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On design of finite-level quantization feedback control

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***Abstract*— This paper studies quantized feedback control of discrete-time linear systems using a finite-level quantizer. Motivated by the fact that most feedback communication channels allow a moderate bit rate, we are not particularly concerned with the problem of finding the minimum bit rate of feedback for a given control objective. Instead, we assume that a moderate bit rate is available. We introduce a dynamic scaling method and combine it with a known logarithmic quantization method. Using this approach, satisfactory control of linear systems can be achieved using a quantizer with a moderate number of quantization levels. Two main advantages of this approach are 1) it is very easy to implement, and 2) the closed-loop system behaves as if there was no limitation on the number of quantization levels when the state of the system is within a "normal" operating range. These features are important for practical applications of quantized feedback control.**

I. Introduction

Motivated by the fact that more and more control designs are implemented using digital communication links, a lot of research has been devoted to quantized feedback control in recent years; see, e.g., [1]-[14]. In quantized feedback control, the feedback signal is quantized and then coded for transmission. From the control design point of view, a fundamental problem is how to design a feedback controller and a quantizer jointly in order to achieve a certain control objective.

Two kinds of quantizers can be deployed in quantized feedback design. A static quantizer is a memoryless nonlinear function, whereas a dynamic quantizer uses memory and thus can be much more complex and more powerful. Existing work using static quantizers includes, e.g., [1], [2], [3], [4], [5]. For quadratic stabilization of a linear system using state feedback, it is shown in [1] that the optimal static quantizer is a *logarithmic quantizer*. This result is generalized in [3] to a number of output feedback problems using a sector bound approach, where logarithmic quantizers are also shown to be optimal.

When a dynamic quantizer is allowed, it is shown in [6] (also see [7]) that stabilization of a SISO LTI system (in some stochastic sense) can be achieved using only a finite number of quantization levels with the minimum number of quantization levels (also known as the minimum *feedback information rate*) explicitly related to the unstable poles of the system. Another type of dynamic quantizers uses dynamic scaling in conjunction with a static quantizer. That is, the input signal is pre-scaled so that its range is more suitable for quantization. Noticeable work along this line includes [11]-[14]. In [11], it is pointed out that if a system is not excessively unstable, by employing a quantizer with various sensitivity a feedback strategy can be designed to bring the closed-loop state arbitrarily close to zero for an arbitrarily long time. The idea of quantizer with sensitivity is extended in [12] where it is shown that there exists a dynamic adjustment of the quantizer sensitivity and a quantized state feedback that asymptotically stabilizes the system. In the case of output feedback, a local (or semi-global) stabilization result is obtained.

This paper is primarily inspired by the work of [6] but also motivated by its limitations. Although it is shown in [6] that stabilization of a linear system can be achieved by feedback with only a finite number of bits per sample and this number is typically very small, the encoder-decoder scheme used for proving this result is impractical and non-robust. It is impractical because the encoder-decoder pair is a very nonlinear operator which would typically result in a large overshoot, and it is non-robust because even a very small amount of noise in the system can drive the closed-loop unstable.

In this paper, we propose a simple *dynamic scaling* method for a logarithmic quantizer. A dynamic *scaling factor* is simply adjusted up or down depending whether the input signal to the quantizer is too "small" or too "large". Using this dynamic scaling method, we show that a linear system can be asymptotically stabilized using a logarithmic quantizer with only a finite number of quantization levels. This number turns out to be very moderate (typically a few bits to a few bytes) and is usually very compatible to the minimum information rate given in [6]. The main advantage of the proposed scheme is that the system behaves as if there were an infinite number of logarithmic quantization levels when the initial state is "moderate" in size, i.e., the state would converge exponentially. Only when the initial state is very large, a transient period of overshoot can be present. The region of exponential convergence can be easily increased by using more quantization levels, and the number of feedback information bits grows only at a $\log(\log(\cdot))$ rate when the size of this region increases. Since most digital communication channels can easily handle a few tens of bytes per sample, the proposed scheme should be very practical.

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II. Infinite-Level Logarithmic Quantization

Consider the following system:

$$x_{k+1} = Ax_k + Bu_k \quad (1)$$
$$y_k = Cx_k \quad (2)$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}$ is the control input, $y_k \in \mathbb{R}$ is the measured output, $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times 1}$ and $C \in \mathbb{R}^{1\times n}$ are given. We will denote the transfer function from u_k to y_k by $G(z)$. Without loss of generality, we assume that A is unstable and (A, B, C) is a minimal realization.

The quantized feedback control problem is to design a feedback quantizer

$$v_k = Q(y_k) \quad (3)$$

which takes values in

$$\mathcal{U} = \{\pm u_i : i = 0, \pm 1, \pm 2, \cdots\} \cup \{0\} \quad (4)$$

and a feedback controller of the form

$$\hat{x}_{k+1} = A_c\hat{x}_k + B_c v_k \quad (5)$$
$$u_k = C_c\hat{x}_k + D_c v_k \quad (6)$$

such that the closed-loop system is stable and that the so-called quantization density [1] is coarsest. The quantization density of $Q(\cdot)$ is defined as follows:

$$\eta_Q = \limsup_{\epsilon\to 0} \frac{\#g[\epsilon]}{-\ln \epsilon} \quad (7)$$

where $\#g[\epsilon]$ denotes the number of quantization levels in the interval $[\epsilon,\ 1/\epsilon]$.

The quantized feedback control problem for the system (1)-(2) is generalized from a quantized state feedback control problem in [1] and has been studied in details in [3]. In particular, it is known [3] that the coarsest quantization density for quadratic stabilization of the system above is achieved by a logarithmic quantizer. Such a quantizer is described by

$$\mathcal{U} = \{\pm\rho^i u_0 :\ i = 0, \pm 1, \pm 2, \cdots\} \cup \{0\},\ u_0 > 0 \quad (8)$$

where $\rho \in (0,\ 1)$. Since a smaller ρ corresponds to a smaller η_Q, we can regard ρ as the quantization density instead. The associated quantizer $Q(\cdot)$ is defined as follows:

$$Q(y) = \begin{cases} \rho^i u_0, & \text{if } \frac{1}{1+\delta}\rho^i u_0 < y \le \frac{1}{1-\delta}\rho^i u_0 \\ 0, & \text{if } y = 0 \\ -Q(-y), & \text{if } y < 0. \end{cases} \quad (9)$$

where

$$\delta = \frac{1-\rho}{1+\rho} \quad (10)$$

Theorem 2.1: Consider the system (1)-(2). For a given logarithmic quantization density $\rho > 0$, the system is quadratically stabilizable via a quantized controller with quantization density ρ if and only if the following auxiliary system:

$$x_{k+1} = Ax_k + Bu_k$$
$$v_k = (1+\Delta)Cx_k, \quad |\Delta| \le \delta \quad (11)$$

is quadratically stabilizable via (5)-(6). It follows that the largest sector bound $\delta_{\sup}$ and the corresponding coarsest quantization density $\rho_{\inf}$ are given by

$$\rho_{\inf} = \frac{1-\delta_{\sup}}{1+\delta_{\sup}} \quad (12)$$
$$\delta_{\sup}^{-1} = \inf_{H(z)} ||(1 - H(z)G(z))^{-1}H(z)G(z)||_\infty \quad (13)$$

where $H(z)$ is the transfer function of the controller.

Furthermore, if $G(z)$ has relative degree equal to 1 and no unstable zeros, then the coarsest quantization density is given by

$$\rho_{\inf} = \frac{\prod_i |\lambda_i^u| - 1}{\prod_i |\lambda_i^u| + 1} \quad (14)$$

where λ_i^u are the unstable eigenvalues of A.

Proof: See [3]. □

Remark 2.1: The result above offers a very convenient tool for studying quantized feedback control designs. The key point of the result is that quantization errors can be described using a sector bound (11) without any conservatism. This essentially converts a quantized feedback control problem into a robust control problem involving a sector bound uncertainty. The latter has been studied in depth in the literature and solutions are known to be related to H_∞ optimization. We also emphasize that this approach, known as the sector bound approach, can be used in many different settings for quantized feedback control and can be extended to deal with performance control and control of systems with uncertain parameters. For more details, please see [3].

III. Finite-Level Quantized Feedback Stabilization

It is obvious that a logarithmic quantizer (9) has an infinite number of quantization levels. This is certainly not implementable practically. One simple approach is to truncate the quantizer using a large saturator and a small dead zone. This will allow the state of the system to converge to a small neighborhood, provided that the initial state is within a known bound. Due to the use of logarithmic quantization, the number of quantization levels required is far less than required by using linear quantization.

In this section, we show that it is possible to dynamically scale the input-output signals of the quantizer so that asymptotic stabilization can be achieved using a finite-level logarithmic quantizer, even without knowing the bound for the initial state. We define an N-level logarithmic quantization with quantization density $\rho > \rho_{\inf}$ as

$$\mathcal{U} = \{\pm\rho^i u_0, i = 0, 1, 2, \cdots, N-1\}, \quad u_0 > 0 \quad (15)$$

The associated quantizer $Q(\cdot)$ becomes:

$$Q(y) = \begin{cases} \rho^i u_0, & \text{if } \frac{1}{1+\delta}\rho^i u_0 < y \le \frac{1}{1-\delta}\rho^i u_0, \\ & \quad 0 \le i < N \\ \rho^{N-1}u_0, & \text{if } 0 \le y \le \frac{1}{1+\delta}\rho^{N-1}u_0, \\ u_0, & \text{if } y > \frac{1}{1-\delta}\rho^i u_0, \\ -Q(-y), & \text{if } y < 0. \end{cases} \tag{16}$$

The basic idea of dynamic scaling is very simple: When the signal y_k is outside of the quantization range, we scale it back by a *scaling factor* (or *gain*) $g_k > 0$ before quantization. The quantized signal is then scaled back by g_k^{-1}. That is, we use

$$v_k = g_k^{-1} Q(g_k y_k) \tag{17}$$

We assume that a controller $H(z)$ and an infinite-level logarithmic quantizer with density $\rho > \rho_{\inf}$ have been designed for quantized feedback stabilization. To simplify bookkeeping, we assume, without loss of generality, that $H(z)$ is absorbed into $G(z)$, or $H(z) = 1$. Following the sector bound approach [3], we can write the closed-loop system as

$$x_{k+1} = A(\Delta_k)x_k := (A + B(1+\Delta_k)C)x_k \tag{18}$$

where

$$\Delta_k y_k = Q(y_k) - y_k, \quad |\Delta_k| \le \delta \tag{19}$$

represents the quantization error. Because (18) is quadratically stable, we have a quadratic Lyapunov function $V(x) = x^T P x$ with $P = P^T > 0$ such that

$$V(x_{k+1}) - V(x_k) = x_k^T (A(\Delta_k)^T P A(\Delta_k) - P) x_k < 0 \tag{20}$$

for all nonzero $x_k \in \mathbb{R}^n$ and admissible Δ_k. It is shown in [3] that the above is equivalent to

$$A(\Delta)^T P A(\Delta) - P < 0, \quad \forall \;\; |\Delta| \le \delta \tag{21}$$

Using the continuity argument, the above is equivalent to

$$A(\Delta)^T P A(\Delta) - P \le -\eta P, \quad \forall \;\; |\Delta| \le \delta \tag{22}$$

for some $0 < \eta < 1$.

We now assume that an N-level logarithmic quantizer with the same density ρ and dynamic scaling (17) is applied instead. We choose two positive scaling factors $0 < \gamma_1, \gamma_2 < 1$ such that

$$\gamma_1^2 A^T P A - P < -\eta_1 P \tag{23}$$

and

$$\gamma_2^{-2}(1+\tau)A(\Delta)^T P A(\Delta) - P < -\eta_2 P, \quad \forall \|\Delta\| \le \delta \tag{24}$$

for some $0 < \eta_1, \eta_2, \tau < 1$. The latter is done by choosing γ_2 close to 1 and τ close to 0 so that $\gamma_2^{-2}(1+\tau)(1-\eta) < 1$ and taking

$$\eta_2 = 1 - \gamma_2^{-2}(1+\tau)(1-\eta) \tag{25}$$

This ensures $\eta_2 > 0$ and makes (24) equivalent to (22).

We initialize g_0 to be any positive value and define g_{k+1} for any $k \ge 0$ as follows:

$$g_{k+1} = \begin{cases} g_k \gamma_1, & \text{if } |Q(g_k y_k)| = u_0 \\ g_k/\gamma_2, & \text{if } |Q(g_k y_k)| = \rho^{N-1}u_0 \\ g_k, & \text{otherwise} \end{cases} \tag{26}$$

Because of the flexibility in g_0, we can normalize $u_0 = 1$ without loss of generality. We will also denote $\bar{\varepsilon} = \rho^{N-1}$. The choice of g_0 does not affect stabilizability, but choosing it according to an estimate of $\|x_0\|$ helps improve the transient performance.

To help analyze the quantized feedback system, we consider the scaled state defined by

$$z_k = g_k x_k \tag{27}$$

and the associated Lyapunov function $V(z) = z^T P z$. We have the following result:

Lemma 3.1: Suppose the scaled N-level logarithmic quantizer (16), (17) and (26) is applied. Then, for any initial state x_0 and any $k \ge 0$,

$$\begin{aligned} & V(z_{k+1}) - V(z_k) \\ \le & \begin{cases} -\eta_3 V(z_k), & \textit{if } \; |Q(Cz_k)| = 1 \\ -\eta V(z_k), & \textit{if } \; \bar{\varepsilon} < |Q(Cz_k)| < 1 \\ -\eta_2 V(z_k) + \eta_4 \bar{\varepsilon}^2, & \textit{if } \; |Q(Cz_k)| = \bar{\varepsilon} \end{cases} \end{aligned} \tag{28}$$

where

$$\begin{aligned} \eta_3 &= \max\{\eta_1, \; 1 - \gamma_1^2(1-\eta)\} > 0; \\ \eta_4 &= \gamma_2^{-2}(1+\tau^{-1})B^T P B. \end{aligned} \tag{29}$$

Proof: The result for the case of $\rho^{N-1} < |Q(Cz_k)| < 1$ follows directly from (18), (22) and $g_{k+1} = g_k$. For the case of $|Q(Cz_k)| = 1$, $g_{k+1} = g_k \gamma_1$. It follows that

$$\begin{aligned} & V(z_{k+1}) - V(z_k) \\ = & \gamma_1^2 (Az_k + B\sigma_k)^T P (Az_k + B\sigma_k) - z_k^T P z_k \end{aligned}$$

where $\sigma_k = \text{sign}(Cz_k)$. Denote

$$f(u) = \gamma_1^2 (Az_k + Bu)^T P (Az_k + Bu) - z_k^T P z_k$$

From (23),

$$f(0) \le -\eta_1 z_k^T P z_k$$

Since $\sigma_k = Q(Cz_k)$, we have $\sigma_k = \theta u_1$ for some $0 < \theta \le 1$, where $u_1 = (1+\Delta_k)Cz_k$ is the unsaturated output of the quantizer. Also from (22), we get

$$\begin{aligned} f(u_1) &= \gamma_1^2 z_k^T A(\Delta_k)^T P A(\Delta_k) z_k - z_k^T P z_k \\ &\le -(1 - \gamma_1^2(1-\eta))V(z_k) \end{aligned}$$

Since $f(u)$ is quadratic and convex (because $f(u) \to +\infty$ when $|u| \to \infty$), it is clear that

$$\begin{aligned} & V(z_{k+1}) - V(z_k) \\ = & f(\sigma_k) \le \max\{f(0), \;\; f(u_1)\} = -\eta_3 V(z_k) \end{aligned}$$

For the case of $|Q(Cz_k)| = \bar{\varepsilon}$, $g_{k+1} = g_k/\gamma_2$. From (15) and (18), we can write

$$x_{k+1} = A(\Delta_k)x_k + B g_k^{-1} \epsilon_k$$

where $|\varepsilon_k| \leq \bar{\varepsilon}$. It follows that

$$\begin{aligned} & V(z_{k+1}) - V(z_k) \\ = \; & \gamma_2^{-2}(A(\Delta_k)z_k + B\varepsilon_k)^T P(A(\Delta_k)z_k + B\varepsilon_k) \\ & -z_k^T P z_k \\ = \; & \gamma_2^{-2} z_k^T A(\Delta_k) P A(\Delta_k) z_k - z_k^T P z_k \\ & +\gamma_2^{-2}(2\varepsilon_k B^T P A(\Delta_k) z_k + \varepsilon_k^2 B^T P B) \\ \leq \; & \gamma_2^{-2}(1+\tau) z_k^T A(\Delta_k) P A(\Delta_k) z_k - z_k^T P z_k \\ & +\gamma_2^{-2}(1+\tau^{-1})\bar{\varepsilon}^2 B^T P B \\ = \; & -\eta_2 z_k^T P z_k + \eta_4 \bar{\varepsilon}^2 \end{aligned}$$

This completes the proof. □

From Lemma 3.1, it is clear that $V(z_k)$ converges to a bounded region. This bound can be computed by solving

$$0 = -\eta_2 V_\infty + \eta_4 \bar{\varepsilon}^2$$

which gives

$$V_\infty = \eta_2^{-1} \eta_4 \bar{\varepsilon}^2 \tag{30}$$

Lemma 3.1 leads to the following result:

Corollary 3.1: Suppose the scaled N-level logarithmic quantizer (16), (17) and (26) is applied. Then, for any initial state x_0, $z_k = g_k x_k$ converges exponentially to the ellipsoid

$$Z_\infty = \{z : \; z \in \mathbb{R}^n, V(z) \leq V_\infty\} \tag{31}$$

From (30) and the corollary above, it is clear that we can choose N to be sufficiently large so that, when k is sufficiently large, $Q(Cz_k)$ will no longer be saturated. This is achieved by choosing N such that

$$|Cz| < 1 \quad \forall z^T P z \leq \eta_2^{-1} \eta_4 \rho^{2(N-1)}$$

Solving this gives $N \geq N_0$, where

$$N_0 = 1 + \frac{\log(\eta_2^{-1}\gamma_2^{-2}(1+\tau^{-1})B^T P B C P^{-1} C^T)}{2\log(\rho^{-1})} \tag{32}$$

The analysis above yields the following main result:

Theorem 3.1: Suppose the scaled N-level logarithmic quantizer (16), (17) and (26) is applied with $N \geq N_0$ in (32). Then, the state x_k converges to zero asymptotically.

Proof: From Corollary 3.1, z_k converges to Z_∞ exponentially. This property and the choice of N_0 imply that $Q(Cz_k)$ will no longer be saturated after a finite number of steps, say k_0 steps. This means that g_k will be non-decreasing for $k \geq k_0$. Note that whenever $g_{k+1} = g_k$, $V(z_k)$ decreases exponentially. If this continues enough number of steps, $|Cz_k|$ will be less than $\bar{\epsilon}$, forcing g_{k+1} to increase by factor of $1/\gamma_2$. This means that g_k cannot converge to a constant. Hence, $g_k \to \infty$ as $k \to \infty$. Since z_k is bounded for $k > k_0$, we conclude that $x_k \to 0$ as $k \to \infty$. □

Remark 3.1: A typical behavior of the system is as follows: If the initial state is very large, the feedback signal tends to be saturated, forcing g_k to decrease fast. This would result in a period of overshoot. Once g_k is sufficiently small, saturation will stop and the state decays exponentially. When the state is sufficiently small, g_k will increase gradually, causing the quantizer to bounce back and forth between the dead zone and logarithmic region. During this phase, the state also decays exponentially, but at a lower rate.

IV. Illustrative Example

In this section, we use an example to illustrate the proposed dynamic scaling method. The example we consider aims at demonstrating the convergence rate of the dynamic scaling method.

Consider the system (1)-(2) with

$$\begin{aligned} A &= \begin{bmatrix} 2.7 & -2.41 & 0.507 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ C &= [1 \quad -0.5 \quad 0.04] \end{aligned}$$

The system is unstable with two unstable open-loop poles at $1.2 \pm i0.5$ but without unstable zero and the relative degree is 1. It follows from Theorem 2.1 that

$$\delta_{\sup} = |1.2 \pm i0.5|^{-2} = 0.5917, \quad \rho_{\inf} = 0.2565$$

Choosing $\delta = 0.2$, we can design the controller $H(z)$ by solving the H_∞ optimal control problem

$$\delta \|(1 - G(z)H(z))^{-1} G(z)H(z)\|_\infty \leq 1$$

as suggested by Theorem 2.1. This gives

$$\begin{aligned} A_c &= \begin{bmatrix} 0.1041 & 0.1615 & -1.2342 \\ 0.1031 & 0.2376 & 0.7151 \\ 0.0874 & 0.1875 & 0.1328 \end{bmatrix} \\ B_c &= \begin{bmatrix} 0.0015 \\ -0.0007 \\ 0.0000 \end{bmatrix} \\ C_c &= [9526 \quad 18043 \quad -12946] \\ D_c &= -1.9250 \end{aligned}$$

Using this controller, we can form the closed-loop matrix. That is, we replace A in (23) with

$$\tilde{A} = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix}$$

For the closed-loop system, we obtain $\eta = 0.5603$ and with the Lyapunov matrix given by

$$P = \begin{bmatrix} 5.2641\times10^{3} & -1.2636\times10^{4} & 8.8965\times10^{3} & 4.8938\times10^{7} & 5.5863\times10^{7} & -5.3009\times10^{8} \\ -1.2636\times10^{4} & 3.0331\times10^{4} & -2.1355\times10^{4} & -1.1747\times10^{8} & -1.3410\times10^{8} & 1.2724\times10^{9} \\ 8.8965\times10^{3} & -2.1355\times10^{4} & 1.5036\times10^{4} & 8.2708\times10^{7} & 9.4412\times10^{7} & -8.9589\times10^{8} \\ 4.8938\times10^{7} & -1.1747\times10^{8} & 8.2708\times10^{7} & 4.5499\times10^{11} & 5.1942\times10^{11} & -4.9281\times10^{12} \\ 5.5863\times10^{7} & -1.3410\times10^{8} & 9.4412\times10^{7} & 5.1942\times10^{11} & 5.9300\times10^{11} & -5.6254\times10^{12} \\ -5.3009\times10^{8} & 1.2724\times10^{9} & -8.9589\times10^{8} & -4.9281\times10^{12} & -5.6254\times10^{12} & 5.3381\times10^{13} \end{bmatrix}$$

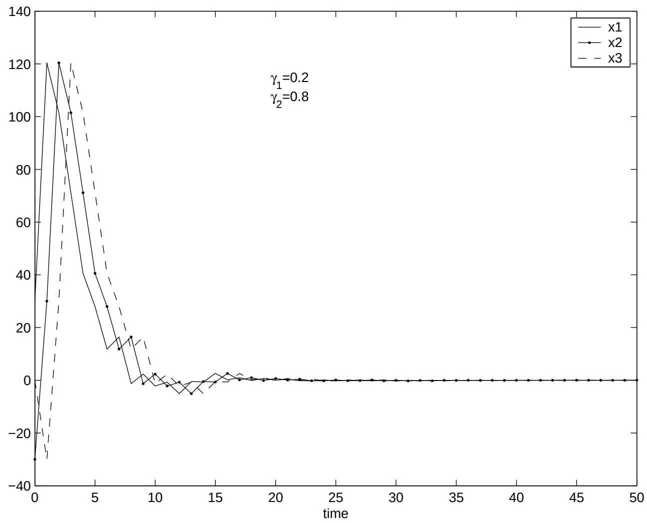

Fig. 1. State response of the closed-loop system

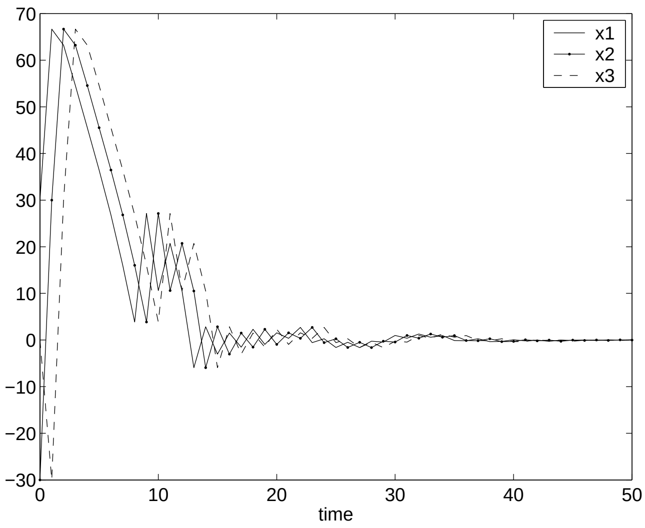

Fig. 3. State response for known initial state

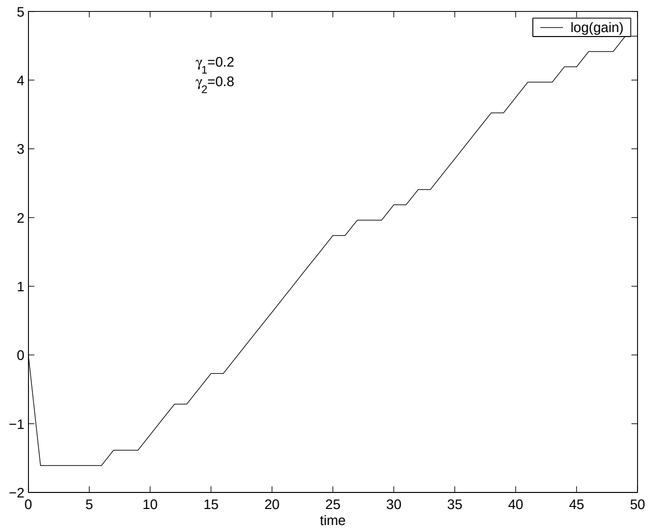

Fig. 2. The scaling factor g_k

Since γ_2 is lower bounded by $\sqrt{1-\eta} = 0.6632$, we choose $\gamma_2 = 0.8$. This gives $N_0 = 6.4256$. Since $N = 7$ and $N = 8$ give the same bit rate (4 bits), we set $N = 8$. Note that the minimal bit rate required for stabilizing this system is 1 bit [6].

It can be easily verified that (23) is satisfied if $\gamma_1 \leq 0.323$. Thus, we take $\gamma_1 = 0.2$. Let the initial state of the controller be $\hat{x}_0 = [0 \;\; 0 \;\; 0]^T$ and the minimal level of the quantizer be 1 (correspondingly, $u_0 = 1/\rho^{M-1}$). The state response of the closed-loop system with the initial state $x_0 = [30 \;\; -30 \;\; 0]^T$ and $g_0 = 1$ is shown in Figure 1. The scaling gain g_k is shown in Figure 2.

If we have a good estimate $\tilde{x}_0$ of the initial state x_0, we may set the initial scaling gain g_0 to improve the transient performance. For example, if we set $g_0 = 1/|C x_0|$ for the given initial condition, the transient performance is improved significantly. The corresponding state response is shown in Figure 3. Note that a similar improvement can be achieved even when only a rough estimate of the initial condition is available.

V. Conclusion

We have proposed a simple dynamic scaling method for quantized feedback control. This allows us to achieve asymptotic stabilization using a very moderate number of quantization levels. The proposed control scheme is easily implementable and has nice convergence properties. The results in this paper represent only preliminary work along this line. Two issues are under further investigation now. One is to work out how to choose relevant design parameters so that the number of quantization levels N can be minimized. The second issue is to study the robustness of the proposed method. Simulation results suggest that the proposed method has good robustness properties with respect to additive noises in the system. Some theoretical analysis is needed to quantify this observation.

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