automatic Control,* 26, 974-975.

[6] Gibbens, P. W., C. Schwartz and M. Fu, (1991) Dynamic decoupling of MIMO systems: linear case, *Proc. American Control Conf.*

[7] Singh, R. P. and K. S. Narendra, (1984) Prior information in the design of multivariable adaptive controllers, *IEEE Transactions on Automatic Control,* AC-29, No. 12, 1108-1111.

[8] Gibbens, P. W., C. Schwartz and M. Fu, (1991) Dynamic decoupling of MIMO systems: nonlinear case, Tech. Report, EE90102, Dept. Electrical and Computer Eng., Univ. of Newcastle, Australia.