On Walsh Filtering Method for Decoding of CPM Signals

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***Abstract*—Walsh filtering has been used as a method to reduce receiver complexity in several coding and modulation systems, especially in continuous phase modulation systems. In this paper, we show that its lowpass filtering ability is poor and alias components arising from an adjacent channel can significantly degrade the maximum-likelihood decoding. Instead, a lowpass filtering method is more robust against adjacent channel interference and thus gives less decoding errors than the Walsh filtering method.**

***Index Terms*—Continuous-phase modulation (CPM), maximum-likelihood detection, signal detection, Walsh filters.**

I. Introduction

THE Walsh filtering method is attractive for receiver complexity reduction, especially in continuous-phase modulation (CPM) systems [1], [2]. The method is depicted in Fig. 1, where the received signal is first converted to two quadrature baseband signals $I(t)$ and $Q(t)$. Each of them is integrated over a sample interval T and then used to form the Walsh coefficients for decoding. In this letter, we analyze the sensitivity of the Walsh filtering method to adjacent channel interference (ACI). This problem is important for applications where ACI is commonly present. We show that the Walsh filtering method is quite sensitive to ACI. In contrast, a simple lowpass filtering method turns out to be more robust against ACI and requires a lower oversampling rate than the Walsh filtering method.

II. Walsh Filtering Method

We first explain briefly how the Walsh filtering method works when applied to a CPM system. An M-ary CPM signal at carrier frequency f_1 with an adjacent channel at f_2 can be expressed as

$$s(t) = A_1\cos[2\pi f_1 t + \phi_1(t,\underline{\alpha}_n)] + A_2\cos[2\pi f_2(t-\Delta) + \phi_2(t-\Delta,\underline{\beta}_n)] \quad (1)$$

for $nT \le t \le (n+1)T$, where Δ represents the time difference between wanted signal and the ACI

$$\phi_1(t,\underline{\alpha}_n) = 2\pi h \sum_{i=n-L+1}^{n} \alpha_i q(t-iT) + \pi h \sum_{i=0}^{n-L} \alpha_i \bmod 2\pi \quad (2)$$

and $\phi_2(t,\underline{\beta}_n)$ is similarly defined. Here

$$\alpha_i, \beta_i \in \{\pm 1, \pm 3, \dots \pm (M-1)\}$$

are data symbols, h is the modulation index, T is the symbol period, and $q(t)$ is the phase response, which is the integral of a frequency pulse $g(t)$ of duration L symbols. The term $\underline{\alpha}_n$ denotes α_n and all previous symbols. For an introduction on CPM, the reader is referred to [3]. In this paper, we study a typical CPM signal with parameters $M=4, L=3, h=1/4, T=1$, and $g(t)$ being the raised cosine (RC) function.

Assuming that the wanted signal and the ACI are originated from the same location (as in the case of the downlink of a cellular system), we can set $\Delta = 0$. Under the assumption of perfect carrier and symbol timing recovery, quadrature demodulation, followed by wideband lowpass filtering to remove high frequency components, gives the in-phase component (see Fig. 1):

$$I(t) = A_1\cos\phi_1(t,\underline{\alpha}_n) + A_2\cos[2\pi(f_2-f_1)t + \phi_2(t,\underline{\beta}_n)] + n(t) \quad (3)$$

for $nT \le t < (n+1)T$, where $n(t)$ is a noise component, normally assumed to be white, and $I_1(t) = A_1\cos\phi_1(t,\underline{\alpha}_n)$ is the wanted component. An ideal CPM receiver would use a large bank of matched filters to process the I and Q signals [3]. This would require a huge amount of computations because the CPM decoding complexity is in the order of pM^L, where p is the number of possible phases of the CPM system at each symbol interval, which depends on the given CPM parameters. Instead, the Walsh filtering method approximates the I and Q signals by projecting them onto a reduced signal space to reduce the receiver complexity. This is done by first oversampling the integrator output at rate $f_s = K/T$ and generating partial integrals $\tilde{I}_{nK+m}, 0 \le m < K$, over the range $(n+m/K)T \le t \le (n+(m+1)/K)T$, giving

$$\tilde{\mathbf{I}}_n = [\tilde{I}_{nK}\tilde{I}_{nK+1}\dots\tilde{I}_{nK+K-1}]^T. \quad (4)$$

Then, the Walsh coefficients are computed by

$$\mathbf{I}_n = [I_0(n)I_1(n)\dots I_{K-1}(n)]^T = \mathbf{W}\tilde{\mathbf{I}}_n \quad (5)$$

where $\mathbf{W}$ is a K-th order Walsh matrix. For $K=4$,

$$\mathbf{W} = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}. \quad (6)$$

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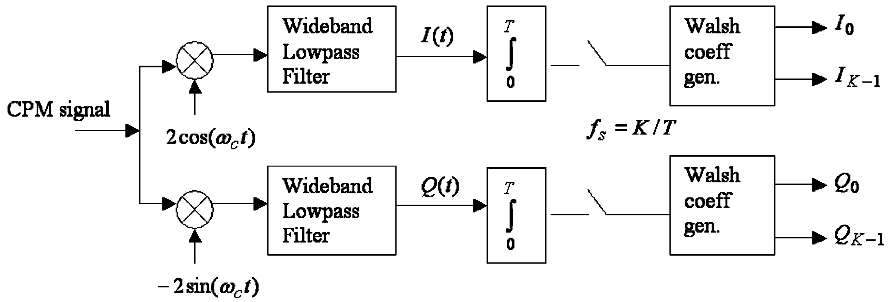

Fig. 1. Walsh filtering method for CPM Detection.

III. Analysis of Walsh Filtering

We remark on two points about the Walsh filtering method. The first point is that the Walsh transformation $\mathbf{W}$ is actually redundant. Indeed, since $\mathbf{W}$ is orthonormal, we have

$$\mathbf{I}_n^T\mathbf{I}_n = \tilde{\mathbf{I}}_n^T\mathbf{W}^T\mathbf{W}\tilde{\mathbf{I}}_n = \tilde{\mathbf{I}}_n^T\tilde{\mathbf{I}}_n. \tag{7}$$

Hence the mapping in (5) is linear and preserves the Euclidean norm. This implies that both $\mathbf{I}_n$ and the integrator output $\tilde{\mathbf{I}}_n$ are equivalent for decoding.

Our second point is that a Walsh filter is a lowpass filter with a poor stopband attenuation. Consider the first partial integral $\tilde{I}_{nK}$. Its lowpass behavior is seen by applying a single tone $I(t) = \cos(2\pi ft)$, giving response

$$C(f) = \int_0^{T/K} \cos(2\pi ft)dt = \frac{1}{2\pi f}\sin(2\pi fT/K) \tag{8}$$

which is a poorly designed lowpass filter. ACI components near iK/T, with an integer i, are particularly troublesome since they will be folded back into the wanted signal. In the absence of specific lowpass filtering, the only way to avoid the effect of the adjacent channel is to increase the oversampling factor K. To explain this point further, we show in Fig. 2 the frequency responses of the Walsh coefficients $\mathbf{I}_1(n)$ and $\mathbf{I}_3(n)$ for $K = 4$. The normalized frequency fT_b is used, where $T_b = T/\log_2 M$ is the bit period. Considering $H_1(f)$, given an ACI carrier at K/T Hz (i.e., $fT_b = 2$) from the wanted carrier at $fT_b = 0$, the ACI sidebands at $fT_b = 1.5$ will only be attenuated by some 13 dB relative to the wanted CPM signals at $fT_b = 0.25$. This can significantly degrade $\mathbf{I}_1(n)$. Similarly, the ACI components near $fT_b = 3$ can significantly degrade $\mathbf{I}_3(n)$. Fig. 3 shows the simulated filter outputs for the composite signal in (3), assuming $K = 4$ and $n(t) = 0$. The four curves are the RMS values of the Walsh filter coefficients as functions of the ACI component which is $+10$ dB relative to the wanted signal and is located at fT_b. Fig. 3 shows a significant increase in coefficient levels as the ACI frequency is reduced, which illustrates the inadequate lowpass filtering property of the Walsh filtering method. In practice, a relatively large K value is required to combat the ACI.

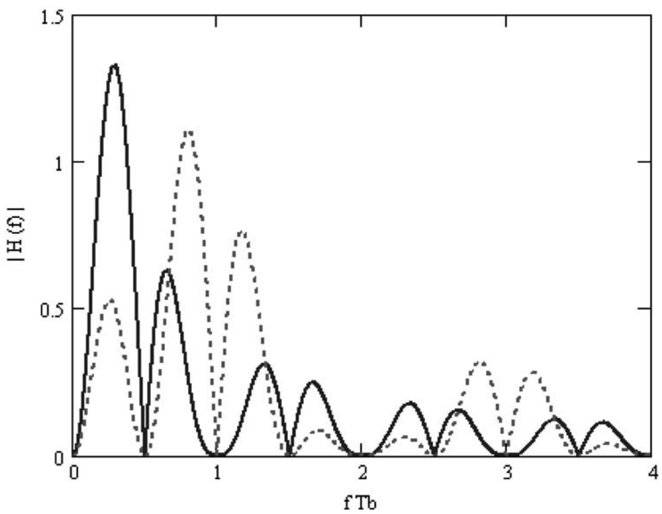

Fig. 2. Frequency responses of Walsh Coefficients $\mathbf{I}_1(n)$ (solid) and $\mathbf{I}_3(n)$ (dotted).

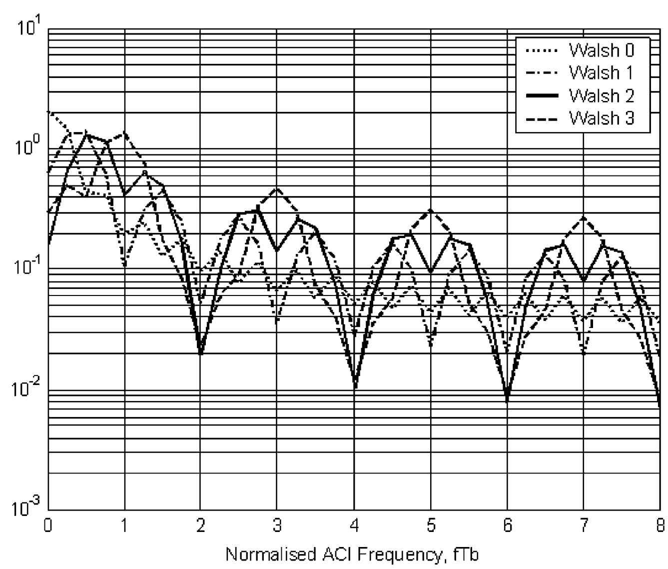

Fig. 3. Walsh coefficients as functions of ACI frequency $(K = 4)$.

IV. Lowpass Filtering Method

The forgoing analysis has shown that the effective lowpass filtering associated with Walsh space is generally inadequate in the presence of ACI. Clearly, the solution is to use better lowpass filtering. Let an ideal lowpass filter, bandwidth $B = K/2T$, replace each filter/integrator combination in Fig. 1. Nyquist sampling of the filter output, yielding samples $I_j(n)$, gives the expansion

$$I(t) \approx \sum_{j=0}^{K-1} I_j(n)\psi_j(t), \quad nT \leq T < (n+1)T \tag{9}$$

where

$$\psi_j(t) = \operatorname{sinc}\left[\frac{K}{T}\left(t-(n+\frac{j}{K}T\right)\right], \qquad j=0,1,\ldots,K-1 \quad (10)$$

and $\operatorname{sinc}(x) = \sin(\pi x)/\pi x$. The filtered signal is then oversampled to obtain the signal for decoding. The system still approximates to ML decoding as K is increased, and an oversampling factor of 4 is generally sufficient. Since the basis is no longer rectangular, the value of K required to adequately represent the signal is generally smaller compared to that for the Walsh basis. Indeed, the required K value is determined by the Nyquist sampling rate of the filtered signal and is independent of the ACI. For practical implementation, the filter should be near-linear phase over the effective CPM bandwidth F with $f_s > 2F+\Delta f$, where Δf is a small guard band. Any residual ACI above $f=F$ would then be folded to be orthogonal to the CPM signal and ignored during decoding.

V. Simulation

We have compared the BER performance of the Walsh filtering method with that of lowpass filtering for the CPM parameters given in the introduction. The effective normalized bandwidth of the CPM signal (taken at the −30-dB point in the power spectrum) is found to be at $FT_b \approx 0.4$, corresponding to $F = 0.8$ Hz. For the lowpass filtering method, the oversampling frequency is therefore $f_s = K/T = K = 2, 3, \ldots$ Hz. Note that $K = 2$ is sufficient by the Nyquist sampling criterion. But sampling frequencies of 2 and 3 Hz are tested here. The −20-dB cutoff frequencies for the corresponding lowpass filters were selected to be $F = 1.2$ Hz and 2.2 Hz, respectively. Their transfer functions chosen to be fifth-order Chebyshev(2) filters, given by

$$\frac{0.377\,01(s^2+62.85)(s^2+164.5)}{(s+5.94)(s^2+7.914s+29.05)(s^2+2.351s+22.6)} \quad (11)$$

$$\frac{0.691\,18(s^2+211.2)(s^2+553.1)}{(s+10.98)(s^2+14.51s+97.64)(s^2+4.31s+75.95)} \quad (12)$$

respectively. The reason for choosing the cutoff frequency of $F = 1.2$ Hz for the first filter is that, when $K = 2$ is used, the alias can only occur above $F > 0.8$ Hz, thus orthogonal to the CPM signal. It is verified that the second filter has an approximately linear phase response over the CPM bandwidth, but even the first filter performs well (i.e., the filtered CPM waveform being almost identical to the original signal). The BER simulation in Fig. 4 used $E_b/N_0 = 9.6$ dB and an ACI component at +10-dB relative to the wanted signal together with Viterbi decoding. This level of ACI is not uncommon in mobile wireless systems. When the ACI is void, the corresponding BER is roughly 4×10^{-3}. As expected, the Walsh filtering method is sensitive to ACI, whereas the lowpass filtering method is much more robust. Also, the difference between 2 Hz sampling $(K = 2)$ and 3-Hz sampling $(K = 3)$ is negligible for the lowpass filtering method. Because a lower K value is required for the lowpass filtering method, it offers more complexity reduction than the Walsh filtering method.

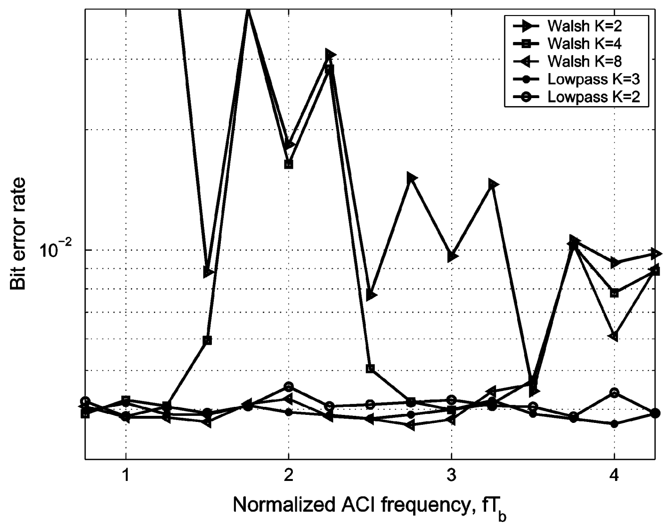

Fig. 4. Tolerance of Walsh and lowpass filtering methods to ACI.

VI. Conclusion

The Walsh filtering approach for reducing receiver complexity (simple integrators and K-times oversampling) has been shown to be susceptable to ACI. Due to the absence of specific lowpass filtering, adjacent channel frequencies close to iK/T with an integer i can significantly degrade the performance of a ML or MAP decoder. The use of lowpass filtering with the same (or lower) oversampling rate removes the ACI while still approximating to ML decoding.

References

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