Seventh International Conference on Control, Automation, Robotics And Vision (ICARCV'02), Dec 2002, Singapore

Integral Quadratic Constraint Approach vs. Multiplier Approach

Minyue Fu[1], Soura Dasgupta[2] and Yeng Chai Soh[3]

[1]School of EECS, University of Newcastle, Callaghan, N.S.W. 2308, Australia.
Currently on leave at School of EEE, Nanyang Technological University, 639798 Singapore

[2] Department of Electrical and Computer Engineering, University of Iowa, U.S.A.

[3] School of EEE, Nanyang Technological University, 639798 Singapore

Email: eemf@ee.newcastle.edu.au

Abstract. Integral quadratic constraints (IQC) arise in many optimal and/or robust control problems. The IQC approach can be viewed as a generalization of the classical multiplier approach in the absolute stability theory. In this paper, we study the relationship between the two approaches for robust stability analysis. The key result shows that for many applications, the existence of an IQC is equivalent to the existence of a multiplier. Because the multiplier approach is typically simpler and more intuitive, this result suggests that the multiplier approach may be more useful than the IQC approach in many applications.

1 Introduction

Integral quadratic constraints (IQCs) often arise in robustness analysis of linear and nonlinear dynamical systems. They are used as a convenient tool for describing parametric uncertainty, time-delays, unmodeled dynamics and nonlinearity of the system, as well as design objectives such as LQG cost or H_∞ performance.

The terminology of IQC was formally introduced by Yakubovich [21, 22] in the 70's for robust stability analysis of systems subject to complicated perturbations. The underlying idea, however, had been around since the seminal work by Popov on absolute stability in early 60's [15]. Popov's idea of using a quadratic constraint to "overbound" sectorial nonlinearity led to a frequency domain condition for absolute stability in terms of a *multiplier* function. The absolute stability theory developed in the 60-70's offers a rich class of multipliers for robustness analysis with various nonlinear functions. Strong connections between multipliers and the network realization theory are well established. Further, a Lyapunov function is associated with each multiplier. See, e.g., Brockett and Willems [4], Narendra and Taylor [14], Desoer and Vidyasagar [5], Safanov [17], Willems [20] and Vidyasagar [18] for details. Many of the classical papers on absolute stability can be found in an edited book by Aggarwal and Vidyasagar [1].

Generalized from the multiplier approach, the IQC approach is able to treat a larger class of uncertainty and nonlinearity. Many IQCs are collected in a paper by Rantzer and Megretski [16]. The examples where IQCs apply include real and complex uncertainties, fast and slow time-varying parameters, time-delays, nonlinearity, H_∞ optimization constraints, etc. The so-called Kalman-Yakubovich-Popov (KYP) Lemma [2, 20] plays a vital role in the analysis of IQCs. Recent development in the IQC approach incorporates the theory of linear matrix inequality (LMI) to derive more advanced robust stability and robust control results; see, e.g., Boyd *et. al.* [3], Gahinet *et. al.* [10], Feron *et. al.* [6], Haddad and Bernstein [11], How and Hall [12], and Fu *et. al.* [9]. The advantage of the LMI approach is that much more complicated uncertainty can be handled using convex optimization, and hence it differs sharply from the traditional absolute stability theory where the main goal was to obtain simple graphical tests.

The purpose of this paper is to study the following converse problem: To what extent does the IQC approach generalize the multiplier approach? In other words, we would like to know under what conditions the existence of an IQC implies the existence of a multiplier. This problem is motivated by the fact that the multiplier approach is simpler and more intuitive. So we want to know when we can apply the simpler approach. We give a technical condition under which the existence of an IQC implies that of a multiplier. Surprisingly, it turns out that this technical condition is satisfied for most applications.

This paper is organized as follows: Section 2 introduces the IQC approach. Section 3 reviews the classical multiplier approach. Section 4 contains the main result of the paper. Section 5 gives some discussions on the main result. Section 6 concludes the paper.

2 IQC Approach

Consider the interconnected system in Figure 1 which is also described by the following equations:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= y + v \\ u &= r + w \\ w &= \Delta(z) \end{aligned} \quad (1)$$

where $\Delta(\cdot) \in \mathbf{\Delta}$ which is a set of linear or nonlinear dynamic operators to be specified later. Denote

$$G(s) = C(sI - A)^{-1}B + D \quad (2)$$

and assume A to be asymptotically stable in the sequel.

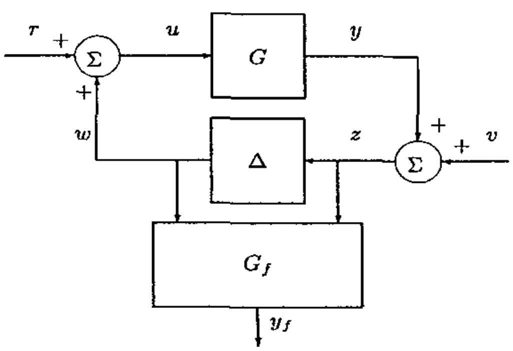

Figure 1: Interconnected Feedback System

The feedback block $\Delta(\cdot)$ is assumed to satisfy an IQC which is constructed via a *filter* given by

$$G_f(s) = C_f(sI - A_f)^{-1}B_f + D_f \quad (3)$$

where A_f is asymptotically stable. It is also assumed that $\mathbf{\Delta}$ is a connected set containing the zero operator.

The IQC used in this paper is then described by the following inequality:

$$\int_{-\infty}^{+\infty} [z^*(j\omega) \quad w^*(j\omega)]\Phi(j\omega) \begin{bmatrix} z(j\omega) \\ w(j\omega) \end{bmatrix} d\omega \geq 0 \qquad \forall\, \Delta \in \mathbf{\Delta} \quad (4)$$

where $z(j\omega), w(j\omega)$ are Fourier transforms of $z(t), w(t)$, respectively, and

$$\Phi(s) = G_f^*(s)\bar{\Phi}G_f(s) \quad (5)$$

We now introduce a notion of stability, absolute total stability, for robust stability analysis with IQC. This stability notion is stronger than asymptotic stability and $\mathcal{L}_2$ BIBO stability.

Definition 1 *The system (1) is called* totally stable *(or simply called stable) if there exists some constant ρ such that for all $r, v \in \mathcal{L}_2[0,\infty)$ and the initial state $x(0)$, the response signals $w(t)$ and $x(t)$ (and hence all other signals) are well-defined at all $t \geq 0$, and the following holds:*

$$\int_0^\infty (x'(t)x(t) + w'(t)w(t))dt \leq \rho\left(x'(0)x(0) + \int_0^\infty (r'(t)r(t) + v'(t)v(t))dt\right) \quad (6)$$

Further, a family of systems of the form (1) is called absolutely totally stable *(or simply called absolutely stable) if there exists a common $\rho > 0$ such that (6) holds for every member system.*

The following result serves the foundation of the IQC approach (see [16]).

Theorem 1 (The IQC Theorem) *Given a set of operators $\mathbf{\Delta}$ for the feedback block of the system (1), the system is absolutely stable if there exists some $\Phi(s)$ of the form (5) and a constant $\epsilon > 0$ such that both (4) and the following condition are satisfied:*

$$[G^*(j\omega) \quad I]\Phi(j\omega)\begin{bmatrix} G(j\omega) \\ I \end{bmatrix} + \epsilon I \leq 0, \ \forall\, |\omega| < \infty \quad (7)$$

3 Multiplier Approach

Let us briefly review the classical multiplier approach to absolute stability analysis. The following result can be found in [5, 18, ?]. We use $\mathcal{U}$ to denote the set of all asymptotically stable square transfer matrices with an asymptotically stable inverse.

Lemma 1 *Consider the system in Figure 1 with $\mathbf{\Delta}$ being a set of $\mathcal{L}_2[0,\infty) \to \mathcal{L}_2[0,\infty)$ operators. Suppose there exist a multiplier $M(s)$ of the following form:*

$$M(s) = M_1^*(s)M_2(s), M_1(s), M_2(s) \in \mathcal{U} \quad (8)$$

and a constant $\epsilon > 0$ such that the following two passivity conditions are satisfied:

$$\int_{-\infty}^{\infty} \mathrm{Re}[z^*(j\omega)M(j\omega)w(j\omega)]d\omega \geq 0, \qquad \forall\, z \in \mathcal{L}_2[0,\infty),\ \Delta \in \mathbf{\Delta} \quad (9)$$

$$M^*(j\omega)G(j\omega) + G^*(j\omega)M(j\omega) \leq -\epsilon I \quad (10)$$

Then, the system in Figure 1 is absolutely stable.

Remark 1 *The physical interpretation of the lemma above is clearly given in Figure 2. It is obvious to see* that Figures 1 and 2 are identical, provided that $\hat{y} = M_1 y$, $\hat{z} = M_1 z$, $\hat{v} = M_1 v$, $\hat{r} = M_2 r$, $\hat{u} = M_2 u$ and $\hat{w} = M_2 w$ are taken. The conditions in (9)-(10) simply mean that the lower block of Figure 2 is passive and the negated upper block is strictly passive.

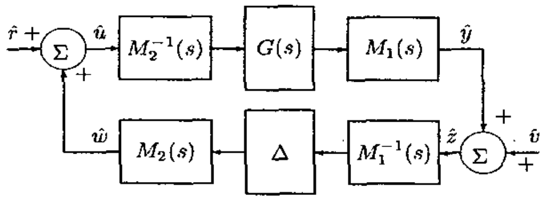

Figure 2: Transformed Feedback System

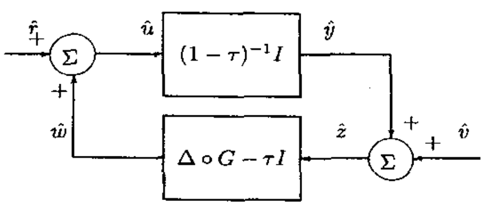

Figure 3: Transformed Feedback System

Now let us consider a modified version of Lemma 1. This modification is obtained by converting Figure 1 into Figure 3, where $\tau \in [0, 1)$ is an arbitrary parameter. It can be verified easily that the signals in Figure 3 are given by

$$(I - \Delta \circ G) \circ (\hat{u} + \hat{v}) = (1-\tau)\hat{r} + (1-\tau)\hat{v} \quad (11)$$

$$(I - G \circ \Delta) \circ (\hat{y} + \hat{v}) = G \circ (1-\tau)\hat{r} + G \circ (1-\tau)\hat{v} \quad (12)$$

In comparison, the signals in Figure 1 are given by

$$(I - \Delta \circ G) \circ u = r + \Delta \circ v \quad (13)$$

$$(I - G \circ \Delta) \circ y = G \circ r + G \circ \Delta \circ v \quad (14)$$

For any $r, v \in \mathcal{L}_2[0, \infty)$, if we take

$$\hat{r} = \frac{1}{1-\tau} r; \quad \hat{v} = \frac{1}{1-\tau} \Delta \circ v \quad (15)$$

then $\hat{r}, \hat{v} \in \mathcal{L}_2[0, \infty)$ and

$$u = \hat{u} + \hat{v}; \quad y = \hat{y} + \hat{v} \quad (16)$$

Hence, the absolute stability of Figure 3 implies that of Figure 1.

Applying Lemma 1 to Figure 3, we obtain the following result.

Lemma 2 *Consider the system in Figure 1 with* $\mathbf{\Delta}$ *being a set of* $\mathcal{L}_2[0,\infty) \to \mathcal{L}_2[0,\infty)$ *operators. Suppose there exists a multiplier* $M(s)$ *of the form (8) and a constant* $\epsilon > 0$ *such that the following conditions are satisfied:*

$$M(j\omega) + M^*(j\omega) \le -\epsilon I \quad (17)$$

$$\int_{-\infty}^{\infty} (\mathrm{Re}[u^*(j\omega)M(j\omega)(w(j\omega) - u(j\omega))] - \epsilon u^*(j\omega)u(j\omega))\, d\omega \ge 0,$$
$$\forall\, u \in \mathcal{L}_2[0,\infty),\ \Delta \in \mathbf{\Delta},\ w = \Delta \circ (G \circ u) \quad (18)$$

Then, the system in Figure 1 is absolutely stable.

Proof: Suppose (17)-(18) hold. For any $\tau < 1$ but sufficiently close to 1, we have from (18)

$$\int_{-\infty}^{\infty} \mathrm{Re}[z^*(j\omega)M(j\omega)(\hat{w}(j\omega) - \tau z(j\omega))]d\omega \ge 0$$

for all $z \in \mathcal{L}_2[0,\infty)$, $\Delta \in \mathbf{\Delta}$. Also, (17) implies

$$M^*(j\omega)\left(\frac{1}{1-\tau}I\right) + \left(\frac{1}{1-\tau}I\right)M(j\omega) \le -\frac{\epsilon}{1-\tau}I$$

Using Lemma 1 on Figure 3, we conclude that the system in Figure 3 is absolutely stable. Hence, so is the system in Figure 1. ■

Remark 2 *Note that (17) is implied by (18) because* $\mathbf{\Delta}$ *contains the zero operator. But we state it to make it explicit.*

4 IQC vs. Multiplier

We are now ready to establish a relationship between IQCs and multipliers. To this end, we consider a more general type of multipliers than (8). Indeed, we allow

$$M(s) = M_1^*(s)M_2(s), \quad (19)$$

where $M_1(s)$ and $M_2(s)$ are asymptotically stable. That is, we do not require $M_1(s)$ and $M_2(s)$ to be invertible. In particular, we allow them to be "tall" to take the advantage of larger dimensions. But we note that they can not be "wide" due to the condition (17).

Also, let us express

$$\Phi(s) = \begin{pmatrix} Q(s) & F(s) \\ F^*(s) & R(s) \end{pmatrix} \quad (20)$$

The key technical condition we require is that $R(j\omega) \le 0$ (negative semidefinite) for all ω. We emphasize that most IQCs experienced in applications satisfy this condition; see Section 5 for discussions. We show that in this case an IQC and a relaxed multiplier in (19) are equivalent.

Theorem 2 *Consider the system in Figure 1 with the assumption that Δ is a set of $\mathcal{L}_2[0,\infty) \to \mathcal{L}_2[0,\infty)$ operators. Suppose there exist some multiplier $M(s)$ of the form (19) and some constant $\epsilon > 0$ such that (9)-(10) are satisfied. Then, (4) and (7) hold with the following $\Phi(s)$:*

$$\Phi(s) = \begin{pmatrix} \frac{\epsilon}{2\|G\|_\infty^2} & M(s) \\ M^*(s) & 0 \end{pmatrix} \tag{21}$$

which can realized in the form of (5) with

$$G_f(s) = \begin{pmatrix} I & 0 \\ M_1(s) & 0 \\ 0 & M_2(s) \end{pmatrix}; \quad \bar{\Phi} = \begin{pmatrix} \frac{\epsilon}{2\|G\|_\infty^2} & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{pmatrix} \tag{22}$$

where $\|G\|_\infty$ is the H_∞ norm of $G(s)$.

Conversely, suppose (4) and (7) hold for some $\epsilon > 0$ and some $\Phi(s)$ of the form (5) with $R(j\omega) \le 0$ for all $\omega \in (-\infty, \infty)$. Then, (17)-(18) hold for

$$M(s) = 2(G^*(s)F(s) + R(s)) \tag{23}$$

which can be realized in the form of (19) with

$$M_1(s) = 2G_f(s)\begin{pmatrix} G(s) \\ I \end{pmatrix}; \quad M_2(s) = \bar{\Phi} G_f(s) \begin{pmatrix} 0 \\ I \end{pmatrix} \tag{24}$$

Remark 3 *The result above shows that the existence of an IQC is equivalent to an multiplier if the IQC is restrited to have negative semidefinite $R(j\omega)$. The advantage of the multiplier is that it is much smaller in dimension and hence in general easier to search for.*

Proof of Theorem 2. Suppose there exist $M(s)$ of the form (19) and some constant $\epsilon > 0$ such that (9)-(10) are satisfied. Using the $\Phi(s)$ in (21), it is trivially verified that (4) and (7) correspond to (9) and (10), respectively. Also, it is easy to check that this $\Phi(s) = G_f^*(s)\bar{\Phi}G_f(s)$ for the $\bar{\Phi}$ and $G_f(s)$ defined in (22). In particular, $G_f(s)$ is asymptotically stable because both $M_1(s)$ and $M_2(s)$ are.

Conversely, suppose (4) and (7) hold for some $\Phi(s)$ in the form of (5) with $R(j\omega) \le 0$ for all ω. Using (20), we rewrite (4) and (7) as follows:

$$\int_{-\infty}^{\infty} (z^*Qz + z^*Fw + w^*F^*z + w^*Rw)d\omega \ge 0 \tag{25}$$

$$G^*QG + G^*F + F^*G + R \le -\epsilon I \tag{26}$$

Take any $u \in \mathcal{L}_2[0,\infty)$. It follows that $z = G \circ u \in \mathcal{L}_2[0,\infty)$ and $w = \Delta \circ G \circ u \in \mathcal{L}_2[0,\infty)$. Then, (25)-(26) become

$$\int_{-\infty}^{\infty} (u^*G^*QGu + u^*G^*Fw + w^*F^*Gu + w^*Rw + \epsilon u^*u)d\omega \ge 0 \tag{27}$$

$$\int_{-\infty}^{\infty} u^*(G^*QG + G^*F + F^*G + R + \epsilon I)u \, d\omega \le 0 \tag{28}$$

The difference between the two integrals above yields

$$\begin{aligned} & \int_{-\infty}^{\infty} (u^*G^*F(w-u) + (w-u)^*F^*Gu \\ & \quad + w^*Rw - u^*Ru - \epsilon u^*u)d\omega \\ = & \int_{-\infty}^{\infty} (u^*(G^*F + R)(w-u) + (w-u)^*R(w-u) \\ & \quad + (w-u)^*(F^*G + R)u - \epsilon u^*u)d\omega \\ \ge & \ 0 \end{aligned}$$

Since $R \le 0$, the above implies

$$\int_{-\infty}^{\infty} (u^*(G^*F + R)(w-u) + (w-u)^*(F^*G + R)u - \epsilon u^*u)d\omega \ge 0$$

which is the same as (18) with $M(s)$ given by (23). It is a trivial matter to verify that $M(s) = M_1^*(s)M_2(s)$ for $M_i(s)$ in (24).

Remark 4 *Observe that the first part of Theorem 2, which generalizes Lemma 1 to allow a multiplier $M(s)$ with "tall" $M_i(s)$, is trivially proved using the IQC Theorem, although the use of such a multiplier seems to be difficult to justify using Figure 2 because $M_i(s)$ are not invertible.*

Corollary 1 *[8] Suppose Δ is a set of causal and asymptotically stable LTI operators containing the zero operator. Then, the following two conditions, both guaranteeing the absolute stability of the system in Figure 1, have the implication that* **i)**$\Longrightarrow$**ii)**.

i). *There exists $\Phi(s)$ of the form (5) and some $\epsilon > 0$ such that (4) and (7) hold and that $R(j\omega) \le 0$ for all $\omega \in (-\infty, \infty)$;*

ii). *There exists a multiplier $M(s)$ of the form (19) such that*

$$M(j\omega)[I - \Delta(j\omega)G(j\omega)] + [I - \Delta(j\omega)G(j\omega)]^*M^*(j\omega) + \epsilon I \le 0 \tag{29}$$

Remark 5 *The problem studied in the corollary above is commonly known as the structured singular value problem when Δ is specially structured. It is known [9, 13] that the multiplier approach gives a less conservative test for robustness analysis than the so-called $D-G$ scaling method given in [7]. In fact, the $D-G$ scaling method amounts to a special multiplier; see details in [8, 13].*

5 Discussions

As we see from Theorem 2 that the technical condition for the existence of a multiplier is that $R(j\omega) \leq 0$. Indeed, most IQCs used in applications satisfy this condition. To see this, we consider a few examples although many more can be found in the literature (see, e.g., [16]).

Example 1: Popov Criterion.

The well-known Popov Criterion [15] considers a single-input single-output system as in Figure 1 with $G(s) = C(sI-A)^{-1}B$ (without the D term) and $\boldsymbol{\Delta}$ being a set of nonlinear functions satisfying $0 \leq z(-\Delta(z)) \leq cz^2$ for some constant $c > 0$. The Popov criterion asserts that such a system is absolutely stable if $(1+ks)(G(s)+c^{-1})$ is SPR for some constant $k \geq 0$. The function $(1+ks)$ is called a multiplier.

The function $\Phi(s)$ for the associated IQC is given by

$$\Phi(s) = \begin{pmatrix} 0 & -(1+ks)^* \\ -(1+ks) & -2c^{-1} \end{pmatrix} \qquad (30)$$

Example 2: Limit Cycles of a Digital Quantizer [19].

Consider a digital quantizer described by

$$w(n) = -sat(z(n)) = \begin{cases} 1, & z(n) < -1 \\ -z(n), & |z(n)| \leq 1 \\ -1 & z(n) > 1 \end{cases} \qquad (31)$$

It follows that $z(n)w(n) \leq 0$ for all n. We may model this as a simple passive device. However, this description is too conservative in general. To overcome this difficulty, we let $H(z)$ be any stable function with L_1 norm less than or equal to 1, i.e.,

$$\sum_0^\infty |h(n)| \leq 1 \qquad (32)$$

where $h(n)$ is the impulse response corresponding to $H(z)$. In addition, it is required that $1 + H(z)$ is invertible. Then, the IQC is given by

$$\Phi(z) = \begin{pmatrix} 0 & -(1+H) \\ -(1+H^*) & -(2+H+H^*) \end{pmatrix}$$

Example 3: Constant Uncertain Parameters

Consider the case

$$w = \Delta z = \text{block diag}\{q_1 I_{k_1}, \cdots, q_p I_{k_p}\}z, \quad q_i \in [-1,\ 1],\ \ i = 1, \cdots, p \qquad (33)$$

where q_i are all constant uncertain parameters. Let us take any

$$\begin{aligned} D(s) &= \text{block diag}\{D_1(s), \cdots, D_p(s)\}; \\ V(s) &= \text{block diag}\{V_1(s), \cdots, V_p(s)\} \end{aligned} \qquad (34)$$

where $D_i(s)$ and $V_i(s)$ are square matrices of dimension k_i, and

$$D(j\omega) = D^*(j\omega) > 0; \quad V(j\omega) = -V^*(j\omega); \quad \forall\, \omega \in (-\infty, \infty) \qquad (35)$$

We can build an IQC with the following $\Phi(s)$:

$$\Phi(s) = \begin{pmatrix} D(s) & V(s) \\ V^*(s) & -D(s) \end{pmatrix} \qquad (36)$$

provided that $D(s)$ and $V(s)$ are such that the $\Phi(s)$ above can be expressed as in (5).

We see in all the examples above, the term $R(j\omega) \leq 0$.

6 Conclusions

In this paper, we have studied the relationship between the IQC approach and the multiplier approach. The main result is that these two approaches are equivalent under a fairly mild condition. It should be pointed out that the purpose of this paper is not to undermine the significance of the IQC approach. Rather, we hope that the work of this paper provides some new insight into these two approaches and can motivate more research in this area.

References

[1] J. K. Aggarwal and M. Vidyasagar, ed., *Nonlinear Systems: Stability Analysis*, Dowden, Hutchinson and Ross, Inc., Pennsylvania, 1977.

[2] B. D. O. Anderson, "A system theory criterion for positive real matrices," *SIAM J. Contr. Optimization*, vol. 6, no. 2, pp. 171-192, 1967.

[3] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, Philadelphia: SIAM, 1994.

[4] R. W. Brockett and J. L. Willems, "Frequency domain stability criterion," *IEEE Trans. Auto. Contr.*, vol. AC-10, no. 4, pp. 225-261, pp. 401-413, 1965.

[5] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*, Academic Press, New York, 1975.

[6] E. Feron, P. Apkarian and P. Gahinet, "S-procedure for the analysis of control systems with parameter uncertainties via parameter-dependent Lyapunov functions," *Proc. 3rd European Contr. Conf.*, Rome, Italy, 1995.

[7] M. K. H. Fan, A. L. Tits and J. C. Doyle, "Robustness in the presence of mixed parametric uncertainty and unmodeled dynamics," *IEEE Trans. Auto. Contr.*, vol. 36, no. 1, pp. 25-38, 1991.

[8] M. Fu and N. E. Barabanov, "Improved upper bounds for the mixed structured singular value," *IEEE Trans. Auto. Contr.*, vol. 42, no. 10, pp. 1447-1452, 1997.

[9] M. Fu, N. E. Barabanov and H. Li, "Robust $\mathcal{H}_\infty$ analysis and control of linear systems," *Proc. 3rd European Contr. Conf.*, Rome, Italy, 1995.

[10] P. Gahinet, P. Apkarian, M. Chilali and E. Feron, "Affine parameter-dependent Lyapunov functions and real parameter uncertainty," *Proc. 3rd European Contr. Conf.*, Rome, Italy, 1995.

[11] W. M. Haddad and D. S. Bernstein, "Parameter-dependent Lyapunov functions, constant real parameter uncertainty, and the Popov criterion in robust analysis and synthesis: parts 1 and 2", *Proc. 31st IEEE Conf. Decision and Contr.*, pp. 2274-2279 and pp. 2262-2263., 1991.

[12] J. P. How and S. R. Hall, "Connections between the Popov stability criterion and bounds for real parameter uncertainty," *Proc. American Control Conf.*, pp. 1084-1089, 1995.

[13] G. Meinsma, Y. Shrivastava and M. Fu, "Some Properties of an Upper Bound for μ," *IEEE Trans. Auto. Contr.*, vol. 41, no. 9, pp. 1326-1330, 1996.

[14] K. S. Narendra and J. H. Taylor, *Frequency Domain Criteria for Absolute Stability*, Academic Press, N. Y., 1973.

[15] V. M. Popov, "Absolute stability of nonlinear systems of automatic control," *Automation and Remote Control*, pp. 857-875, March 1962.

[16] A. Rantzer and A. Megretski, "System analysis via integral quadratic constraints," *IEEE Trans. Auto. Contr.*, vol. 42, no. 6, pp. 819-830, June 1997.

[17] M. G. Safanov, *Stability and Robustness of Multivariable Feedback Systems*, MIT Press, 1980.

[18] M. Vidyasagar, *Nonlinear Systems Analysis*, Prentice-Hall, Englewood Cliffs, 2nd ed., 1992.

[19] L. Xie, M. Fu and H. Li, "Passivity analysis and passification for uncertain signal processing systems," submitted.

[20] J. Willems, *The Analysis of Feedback Systems*, MIT Press, 1971.

[21] V. A. Yakubovich, "Frequency conditions of absolute stability of control systems with many nonlinearities," *Automatica i Telemekhanica*, vol. 28, pp. 5-30, 1967.

[22] V. A. Yakubovich, "S-procedure in nonlinear control theory," *Vestnik Leninggradskogo Universiteta, Ser. Matematika*, pp. 62-77, 1971.