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Stability Analysis of a Distributed State Estimator for Linear Systems

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***Abstract*—A distributed steady-state estimator is presented for linear interconnected systems. Stability of the distributed estimator is investigated. Sufficient conditions of stability are deduced based on the state and observation models. Some examples are provided to illustrate the relationship between the stability of the estimator and that of the original dynamics.**

I. Introduction

Complex large-scale systems, such as power systems [1] and military command systems [2], usually involve high-order dynamics and deploy over a large geographical region. If all the measurements of the whole system can be collected together, optimal state estimation can be obtained by the conventional Kalman filtering method. But for most large-scale systems, it is difficult or impossible to centralize all the measurements. Therefore, distributed algorithms are often adopted.

Distributed Kalman filtering has many results [3–19]. Most work in the literature discusses the situation in which observation systems enjoy a common state dynamic [3–11]. Optimal [5, 7] and sub-optimal [6, 9, 10] algorithms are proposed in this case. However, a complex large-scale system usually comprises many subsystems which are interconnected [12, 13, 15–17, 19] and the states among subsystems are coupled. Each subsystem has its own dynamics and measurements. States of subsystems are estimated based on local measurements and information of neighboring subsystems. [12] and [13] presented unbiased filters when measurements include some information of states of other subsystems. [15–18] discussed the design of state estimator for the systems with sparsely connected states. A steady-state filter is presented in [17, 18] based on a consensus strategy.

In this paper, we consider a distributed estimation problem for a class of weakly interconnected linear systems. Each subsystem depends on its own state and a part of the states of other subsystems. The dimension of the partial state which influences the neighboring subsystems is much smaller than the number of measurements for each subsystems. Therefore, it is not a good idea to transmit all the measurements from one subsystem to another. We will design a distributed algorithm by transmitting just necessary information about local estimation. In this case, a steady-state distributed estimator is presented. Stability conditions for the estimator are provided based on the state and observation models. It is worth pointing out that the stability of the estimator is unrelated to the stability of the system. For simplicity we only consider two interconnected subsystems in the paper.

The rest of this paper is organized as follows. Problem statement is given in Section II. A distributed steady-state filters is designed in Section III and stability conditions of the distributed state estimator are derived in Section IV. Several examples are provided to illustrate the relationship between the stability of the estimator and that of the original dynamics. in Section V. Some conclusions are drawn in Section VI.

II. Problem Statement

Consider the following dynamics composed of two interconnected subsystems:

$$\begin{aligned} \mathbf{S1}: x_1(k+1) &= A_{11}x_1(k) + B_{12}z_1(k) + B_1w_1(k), && (1)\\ z_1(k) &= C_{12}x_2(k), && (2)\\ \mathbf{S2}: x_2(k+1) &= B_{21}z_2(k) + A_{22}x_2(k) + B_2w_2(k), && (3)\\ z_2(k) &= C_{21}x_1(k), && (4) \end{aligned}$$

where $x_i(k) \in \Re^{n_i}$ is the state vector of i-th subsystems, $z_i(k) \in \Re^{r_i}$ is the input from the other subsystem and $w_i(k) \in \Re^{m_i}$ is the process noise or exogenous disturbance signal. $A_{ii}, B_i, B_{ij}, C_{ij}$ are constant matrices of appropriate dimensions. In our problem setting, we assume that z_i has much smaller dimension than x_j $(i \neq j)$, thus the two subsystems are weakly coupled. This is motivated by applications where a large system (such as power network) is divided into several subsystems geographically and with coupling between subsystems happening only through the boundaries. Each subsystem has its own measurement system as follows:

$$\begin{aligned} y_1(k) &= C_1x_1(k) + v_1(k), && (5)\\ y_2(k) &= C_2x_2(k) + v_2(k), && (6) \end{aligned}$$

where $y_i(k) \in \Re^{p_i}$ are measured outputs about corresponding subsystems, $v_i(k) \in \Re^{p_i}$ are measurement noises, and C_i are constant measurement matrices. We adopt a standard assumption on the statistical characteristics of the noises:

Assumption 1. *The process noise $w_i(k)$ and the measurement noise $v_i(k)$ are uncorrelated Gaussian white noises with zero* *mean and covariance*

$$E\left\{\begin{bmatrix} w_i(k) \\ v_i(k) \end{bmatrix}\begin{bmatrix} w_j^T(l) & v_j^T(l) \end{bmatrix}\right\} = \begin{bmatrix} Q_{w_i} & 0 \\ 0 & Q_{v_i} \end{bmatrix}\delta_{kl}\delta_{ij}, \quad (7)$$

where δ_{kl} and δ_{ij} are the Kronecker delta and $Q_{w_i} > 0$ and $Q_{v_i} > 0$.

The structure of the dynamics and measurements are described in Fig. 1.

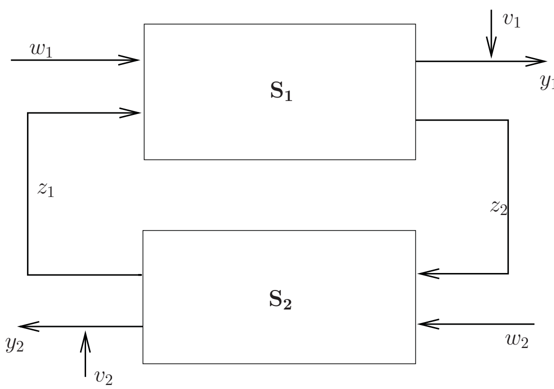

Fig. 1: Coupled System

III. Distributed State Estimator

If $z_1(k)$ is known to the estimator of system $\mathbf{S1}$, it is easy to obtain the following optimal estimation of $x_1(k+1)$ by standard Kalman filtering:

$$\begin{aligned}\hat{x}_1(k+1|k+1) &= A_{11}\hat{x}_1(k|k) + A_{12}x_2(k) \\ &\quad +K_1(k+1)\varepsilon_1(k+1), \end{aligned} \quad (8)$$

where

$$A_{12} \triangleq B_{12}C_{12}, \quad (9)$$

$$\begin{aligned}\varepsilon_1(k+1) &= y_1(k+1) - C_1(A_{11}\hat{x}_1(k|k) \\ &\quad +A_{12}x_2(k)). \end{aligned} \quad (10)$$

Similarly, there is an optimal estimator of system $\mathbf{S2}$ if $z_2(k)$ is known:

$$\begin{aligned}\hat{x}_2(k+1|k+1) &= A_{21}x_1(k) + A_{22}\hat{x}_2(k|k) \\ &\quad +K_2(k+1)\varepsilon_2(k+1), \end{aligned} \quad (11)$$

where

$$A_{21} \triangleq B_{21}C_{21}, \quad (12)$$

$$\begin{aligned}\varepsilon_2(k+1) &= y_2(k+1) - C_2(A_{21}x_1(k) \\ &\quad +A_{22}\hat{x}_2(k|k)). \end{aligned} \quad (13)$$

$K_1(k)$ and $K_2(k)$ in (8) and (11) are computed as follows:

$$K_i(k+1) = P_i(k+1|k)C_i^T Q_{P_i}^{-1}(k+1), \quad (14)$$

$$Q_{P_i}(k+1) = C_i P_i(k+1|k)C_i^T + Q_{v_i}, \quad (15)$$

where $P_i(k+1|k)$ satisfies the following Ricaati equations:

$$\begin{aligned}&P_i(k+1|k) \\ &= A_{ii}(k)P_i(k|k)A'_{ii}(k) + B_i(k)Q_{w_i}B'_i(k), \end{aligned} \quad (16)$$

$$\begin{aligned}&P_i(k+1|k+1) \\ &= P_i(k+1|k) - K_i(k+1)Q_{P_i}(k+1)K'_i(k+1). \end{aligned} \quad (17)$$

However, it is almost impossible to know the exact value of $z_1(k)$ for the estimator of system $\mathbf{S1}$. Therefore we have to consider to use an estimated value of $z_1(k)$ to estimate $x_1(k+1)$. A natural idea is to utilize systems (3) and (6) to obtain the estimated value of $x_2(k)$ which will replace the exact value of $x_2(k)$. In this case, a distributed estimator for system $\mathbf{S1}$ and $\mathbf{S2}$ can be described as Fig. 2.

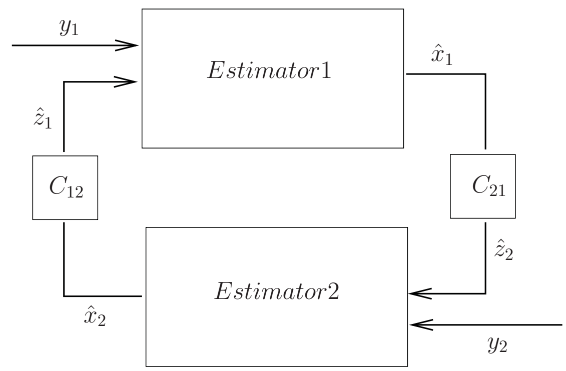

Fig. 2: Distributed Estimator

Define

$$e_i(k+1) = y_i(k+1) - C_i(A_{i1}\hat{x}_1(k|k) + A_{i2}\hat{x}_2(k|k)). \quad (18)$$

Then the estimated values of $x_1(k)$ and $x_2(k)$ can be iteratively computed as follows:

$$\begin{aligned}&\hat{x}_1(k+1|k+1) \\ &= A_{11}\hat{x}_1(k|k) + A_{12}\hat{x}_2(k|k) + K_1(k+1)e_1(k+1) \\ &= F_1(k+1)A_{11}\hat{x}_1(k|k) + F_1(k+1)A_{12}\hat{x}_2(k|k) \\ &\quad +K_1(k+1)y_1(k+1), \end{aligned} \quad (19)$$

$$\begin{aligned}&\hat{x}_2(k+1|k+1) \\ &= A_{21}\hat{x}_1(k|k) + A_{22}\hat{x}_2(k|k) + K_2(k+1)e_2(k+1) \\ &= F_2(k+1)A_{21}\hat{x}_1(k|k) + F_2(k+1)A_{22}\hat{x}_2(k|k) \\ &\quad +K_2(k+1)y_2(k+1), \end{aligned} \quad (20)$$

where $K_i(k+1)$ are computed by (14) – (17) and

$$F_i(k+1) \triangleq I - K_i(k+1)C_i.$$

Obviously, the estimations in (19) and (20) are sub-optimal. From the point of view of practical applications and simplification, what attracts us is to design steady-state filters. Firstly, the following assumption is presented for $i = 1$ or 2 in order to describe clearly.

Assumption 2. *$(A_{ii}, B_iQ_{w_i}^{\frac{1}{2}})$ is completely stabilizable. (A_{ii}, C_i) is completely detectable.*

According to the design of the steady-state filter in [20] and refering to (8) and (11), we have the following lemma.

Lemma 3. *If Assumption 2 is satisfied, we have the following steady-state filters of systems (1) – (6)*

$$\begin{aligned}&\hat{x}_1(k+1|k+1)\\ =\;& (I-K_1C_1)A_{11}\hat{x}_1(k|k)+K_1y_1(k+1)\\ &+(I-K_1C_1)A_{12}x_2(k), \qquad (21)\\ &\hat{x}_2(k+1|k+1)\\ =\;& (I-K_2C_2)A_{22}\hat{x}_2(k|k)+K_2y_2(k+1)\\ &+(I-K_2C_2)A_{21}x_1(k), \qquad (22)\end{aligned}$$

where

$$K_i = R_iC_i^T(C_iR_iC_i^T+Q_{v_i})^{-1}, \qquad (23)$$

and R_i is the unique positive definite solution to the following algebraic Riccati equation

$$\begin{aligned}R_i =\;& A_{ii}R_iA_{ii}^T - A_{ii}R_iC_i^T(C_iR_iC_i^T+Q_{v_i})^{-1}C_iR_iA_{ii}^T\\ &+B_iQ_{w_i}B_i^T. \qquad (24)\end{aligned}$$

Note that $x_2(k)$ in (21) and $x_1(k)$ in (22) are the inputs of the filters. If they are known, then the optimal steady-state filter can be obtained as in Lemma (3). However, it is difficult to know $x_1(k)$ and $x_2(k)$. Therefore, we will consider a sub-optimal steady-state filter. In addition, R_i means the covariance of the predicted error in the steady-state filter. If P_i is denoted as the covariance of the estimation error, then the following relation can be obtained by information filters ([20])

$$P_i^{-1} = R_i^{-1}+C_i^TQ_{v_i}^{-1}C_i. \qquad (25)$$

In view of Lemma 3, we present:

Algorithm 4. *A distributed steady-state filters for systems (1) – (6) is as follows:*

$$\begin{aligned}\hat{x}_1(k+1|k+1) =\;& F_1A_{11}\hat{x}_1(k|k)+F_1A_{12}\hat{x}_2(k|k)\\ &+K_1y_1(k+1), \qquad (26)\\ \hat{x}_2(k+1|k+1) =\;& F_2A_{21}\hat{x}_1(k|k)+F_2A_{22}\hat{x}_2(k|k)\\ &+K_2y_2(k+1), \qquad (27)\end{aligned}$$

where $F_i = I-K_iC_i$ and K_i are presented as (23).

Algorithm 4 is easy to obtain. The main work includes the analysis about stability of the algorithm and it will be discussed in the next section.

IV. Stability Analysis

The algorithm presented by (26) and (27) is simple though it is sub-optimal. Now we will discuss the stability of error systems from the filters (26) and (27). Define $\widetilde{x}_i(k|k) = x_i(k)-\hat{x}_i(k|k)$. Then the error equations of dynamic systems **S1** – **S2** and filters (26)-(27) are as follows.

$$\begin{aligned}&\begin{pmatrix}\widetilde{x}_1(k+1|k+1)\\ \widetilde{x}_2(k+1|k+1)\end{pmatrix}\\ =\;& FA\begin{pmatrix}\widetilde{x}_1(k|k)\\ \widetilde{x}_2(k|k)\end{pmatrix}+\begin{pmatrix}F_1B_1w_1(k)\\ F_2B_2w_2(k)\end{pmatrix}\\ &-\begin{pmatrix}K_1v_1(k+1)\\ K_2v_2(k+1)\end{pmatrix}, \qquad (28)\end{aligned}$$

where $F \triangleq \begin{pmatrix}F_1 & 0\\ 0 & F_2\end{pmatrix}$ and $A \triangleq \begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}$. What we will consider is the internal stability of the error system (28), i.e., the stability of FA.

Remark 5. *Assumption (2) can not guarantee the internal stability of error systems (28). An example (Example (13)) is provided to illustrate that in the next section.*

With the help of the bounds of the solution to the algebraic Riccati equation (24), we can obtain sufficient conditions of the internal stability of the systems (28). Denote $\lambda_M(A)$ and $\lambda_m(A)$ as the maximal and minimal eigenvalue of matrix A, respectively. Then we have the following result.

Theorem 6. *If*

$$\lambda_M(AA^T)(\min_i\{\lambda_m(L_{R_i})\})^{-2}(\max_i\{\lambda_M(U_{P_i}\})^2 < 1, \quad (29)$$

then error equation (28) is internally stable, where L_{R_i} and U_{P_i} are any lower and upper bounds of R_i and P_i, respectively.

Proof. It is easy to know $F_i = P_iR_i^{-1}$ by the standard Kalman filtering [21]. Then we have $F = PR^{-1}$, where $P \triangleq \begin{pmatrix}P_1 & 0\\ 0 & P_2\end{pmatrix}$ and $R \triangleq \begin{pmatrix}R_1 & 0\\ 0 & R_2\end{pmatrix}$. Noting P and R are positive-definite matrices, then we have

$$\begin{aligned}\|FA\|_2^2 =\;& \lambda_M(FAA^TF^T) = \lambda_M(PR^{-1}AA^TR^{-1}P)\\ \le\;& \lambda_M(R^{-1}AA^TR^{-1})\lambda_M(P^2)\\ \le\;& \lambda_M(AA^T)\lambda_M(R^{-2})\lambda_M(P^2),\end{aligned}$$

where $\|FA\|_2$ is the spectral norm of FA. Noting that $\lambda_M(R^{-1}) = (\lambda_m(R))^{-1}$, then we know $\|FA\|_2 \le 1$ when the condition is satisfied. It is well known, the spectral radius of a matrix is not greater any norm of the matrix ([22]). Therefore, error equation (28) is internally stable when $\|FA\|_2 \le 1$. □

Remark 7. *The stability of A_{11}, A_{22} or A is not necessary for the condition (29); see Example 14.*

Next we present another sufficient condition for the stability of (28). First, two well-known lemmas are listed as follows.

Lemma 8. *Let $X = \begin{pmatrix}X_{11} & X_{12}\\ X_{12}^T & X_{22}\end{pmatrix}$, Then following statements are equivalent:*

i) $X > 0$,
ii) $X_{11} > 0$ and $X_{12}^TX_{11}^{-1}X_{12} < X_{22}$,
iii) $X_{22} > 0$ and $X_{12}X_{22}^{-1}X_{12}^T < X_{11}$.

Lemma 9. *[23] Let* $\begin{pmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{pmatrix} (\in R^{2n\times 2n}) > 0$*, where* $X_{11}, X_{12}, X_{22} \in R^{n\times n}$*. If* $X_{11} \geq X_{22}$*, then* $X_{11}^{-1}X_{12}^T$ *is Schur stable.*

Now we can present the following sufficient condition of internal stability of the error equation (28).

Theorem 10. *If the following holds:*

$$A\begin{pmatrix} U_{P_1} & 0 \\ 0 & U_{P_2} \end{pmatrix} A^T < \begin{pmatrix} L_{R_1} & 0 \\ 0 & L_{R_2} \end{pmatrix}, \tag{30}$$

then error system (28) is internally stable, where U_{P_i} *and* L_{R_i} *are any upper and lower bounds of* P_i *and* R_i*, respectively.*

Proof. It is easy to know $R^{-1} \leq P^{-1}$ for $P \leq R$ [20]. Then (30) implies $APA^T < R$ and $R^{-1}APA^TR^{-1} < R^{-1}$. From Lemma 8, we have

$$\begin{pmatrix} P^{-1} & A^TR^{-1} \\ R^{-1}A & R^{-1} \end{pmatrix} > 0.$$

Therefore, $PR^{-1}A = FA$ is asymptotically stable according to Lemma 9. □

Taking $L_{R_i} = R_i$ and $U_{P_i} = P_i$ is simple and direct. However, this does not allow us to make a judgement about stability before R_i is computed. We hope to obtain some stabilizable conditions only via system models. There are many papers in the literature to discuss the bounds of the solution to the Riccati equation (24). If A_{ii} is stable, the upper bounds are obtained via the unique positive semi-definite solution to corresponding Lyapunov equation in [24]. If A_{ii} is not stable and $(A_{ii}, C_i^TQ_{v_i}^{-1/2})$ is stabilizable, [25] presents the upper bounds by using a feedback gain L_i stabilizing $A_{ii}+C_i^TQ_{v_i}^{-1/2}L_i$. The following lemma synthesizes the results of Theorems 3.1 – 3.4 in [26] and presents a simpler form about the bounds.

Lemma 11. *Define*

$$\begin{aligned} \Phi_{i,j} &= A_{ii}(\Phi_{i,j-1}^{-1} + C_i^TQ_{v_i}^{-1}C_i)^{-1}A_{ii}^T + B_iQ_{w_i}B_i^T, \\ & \quad i = 1, 2,\ j = 1, 2, \ldots, \\ \Phi_{i,0} &= B_iQ_{w_i}B_i^T, \\ \Psi_{i,j} &= A_{ii}(\Psi_{i,j-1}^{-1} + C_i^TQ_{v_i}^{-1}C_i)^{-1}A_{ii}^T + B_iQ_{w_i}B_i^T, \\ & \quad i = 1, 2,\ j = 1, 2, \ldots, \\ \Psi_{i,0} &= A_{ii}(C_i^TQ_{v_i}^{-1}C_i)^{-1}A_{ii}^T + B_iQ_{w_i}B_i^T. \end{aligned}$$

If the DARE (24) has a unique positive definite solution, then the following statements hold:

1) $\Phi_{i,j}$ *is the lower bound of* R_i*. Moreover, it satisfies* $\Phi_{i,j} \geq \Phi_{i,j-1}$ *and* $R_i = \lim_{j\to\infty} \Phi_{i,j}$*.*
2) $\Psi_{i,j}$ *is the upper bound of* R_i*. Moreover, it satisfies* $\Psi_{i,j} \leq \Psi_{i,j-1}$ *and* $R_i = \lim_{j\to\infty} \Psi_{i,j}$*.*

Proof. Riccati equation (24) can be rewritten as

$$\begin{aligned} R_i &= A_{ii}R_iA_{ii}^T - A_{ii}R_iC_i^TQ_{v_i}^{-1/2}(Q_{v_i}^{-1/2}C_iR_iC_i^TQ_{v_i}^{-1/2} \\ & \quad +I)^{-1}Q_{v_i}^{-1/2}C_iR_iA_{ii}^T + B_iQ_{w_i}B_i^T. \end{aligned}$$

Comparing to the Riccati equation in [26], we obtain the result. □

Noting formula (25), we can obtain the following lower and upper bounds of P_i:

$$P_i \geq ((\Phi_{i,j})^{-1} + C_i^TQ_{v_i}^{-1}C_i)^{-1} \tag{31}$$

and

$$P_i \leq ((\Psi_{i,j})^{-1} + C_i^TQ_{v_i}^{-1}C_i)^{-1}, \tag{32}$$

where $\Phi_{i,j}$ and $\Psi_{i,j}$ are the lower and upper bounds of R_i in Lemma 11. Now a sufficient condition for the stability of system (28) is presented based on the models of system dynamics and measurements in the following corollary.

Corollary 12. *If the following inequalities are satisfied*

$$\begin{aligned} &\lambda_M(AA^T)(\min_i\{\lambda_m(\Phi_{i,j})\})^{-2} \\ &\times(\min_i\{\lambda_m(\Psi_{i,j}^{-1} + C_i^TQ_{v_i}^{-1}C_i)\})^{-2} < 1, \end{aligned} \tag{33}$$

or

$$A\begin{pmatrix} \Omega_1^{-1} & 0 \\ 0 & \Omega_2^{-1} \end{pmatrix} A^T < \begin{pmatrix} \Phi_{1,j} & 0 \\ 0 & \Phi_{2,j} \end{pmatrix}, \tag{34}$$

where $\Omega_1 = (\Psi_{1,j})^{-1} + C_1^TQ_{v_1}^{-1}C_1$*,* $\Omega_2 = (\Psi_{2,j})^{-1} + C_2^TQ_{v_2}^{-1}C_2$*,* $\Phi_{i,j}$ *and* $\Psi_{i,j}$ *are denoted in Lemma 11, then (28) is internally stable.*

Proof. The result is obvious by the Theorem 6 and Theorem 10 after considering the bounds provided by Lemma 11, (31) and (32). □

V. Examples

We provide two examples to explain Remarks 5 and 7 about the relations of the system models and the filter we proposed in the section.

Example 13. *Consider the following systems:*

$$\begin{aligned} \mathbf{S1}:\ x_1(k+1) &= 0.8x_1(k) + 0.5x_2(k) + w_1(k), \\ \mathbf{S2}:\ x_2(k+1) &= 0.5x_1(k) + 0.9x_2(k) + w_2(k), \\ y_1(k) &= 0.4x_1(k) + v_1(k), \\ y_2(k) &= 0.3x_2(k) + v_2(k), \end{aligned}$$

with $Q_{w_1} = Q_{w_2} = Q_{v_1} = Q_{v_2} = 1$*.*

Obviously, the subsystem **S1** *is stable for* $x_2(k)$ *and* $w_1(k)$ *are viewed as inputs. So does subsystem* **S2***. However, the error systems (28) is not stable since the maximum eigenvalue of* FA *is* 1.0571*. Therefore, the stability of each subsystem can not guarantee the stability of the estimation error system.*

Example 14. *Consider the following systems:*

$$\begin{aligned} \mathbf{S1}:\ x_1(k+1) &= 1.1x_1(k) + z_1(k) + w_1(k), \\ z_1(k) &= x_2(k), \\ \mathbf{S2}:\ x_2(k+1) &= z_2(k) + 1.1x_2(k) + w_2(k), \\ z_2(k) &= x_1(k), \\ y_1(k) &= x_1(k) + v_1(k), \\ y_2(k) &= x_2(k) + v_2(k), \end{aligned}$$

with $Q_{w_1} = Q_{w_2} = Q_{v_1} = Q_{v_2} = 1$.

The subsystem $\mathbf{S1}$ *and* $\mathbf{S2}$ *are not stable. Neither is the system* $\mathbf{S}$ *composed of* $\mathbf{S1}$ *and* $\mathbf{S2}$*. However, after taking* Φ_{11}*,* Φ_{21}*,* Ψ_{10} *and* Ψ_{20} *in (33), the left of (33) is* 0.8115*, then the condition in Theorem 6 is satisfied. The maximum eigenvalue of* FA *is* 0.7571 *at the same time. Therefore, the error system (28) derived by Algorithm 4 is internally stable (Figure 3).*

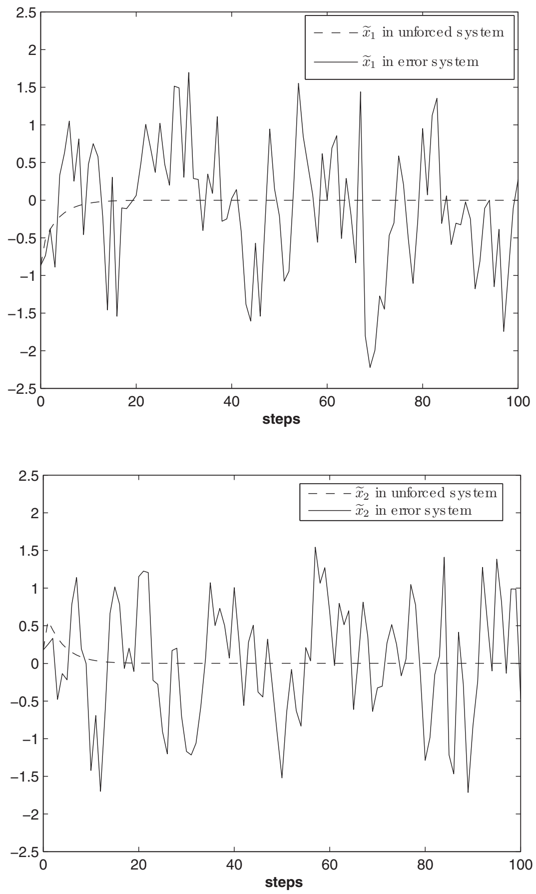

Fig. 3: State trajectories of error system (28) and corresponding unforced system

Example 15. *Consider the following system:*

$$
\begin{aligned}
\mathbf{S1}: \quad x_1(k+1) &= 1.1x_1(k) + 0.1z_1(k) + w_1(k),\\
z_1(k) &= x_2(k),\\
\mathbf{S2}: \quad x_2(k+1) &= z_2(k) + 1.1x_2(k) + w_2(k),\\
z_2(k) &= 0.1x_1(k),\\
y_1(k) &= x_1(k) + v_1(k),\\
y_2(k) &= x_2(k) + v_2(k),
\end{aligned}
$$

with $Q_{w_1} = Q_{w_2} = Q_{v_1} = Q_{v_2} = 1$.

The subsystems $\mathbf{S1}$ *and* $\mathbf{S2}$ *are not stable. Neither is the system composed of* $\mathbf{S1}$ *and* $\mathbf{S2}$*. However, after taking* Φ_{10}*,* Φ_{20}*,* Ψ_{10} *and* Ψ_{20} *in (34), the condition in Theorem 10 is satisfied. The maximum eigenvalue of* FA *is* 0.4326 *at this time. Therefore, the estimation error system (28) derived by Algorithm 4 is internally stable (Figure 4).*

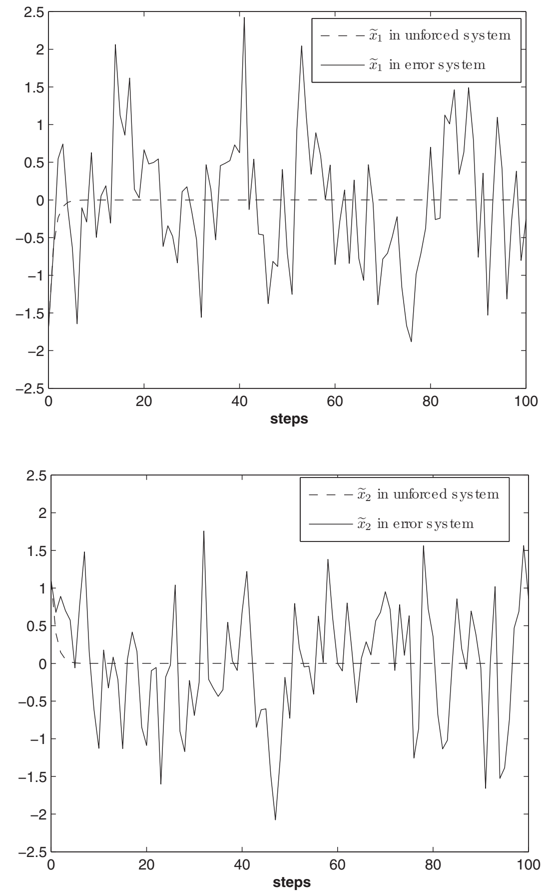

Fig. 4: State trajectories of error system (28) and corresponding unforced system

VI. CONCLUSIONS

A distributed steady-state estimation algorithm for a class of interconnected systems is proposed. Stability conditions for estimation error dynamics are provided based on the state and observation models. It is worth pointing out that the stability of the distributed state estimator is not related to the stability of the given dynamical systems.

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