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Localization based Switching Adaptive Controllers*

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Abstract: Many different types of adaptive or "universal controllers", capable of dealing with a very broad range of linear time-invariant systems, have been proposed. One class of such controllers uses switching between a finite, or at least countable, number of fixed controllers until stability is detected. Such controllers are very attractive from a theoretical viewpoint, in providing stabilization and asymptotic performance for a broad class of plants. However, such controllers are also known to have very poor transient properties, due to the long time required to search for a stabilizing feedback. The key contribution of this paper is to introduce the "method of localization" which can greatly improve the speed of the search. The method is described for linear time-invariant discrete time systems of known nominal order, with disturbances and noise present. Analysis and simulations demonstrate the potential for greatly improved transient performance.

1 Introduction

Significant research into adaptive control has been conducted by many authors for some time, see for example [1]-[15]. One fundamental motivation for this research is the desire to construct a "universal controller" which relaxes the assumptions of "classical" adaptive control; such as minimum phase, known plant order and relative degree, size of exogenous disturbances. Since the early 1980s the assumptions required have been weakened significantly [4]-[15], in many cases by using a switching adaptive controller. For example, in [8] it was shown that the only a priori information which is needed for adaptive stabilization is the order of a linear time-invariant stabilizing controller. A switching control algorithm proposed in [4] and its discrete-time generalization [1] not only provide global stability of the closed-loop system, but do so in the sense of Lyapunov. Some later results on this subject include work done in [9] and its discrete-time extension given in [7]. However, as has been pointed out by many researchers (see, for example, [7]) these "universal controllers" are far from being ideal. One of the main drawbacks of the proposed algorithms is that of the poor transient performance of the algorithms described in [1, 4, 7, 8, 9]. This poor transient performance can be attributed in part to the exhaustive search procedures utilised.

To alleviate the problem of exhaustive search, two approaches have been proposed recently. The first approach is called supervisory control [11, 13, 6] which employs a family of on-line plant estimators (predictors) and determines the switching strategy based on the prediction errors. This supervisory control approach guarantees exponential stability. Also, simulation results demonstrate that supervisory control usually gives better performance than "non-supervisory" switching control methods. A second approach to avoiding excessive search times is to reduce the number of controllers required to stabilize the plant family, see for example [3].

We develop a new method of discrete-time adaptive control design, namely, the method of localization. The properties of compactness [4] and robustness of the family of the controlled plants are essential ingredients in the proposed method. The main idea of this method is to reduce the problem of adaptive stabilization to effective localization on a finite set of potentially stabilizing feedbacks. The main concepts of the proposed method are developed in this paper for the class of linear single-input/single-output uncertain systems. We show that typical assumptions such as minimum phase, known plant order and relative degree, availability of information about exogenous disturbances may not be necessary for rapid localization.

2 Preliminaries

We consider linear time invariant discrete-time plants:

$$D(q)\tilde{y}_t = N(q)u_t + \zeta_t \quad (1)$$

$$y_t = \tilde{y}_t + v_t \quad (2)$$

where $D(q)$ and $N(q)$ are polynomial operators in the forward shift operator, q:

$$D(q) = q^n + d_{n-1}q^{n-1} + \cdots d_0 \quad (3)$$

$$N(q) = n_{n-1}q^{n-1} + \cdots + n_0 \quad (4)$$

ζ_t is an input disturbance term, v_t is a measurement noise term, u_t is the system input and y_t is the measurement of the system output $\tilde{y}_t$.

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We consider a set of plants of the form (1), (2), which we classify by the set:

$$\Omega = \{(D(q), N(q)) : \; D(q) \in \Omega_D, N(q) \in \Omega_N\} \qquad (5)$$

We would like to construct an adaptive controller such that the closed-loop system is globally stable for any $(D, N) \in \Omega$. Two important, though not restrictive, assumptions we require are as follows:

Assumption 2.1 *The set Ω is compact.*

Assumption 2.2 *For all $(D, N) \in \Omega$, there exist no common zeros of $D(q)$ and $N(q)$ on or outside the unit disk.*

In other words, Ω contains only a set of output stabilizable plants. Furthermore, since Ω is compact, then it is a set of "uniformly" output stabilizable plants. We introduce an equivalent non-minimal (though in view of Assumption 2.2 stabilizable and detectable) state space description of the plant (1)-(2):

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + e_{2n-1}\epsilon_t &(6)\\ y_t &= e_{2n-1}^T x_t &(7)\end{aligned}$$

where

$$x_t^T = [u_{t-n+1}, \cdots u_{t-2}, u_{t-1}, y_{t-n+1}, \cdots y_{t-1}, y_t] \qquad (8)$$

$$\epsilon_t = \zeta_{t-n+1} + \Sigma_{i=0}^{n-1} d_i v_{t-i} \qquad (9)$$

$$A = \begin{bmatrix} I^+_{(n-1)\times(n-1)} & 0 \\ 0 & I^+_{n\times n} \\ \underline{n} & -\underline{d} \end{bmatrix} \qquad (10)$$

($\underline{n}=[\; n_0 \;\; n_1 \;\; \cdots \;\; n_{n-2} \;]$, $\underline{d}=[\; d_0 \;\; d_1 \;\; \cdots \;\; d_{n-1} \;]$, $I^+_{mxm} \in \mathbb{R}^{m\times m}$ is a square matrix of zeros, except for ones on the super-diagonal, that is $I^+_{mxm} = [\delta_{i,j-1}]$);

$$B^T = \left[0 \; \cdots\cdots \; 0 \vdots 1 \vdots 0 \; \cdots\cdots \; 0 \vdots n_{n-1} \right] \qquad (11)$$

and e_i is the i^{th} standard basis vector for $\mathbf{R}^{2n-1}$.

Under Assumptions 2.1 and 2.2 there exist finite decompositions of the set Ω into possibly intersecting sets $\Omega_i, i = 1, \cdots s$ so that for each i, there exists a single controller transfer function $H_i(q)$, such that for all $(D, N) \in \Omega_i$, the controller $u_t = -H_i(q)y_t$ stabilizes (1),(2).Any linear time invariant controller $H(q)$ of order $(n-1)$ can be rewritten as:

$$u_t = -Kx_t \qquad (12)$$

where

$$K = [f_0 \cdots f_{n-2}, -g_0 \cdots - g_{n-1}] \qquad (13)$$

and $H(q)$ is

$$H(q) = \frac{g_{n-1}q^{n-1} + g_{n-2}q^{n-2} + \cdots + g_0}{q^{n-1} + f_{n-2}q^{n-2} + \cdots + f_0} \qquad (14)$$

The main idea behind the switching adaptation method is as follows. It is possible to define a finite number of potentially stabilizing linear time-invariant controllers $\{K_i\}_{i=1}^s$ where $s \in \mathbf{N}$ can be (in principle) arbitrarily large. In switching adaptive controls (such as those in [2], [4], [8]) an 'exhaustive' search for a stabilizing K_i is conducted. In the method of localization, however, we replace a regular switching of the controller $K_1 \to K_2 \to \cdots$ with a purposeful search over the space of potentially stabilizing controllers. To facilitate this, we introduce the notion of "stabilizing sets". We first define an auxiliary output, z_t, as

$$z_t = Cx_t, \;\; C^T \in \mathbf{R}^{2n-1}, \qquad (15)$$

and the inclusion:

$$\mathcal{I}_t : \quad |z_t| \le \Delta\|x_{t-1}\| + c_0 \qquad (16)$$

Definition 1 *$\mathcal{I}_t$ is said to be a stabilizing inclusion of the system (6), (7) if $\mathcal{I}_t$ being satisfied for all $t > t_0$ and boundedness of $\epsilon_t(\epsilon_t \in \ell_\infty)$, implies boundedness of the state, x_t, and in particular, there exist α_0, β_0 and $\sigma \in (0, 1)$ such that $\|x_t\| \le \alpha_0\sigma^{t-t_0}\|x_{t_0}\| + \beta_0\|\epsilon_t\|_{\ell_\infty}$.* ∎

Definition 2 *The uncertain system (6),(7) is said to be globally $\{C, \Delta\}$ stabilizable if for the given values of C in (15) and Δ in (16):*

1. *$\mathcal{I}_t$ is a stabilizing inclusion of the system (6), (7) and*
2. *there exists a control, $u_t = -Kx_t$, such that after a finite time, $\mathcal{I}_t$ is satisfied.* ∎

Using the following preliminary results, we later show that stabilizing sets can be effectively used in the process of localization.

Lemma 1 *Let $\sup_{t\ge t_0}|\epsilon_{t+1}| < \infty, CB > 0$. Then there exists a c_0 such that the system (6), (7) is globally $\{C, 0\}$ stabilizable if and only if*

$$|\lambda_{\max}(PA)| < 1 \qquad (17)$$

where

$$P = I - (CB)^{-1}BC \qquad (18)$$

Proof:*(See full version of this paper)* ∎

Remark 1 *The stability condition (17), (18) is equivalent to the condition that the transfer function from u_t to z_t, $C(zI - A)^{-1}B$, be relative degree 1, and minimum phase.* ∎

Remark 2 *If the original plant transfer function from u_t to y_t, (1), (2) is known to be minimum phase, and relative degree 1 then it suffices to take $C = e_{2n-1}^T$, and the system is then c_0 stabilizable for any $c_0 \ge 0$.*

If the original plant transfer function is non-minimum phase, then let:

$$C = [f_0, f_1 \cdots f_{n-2}, g_0, g_1 \cdots g_{n-1}] \qquad (19)$$

The transfer function from u_t, via (6) to z_t is then:

$$z_t = F(q)u_t + G(q)y_t = \left(\frac{A(q)F(q)+G(q)B(q)}{A(q)}\right)u_t \quad (20)$$

where $F(q) = (f_0 + f_1 q + \cdots + f_{n-2}q^{n-2})$ *and* $G(q) = (g_0 + g_1 q + \cdots + g_{n-1}q^{n-1})$.

Therefore, for a non-minimum phase plant, knowledge of a C such that $\mathcal{I}_t$ is a stabilizing inclusion is equivalent to knowledge of a (possible improper) controller $\{u_t = -G(q)/F(q)y_t\}$ which stabilizes the system. Because we are dealing with discrete time systems, it is not clear whether this corresponds to knowledge of a proper, stabilizing controller for the set. ∎

Remark 3 *Because of the robustness properties of exponentially stable linear time invariant systems, Lemma 1 can easily be generalized to include non-zero, but sufficiently small Δ.* ∎

Lemma 2 *Any Ω which satisfies Assumptions 2.1 and 2.2 has a finite decomposition into compact sets:*

$$\Omega = \bigcup_{\ell=1}^{L} \Omega^{\ell} \quad (21)$$

such that for each ℓ, there exists a C_ℓ, Δ_ℓ and $c_{0,\ell}$ such that, for all $(A,B) \in \Omega^\ell, \mathcal{I}_t$ is a stabilizing inclusion, and $C_\ell B$ has constant sign.

Proof: (Outline) It is well known that (e.g. [4]) that Ω has a finite decomposition into sets stabilized by a fixed controller. From Remark 2, the requirements for knowledge of a C_ℓ such that $\mathcal{G}(C_\ell,\cdot,\cdot)$ is a stabilizing set on Ω are less stringent than knowledge of a stabilizing controller for the set Ω^ℓ. ∎

3 Localization-known ε bound

We now introduce our control method, including the method of localization for determining which controller to use. The first case we consider, is the simplest case where there is a single set to consider:

Case 1: $L = 1$, $sgn\,(CB)$ known & ε bound known:

In this case we decompose Ω as:

$$\Omega = \Omega^1 = \bigcup_{i=1}^{s} \Omega_i \quad (22)$$

For $i = 1 \dots s$ we define a control law:

$$u_t^i = -K_i x_t \triangleq -\frac{1}{CB_i} C A_i x_t \quad (23)$$

where the plant model described by, A_i, B_i is in the set Ω_i. We require knowledge of a Δ such that:

$$\|C(A - A_i\left(\frac{CB}{CB_i}\right))\| \leq \Delta; \forall i, \forall (A,B) \in \Omega_i \quad (24)$$

and $\mathcal{I}_t$ is a stabilizing inclusion on Ω_i for all i. Note that for any bounded Ω, for which we can find a single C which gives $C(zI-A)^{-1}B$ minimum phase and relative degree 1 we can always find, for s large enough, a Δ with the required properties. (see for example [4])

At any time $t > 0$, the auxiliary output z_{t+1}^i which would have resulted if we applied $u_t^i = -K_i x_t$ to the true plant is, using (6):

$$z_{t+1}^i = CAx_t + CBu_t^i + Ce_{2n-1}\epsilon_t \quad (25)$$

$$= z_{t+1} - CB\left(u_t - u_t^i\right) \quad (26)$$

Note that if the true plant is in the set Ω_i, then from (25) and (23)

$$z_{t+1}^i = C\left(A - A_i\left(\frac{CB}{CB_i}\right)\right)x_t + Ce_{2n-1}\epsilon_t \quad (27)$$

and therefore, if the true plant is in Ω_i, then from (24), and with $c_0 = |Ce_{2n-1}|$

$$|z_{t+1}^i| \leq \Delta\|x_t\| + c_0 \quad (28)$$

Our proposed control algorithm for Case 1 is as follows (where, without loss of generality, we take $CB > 0$).

Algorithm A: Known ε bound and $L = 1$

Step 1.1 Initialization:

$$S_0 = \{1, 2, \cdots s\} \quad (29)$$

Step 1.2 Localization:

If $t > 0$, perform the following; else skip to Step 1.3.
If $z_t > \Delta|x_{t-1}| + c_0$ then set
$S_t = S_{t-1} - \{k, \dots j_{s-1}, j_s\}$
If $-z_t > \Delta|x_{t-1}| + c_0$ then set
$S_t = S_{t-1} - \{j_1, j_2, \dots, k\}$
otherwise, $S_t = S_{t-1}$.
where $k, j_1 \dots j_s$ and s are integers from the previous time instant (see steps 1.4, 1.5).

Step 1.3 Possible Control Computations

For all $i \in S_t$, compute u_t^i as in (23).

Step 1.4 Control Sorting

Order $u_t^i, i \in S_t$ such that:

$$u_t^{j_1} \leq u_t^{j_2} \leq \dots \leq u_t^{j_s} \quad (30)$$

Then apply the "median" control:

$$u_t = u_t^k \text{ where } k = j_{\lfloor s/2 \rfloor}. \quad (31)$$

Step 1.5 Done

Wait for the next sample and return to Step 1.2.

We then have the following stability result for this control algorithm.

Theorem 3 *The control algorithm, (29) - (31), applied to a plant where C is known, and where the decomposition (22) has the properties that (24) is satisfied and $\mathcal{I}_t$ is a stabilizing inclusion, has the following properties:*

(a) *The inclusion:*

$$\mathcal{I}_t : |z_t| \leq \Delta\|x_{t-1}\| + c_0 \tag{32}$$

is violated no more than $N = \lfloor \log_2(s) \rfloor$ times.

(b) *All signals in the closed loop system are bounded. In particular there exist constants $\alpha, \beta < \infty, \sigma \in (0,1)$ such that all trajectories satisfy, for any $t_0, x_{t_0}, T > 0$,*

$$\|x_{t_0+T}\| \leq \alpha\sigma^T \|x_{t_0}\| + \beta \tag{33}$$

Proof:*(Outline-see full version for details)*

(a) We can show that if (32) is violated at time t, then $s_{t+1} \leq \frac{1}{2}s_t$ from which (a) follows.

(b) Exponential stability follows from (a) since whenever (32) is satisfied, the system is exponentially stable. ∎

Case 2: $L > 1$ & known ε bound:
Suppose that we do not know a single C such that both $\mathcal{I}_t$ is a stabilizing inclusion, and CB is of known sign, then using finite covering ideas [4] as in Remark 2 let

$$\Omega = \bigcup_{\ell=1}^{L} \Omega^\ell = \bigcup_{\ell=1}^{L} \bigcup_{m=1}^{s^\ell} \Omega_m^\ell \tag{34}$$

where for each ℓ, we know $C_\ell, \Delta_\ell, c_0^\ell$ such that $\mathcal{I}_t$ is a stabilizing inclusion on Ω^ℓ and the sign of $(C_\ell B)$ is constant for all plants in Ω^ℓ.
At this point we need to be careful since if Ω^ℓ does not contain the true plant, $\mathcal{I}_t$ need not be a stabilizing inclusion. As a result of this, divergence of the states may occur without violating (32). To alleviate this problem, we use the exponential stability result, (33), in our subsequent development (see equation (35) below).
Algorithm B: Known ε bound and $L > 1$
We initialize $t(i) = 0, R_0 = \{1, 2, \ldots L\}$ and take any $\ell = \ell_0 \in R_0$. We then perform localization using Algorithm A on Ω^ℓ, with the following additional[1] steps: If at any time

$$\|x_t\| > \alpha\sigma^{t-t(i)}\|x_{t(i)}\| + \beta \tag{35}$$

(where α, σ, β are the appropriate constants for Ω^ℓ from Theorem 3), then we set $S^\ell = \{\}$.

[1] In fact, we can localize simultaneously within other $\Omega^i, i \neq \ell$, however for simplicity and brevity we analyse only the case where we localize in one set at a time.

Also, if at any time t, S^ℓ becomes empty, we set $R_t = R_{t-1} - \{\ell\}, t(i) = t$, and we take a new ℓ from R_t. With these modifications, it is clear that Theorem 3 can be extended to cover this case as well:

Corollary 4 *The control algorithm B applied to a plant with decomposition as in (34) satisfies:*

(a) There are no more than: $L - 1 + \sum_{\ell=1}^{L} \lfloor \log_2(s_\ell) \rfloor$ instances such that

$$|z_{t+1}^{\ell_t}| \geq \Delta_{\ell_t}\|x_t\| + c_{0,\ell_t} \tag{36}$$

(where ℓ_t denotes the value of ℓ at time t).

(b) All signals in the closed loop are bounded. In particular, there exist constants $\bar{\alpha}, \bar{\beta} < \infty, \bar{\sigma} \in (0,1)$ such that for any $t_0, x_{t_0}, T > 0$

$$\|x_{t_0+T}\| \leq \bar{\alpha}\bar{\sigma}^T \|x_{t_0}\| + \bar{\beta} \tag{37}$$

Proof: *Follows from Theorem 3.* ∎

4 Localization-unknown ε bound

In this section we consider the problem of localization based switching control for linear uncertain plants where an upper bound on the disturbance, $\sup_{t \geq t_0} |\epsilon_t|$, is not known. To facilitate this extension, we will make the following observation. The control law described by Algorithms A, B is well defined, in the following sense. If for some $\ell \in \{1, \ldots L\}$ and for all t we have $c_0^\ell \geq \sup_t |C_\ell e_{2n-1}\epsilon_t|$, then $R_t \neq \{\}$. This is the key point allowing us to construct an algorithm of on-line identification of the parameters $c_0^\ell, \ell = 1, \ldots L$. To take advantage of this fact, if R_t is empty, then we know that our assumed value(s) for c_0^ℓ are too small, and so we compute R_t with different values of c_0^ℓ over a finite interval, until we reach the point where R_t is non-empty. We now propose the following modification of Algorithm B:
Let $R_{t,t_2}\left(c_0^\ell(t)\right)$ denote the localization set R_t computed for a finite time interval $[t_2, t]$, (where t_2 is implicitly a function of t to be defined) based on an assumed disturbance bound, $c_0^\ell(t)$, with the initial condition $R_{t_2,t_2}\left(c_0^\ell(t)\right) = R_0$. For simplicity of notation we will mostly supress the dependence of R_{t,t_2} on $c_0^\ell(t)$ in the following:
Algorithm C. Unknown ε bound and multiple sets

1.1 Initialization:
For $\ell = 1, \ldots L$ set $t_2 = t_0$; and $c_0^\ell(t_0) = 0$
$R_{t_0,t_0} = R_0$

1.2 Modify the algorithm of localization as follows:

$$R_{t,t_2} = \begin{cases} R_{t-1,t_2} - \{\ell\} \text{ if } t > t_2, \\ R_0 \text{ otherwise,} \end{cases} \tag{38}$$

where

$$t_2 = \begin{cases} t \text{ if } R_{t-1,t_2} = \{\} \text{ and } t > t_2 + \tau, \\ t_2 \text{ otherwise,} \end{cases} \quad (39)$$

and τ is an integer choosen such that $\bar{\alpha}\bar{\sigma}^{\tau} < 1$.

1.3 If $R_{t,t_2} = \{\}$, estimate new disturbance upper bound parameters $c_0^{\ell}, \ell = 1, \ldots L$:

$$c_0^{\ell}(t) = \begin{cases} c_0^{\ell}(t-1) + m_t \epsilon_0 \text{ if } t \leq t_2 + \tau, \\ c_0^{\ell}(t-1) + \epsilon_0 \text{ if } t > t_2 + \tau, \end{cases} \quad (40)$$

where ϵ_0 is a positive constant and:

$$m_t = \arg \min_{m \in \{1,2,\ldots\}} \{m | R_{t,t_2}(c_0^{\ell}(t-1) + m\epsilon_0) \neq \{\}\} \quad (41)$$

The main idea behind the proposed algorithm consists of on-line estimation (40), (41) of the parameters of the localization sets and the resetting of the localization set R_{t,t_2} (38), (39) if the process of localization is exhausted, that is, when R_{t,t_2} is empty. ϵ_0 is used to ensure that a localization can only be exhausted a finite number of times. τ is used to ensure that if we need to compute m_t (41), that only a bounded amount of data needs to be considered.

Theorem 5 *The control law in algorithm C (38)-(41) applied to a plant subject to unknown but bounded exogenous disturbance with decomposition as in (34) satisfies:*

(a) *There are no more than a finite number of instances such that $R_t = \{\}$;*

(b) *All signals in the closed-loop system are bounded. Moreover, there exist constants $\tilde{\alpha}, \tilde{\beta} < \infty, \tilde{\delta} \in (0,1)$ such that for any $t_0 > 0$:*

$$\|x_{t_0+T}\| \leq \tilde{\alpha}\tilde{\sigma}^T \|x_{t_0}\| + \tilde{\beta} \quad (42)$$

Proof: (Outline)The proof of global stability follows that of Theorem 3 with one slight difference. Since the relation $R_{t,t_2}(c_0^{\ell}) \neq \{\}$ is valid for any $t \geq t_2$ if $c_0^{\ell} \geq \max_{l \leq \ell \leq L} \left\{ \sup_{t \geq t_0} |C_\ell e_{2n-1} \epsilon_t| \right\}$; the total number of times that c_0^{ℓ} is altered does not exceed:

$$\mathcal{S} = \left\lceil \frac{\max_{1 \leq \ell \leq L} \sup_{t \geq t_0} |C_\ell e_{2n-1} \epsilon_t|}{\epsilon_0} \right\rceil + 1 < \infty$$

This, in turn, guarantees that the inequality (37) cannot be violated more than $\mathcal{S}$ times. Relying on this fact the bound (42) can be easily proven through simple algebraic manipulations. ∎

5 Illustrative Example

An example is presented in this section to illustrate the behaviour of an uncertain discrete-time system with an unobservable exogenous disturbance.
The set of possible SISO plants is described as:

$$\begin{aligned} y_{t+1} &= a_0 y_t + a_1 y_{t-1} + a_2 y_{t-2} + u_t + \zeta_t \\ a_i &\in [-1.75, 1.75] \quad, i = 0,1,2 \;, \\ \zeta &: \sup_{t \geq t_0} |\zeta_t| \leq \gamma \end{aligned}$$

Choosing the vector C and the stabilizing set $\mathcal{I}_t$ as prescribed in Section 3, we obtain

$$\mathcal{I}_t : |z_{t+1}| \leq \Delta \|x_t\| + \gamma$$

where $C = (0,0,1)$ and $\Delta = 0.9$

We decompose the set $\Omega_D = ([-1.75, 1.75], [-1.75, 1.75], [-1.75, 1.75])$ into 216 subsets with the basic vectors of decomposition $\{K_i\}_{i=1}^{216}$, $K_i = (K_{1i}, K_{2i}, K_{3i})$. Each element of the gain vector, K_{ij} , $j \in \{1,2,3\}$, $i \in \{1, \cdots 216\}$ takes a value in the set of six elements $\{\pm 1.5, \pm 0.9, \pm 0.3\}$ giving $6^3 = 216$ different possibilities. In Figure 1 the results of

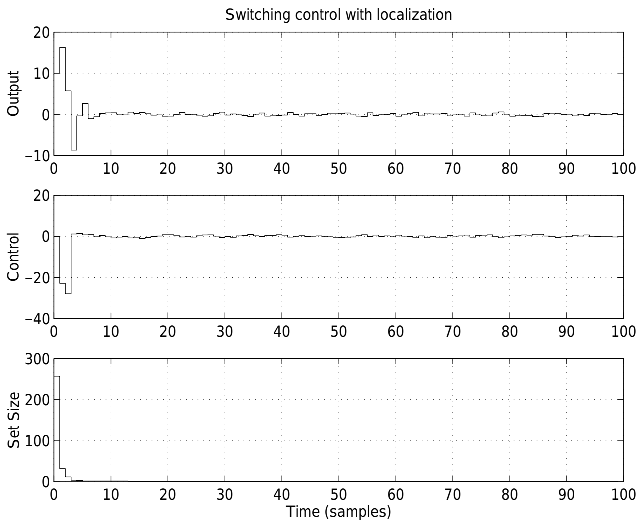

Figure 1: Localization based control with random disturbance.

computer simulation of the closed-loop system subject to a uniformly distributed exogenous disturbance, are presented. Algorithm A has been used for this study. These results are compared with those obtained by simulation of a more conventional switching controller (Figure 2)

$$u_t = K_{i(t)} x_t$$

$$i(t) = \left\{ \begin{array}{l} i(t-1) \text{ if } \mathcal{I}_t \text{ is satisfied} \\ i(t-1) + 1 \text{ otherwise.} \end{array} \right\}$$

Clearly, the localization based control has a vastly superior transient response in this example.

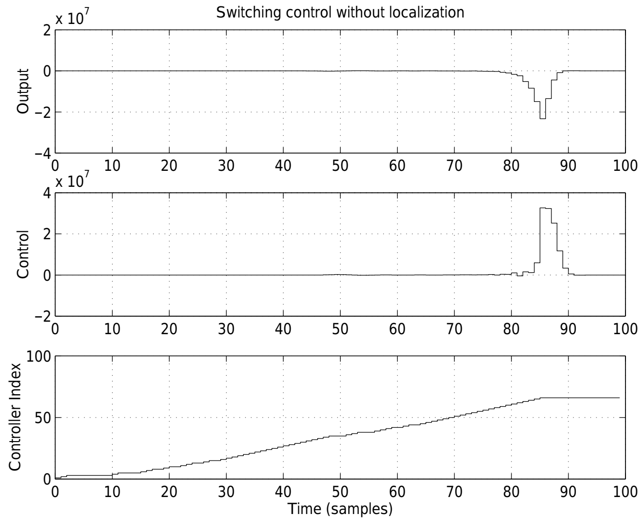

Figure 2: Switching Control without Localization

6 Conclusion

This paper develops a new method of adaptive discrete-time control. The main idea of the method consists in effective reducing the problem of adaptive stabilization to localization on a finite set of potentially stabilizing feedbacks. To avoid serious practical limitations of standard switching-type "universal controllers" associated with an excessive overshoot different types of localization algorithms are presented. The method of localization is developed in this paper for the class of uncertain linear discrete-time SISO plants subjected to unknown exogenous disturbance, input disturbance, and measurement noise. We show that such typical assumptions as minimum phase, known plant order and relative degree, and availability of information about disturbances are not necessarily needed for fast localization.

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