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Comparison of Node Localization Methods for Sensor Networks

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***Abstract*—A randomly deployed sensor network is typically not completely localizable using distance-based measurements only. Though a necessary and sufficient condition for testing whether a network is localizable has been given in the literature, how to find localizable nodes from a not fully localizable network is still open. In this paper, we try to address a connection between two well-known localization methods, the trilateration method and the WHEEL extension method, by using a graphical tool named *Henneberg operations*. We also study whether Henneberg operations always guarantee the localizability of a network. The localizability by a Henneberg operation-based algorithm is given. Simulation shows that the performance of this algorithm for finding localizable nodes is very close to a well-known necessary condition called 3-path condition.**

I. Introduction

Location based service (LBS) is a fundamental research topic in applying sensor networks. The process of computing the location of a sensor node is called a localization problem. This process usually contains two steps: distance estimation and localization algorithm. The distance between two nodes can be either measured directly through radio signals [4] or estimated through hop-counting [3]. The localization algorithm then utilizes the distance estimates to compute the localization. Most commonly used localization algorithm is the so-called *trilateration* scheme. In the 2-D case, one node's location can be uniquely located through three direct connections with three position-known nodes called anchor nodes. Trilateration can run in a sequential way through adding localizable nodes into the set of anchor nodes one by one.

But for a randomly deployed sensor network, there may be only a small part of the whole sensor network that can be localized using the trilateration scheme because not all nodes in the sensor network have three direct connections with position-known nodes [1] [9]. A newly reported scheme named *WHEEL extension* can find more nodes than trilateration. They explore the whole network to find a specialized structure formed by 6 nodes, 3 of which are position-known nodes, and the other 3 nodes in this structure are jointly localizable.

In this paper, we will introduce a bridging technique that can connect trilateration scheme and WHEEL extension. We will show that the WHEEL extension can be obtained through applying a series of Henneberg operations onto a localized graph. Inspired by a comment about WHEEL extension in [11], we will also discuss whether Henneberg operations always guarantee localizability of a network. Finally, we will give a Henneberg operation-based algorithm and evaluate the gap to a known necessary condition called 3-path condition.

II. Problem Statement

A sensor network is usually abstracted to be a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a non-empty node set and, for $\forall j \in \mathcal{V}$, $(i, j) \neq 0$ and $(i, j) \in \mathcal{E}$. In graph theory, a graph is called rigid if it can not be continuously deformed without changing the distances. A graph is called redundantly rigid if it is still rigid after removing any one edge. A graph is called 3-connected if it is still connected after removal of any two nodes. A graph is called globally rigid if there is a unique realization with the given distance constraints. A globally rigid graph is called minimally globally rigid if it is no longer globally rigid after removing any edge. More detailed definition of these conceptions can be referenced in [1] and [10].

We introduce a necessary and sufficient condition for global rigidity of a graph, which first appeared in [14].

Lemma 1: A graph is globally rigid if and only if it is 3-connected and redundantly rigid.

We also introduce a necessary and sufficient condition for localizability of a network, which was originally pointed out in [6].

Lemma 2: A network is localizable if and only if the graph is globally rigid and there are at least three anchor nodes.

For a network, we can determine whether it is wholly localizable through above necessary and sufficient condition. But a randomly deployed network is hardly fully localizable. In this case we want to know which nodes are still localizable.

There are two competitive schemes to this aim. The first one is the commonly used trilateration scheme, which tests one node at a time for three possible connections with position-known nodes until no more localizable nodes can be found. The advantage of this scheme is that it is easy to realize. But the requirement of three direct connections with position-known nodes is so strong that many localizable nodes will be missed. The other scheme is named WHEEL extension. It also can detect localizable nodes in a sequential way. But its topology requirement is a little more complex than trilatertaion. It aims to detect a specialized 6-node subgraph from the whole network. If this subgraph also contains at least three location known nodes, the graph is localizable. In [8], *Yang et al.* proved that WHEEL extension shows improvement over the trilateration method.

The trilateration scheme and WHEEL extension are both sequential methods and contain sufficient conditions for localizability. A natural question is: What is the connection between these two schemes? In this paper, we will try to build a connection between these two schemes by using a graphical tool called Henneberg operations.

One emphasized character of Henneberg sequence is its ability to maintain global rigidity. However, *Huang et al.* [11] mentioned that WHEEL extension cannot always guarantee the global rigidity of a network and gave a counter example to prove this statement. We will discuss this counter example and study whether a Henneberg sequence can always maintain the global rigidity of a network.

Since there is still no computationally tractable necessary and sufficient condition to test whether a node is localizable, we want to explore the *gap* between a necessary condition known as the 3-path condition [7] and the localization scheme based on Henneberg operations.

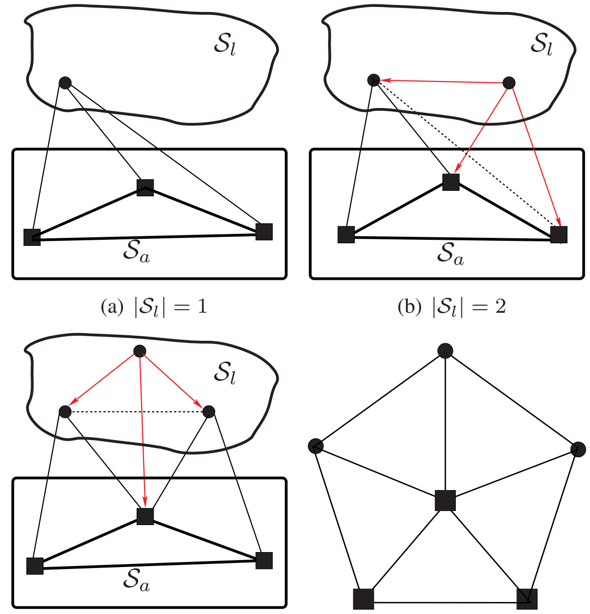
III. From Trilateration to WHEEL extension

A graph $\mathcal{G}$ to be localized consists of m anchor nodes, $m \geq 3$, and n other nodes. When we use the trilateration scheme to detect localizable nodes in $\mathcal{G}$, we start with the neighbors of m anchor nodes. If a node has three direct connections with anchor nodes, it will be treated as a localizable node and added into the set of position known nodes. Then check the next node. This process will keep on until no more localizable nodes can be found. One example is shown in Fig. 1(a)the sets $\mathcal{S}_a$ and $\mathcal{S}_l$ indicate a set of anchor nodes and a set of nodes to be localized, respectively. Anchor nodes and other nodes are indicated by solid circles and squares, respectively. We notice that all nodes in $\mathcal{S}_l$ are localizable since each has three direct connections with anchor nodes.

The WHEEL extension aims to find a 6-node structure, as shown in Fig. 1(d). This 6-node structure can be easily checked to be localizable according to Lemma 2. Note that, though this graph is localizable, we cannot find a trilateration sequence in this graph. This is caused by the fact that the trilateration scheme is only a sufficient but not necessary condition for localizability.

Now, we introduce the conception of *Henneberg operations*. As defined in [13], for a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, a Henneberg operation is to firstly add a new vertex $u \notin \mathcal{V}$ and edges uv, uw, ut with $v, w, t \in \mathcal{V}$ and then delete one edge among v, w and t.

The operations from Fig. 1(a)-(c) is an example of Henneberg operations. The graph shown in Fig. 1(a) is localizable. Then we add a new node onto this graph with three new edges



(a) $|\mathcal{S}_l| = 1$ (b) $|\mathcal{S}_l| = 2$ (c) $|\mathcal{S}_l| = 3$ (d) WHEEL case

Fig. 1. Henneberg sequence from $|\mathcal{S}_1| = 1$ to $|\mathcal{S}_1| = 3$

as in Fig. 1(b) and then delete an existing edge, which is indicated by the dashed line. After that, we can obtain a new graph as shown in Fig. 1(b) without the dashed line. Operate similarly on this graph, we can obtain a new one as shown in Fig. 1(c) without the dashed line.

The graph shown in both Fig. 1(b) and Fig. 1(c) can be checked to be localizable according to Lemma 2. This is due to a property of Henneberg sequence that every globally rigid graph can be obtained from a fully connected graph through Henneberg operations [14].

The graph shown in Fig. 1(c) is identical with the WHEEL 6-node structure shown in Fig. 1(d). Consider the fact that Fig. 1(a) a single trilateration scheme, we have our first conclusion as follows:

Conclusion 1: A 6-node WHEEL graph as shown in Fig. 1(d) can be formed using a series of Henneberg operations starting from a localizable subgraph in Fig. 1(a) by the trilateration scheme.

IV. Can Henneberg Operations Always Guarantee the Localizability?

In [11], *Huang et al.* provided an encounter example to show that the graph formed by WHEEL extension may not always be globally rigid. Since WHEEL extension can be obtained by adding a series of Henneberg sequences to a globally rigid graph, a natural question is whether Henneberg operations can guarantee globally rigidity of the resulting graph $\mathcal{G}'$.

The answer is Henneberg operations *almost surely* preserve the global rigidity of the graph, as shown below:

Theorem 1: If graph $\mathcal{G}'$ is obtained from graph $\mathcal{G}$ by Henneberg operations and $\mathcal{G}$ is 3-connected and redundantly rigid, then $\mathcal{G}'$ is almost surely globally rigid.

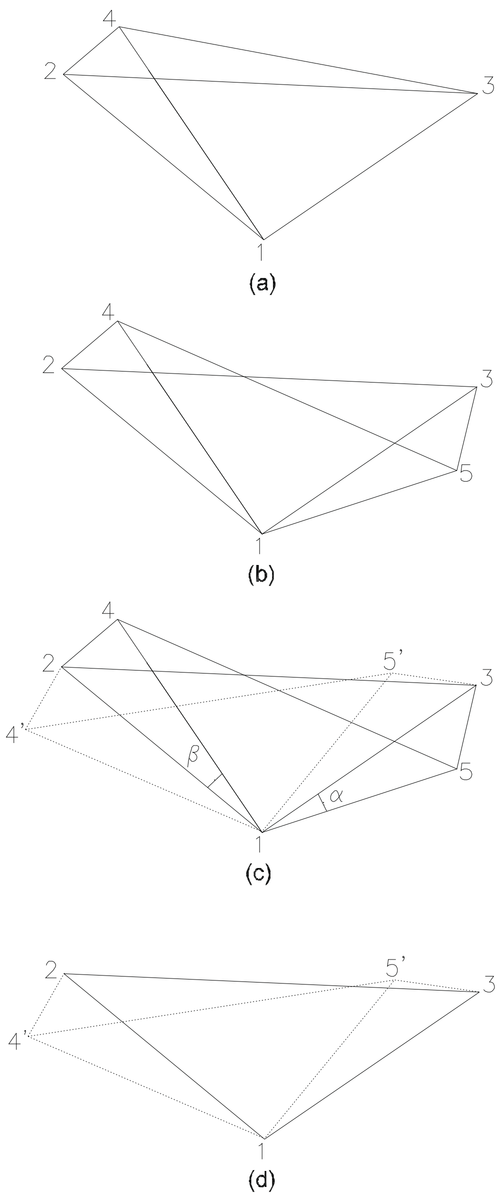


Fig. 2. A counter example. Subfigure (a), (b) and (d) illustrate graph $\mathcal{G}$, $\mathcal{G}^{'}$ and $\mathcal{G}^{''}$, respectively.

Proof: Firstly, we describe the extreme case that $\mathcal{G}'$ are not globally rigid. We choose an arbitrary 4-node fully connected graph $\mathcal{G}$, as shown in Fig. 2(a). It is easily to check that a 4-node fully connected graph is 3-connected and redundantly rigid.

Then we construct a new graph $\mathcal{G}'$ through Henneberg operation on $\mathcal{G}$. The graph after a Henneberg operation is shown in Fig. 2(b). As described before, Henneberg operation adds a new vertex and new edges with three different vertices (vertices 1, 3 and 4) on $\mathcal{G}$. Also, an edge between these three vertices in $\mathcal{G}$ (edge 34) will be deleted. So far, we have finished a standard process of Henneberg operation on a 4-node fully connected graph.

Next, we will examine if this newly obtained graph $\mathcal{G}'$ is globally rigid. At first, we find an axisymmetric point for vertices 4 and 5 with respect to edges 12 and 13 respectively. As shown in Fig.2(c), the axisymmetric point are $4'$ and $5'$. From the definition of axisymmetry, we can obtain that edges 15 and 14 are identical with $15'$ and $14'$, respectively. Also, the angle $\angle\alpha$ and $\angle\beta$ are equal to angle $\angle 315'$ and $\angle 214'$, respectively.

Now we divide the graph $\mathcal{G}'$ into two cases: 1) $\angle\alpha = \angle\beta$; 2)$\angle\alpha \neq \angle\beta$.

Suppose $\angle\alpha = \angle\beta$. We can obtain $\angle 515'$=$\angle 414'$. Because $\angle 415 = \angle 415' + \angle 515'$ and $\angle 4'15' = \angle 4'14 + \angle 415'$, we get $\angle 415$=$\angle 4'15'$.

So far, for $\triangle 415$ and $\triangle 4'15'$, we have two pairs of edges and one angle between them that are equal, i.e., edge $15 = 15'$, edge $14 = 14'$ and angle $\angle 415$=$\angle 4'15'$. Hence $\triangle 415 = \triangle 4'15'$ and edge $45 = 4'5'$. Because vertices $4'$ and $5'$ are axisymmetric with 4 and 5, we also get edges $24 = 24'$ and $35 = 35'$.

Define a graph $\mathcal{G}''$ formed by vertices 1, 2, 3, $4'$, $5'$ and the connecting edges between them. We redraw $\mathcal{G}''$ in Fig. 2(d). We notice that each edge in $\mathcal{G}''$ has the same length with one corresponding edge in $\mathcal{G}'$. So, graph $\mathcal{G}'$ has more than one realization. It is also clear to compare the Fig. 2(d) with Fig. 2(b). Since a globally rigid graph should have unique realization, we can conclude that $\mathcal{G}'$ is not globally rigid if and only if $\angle\alpha = \angle\beta$.

For a randomly deployed network, once the position of vertices $1, 2, 3, 4$ are fixed, it has a zero measure for vertex 5 to be deployed at a position such that $\angle\alpha = \angle\beta$. Thus, we can conclude that the graph $\mathcal{G}'$ is almost surely globally rigid. ■

Remark 1: Note that there is also a *concave* form of 4-node fully connected graph $\mathcal{G}$, such as shown in Fig. 3(a). For this case, we can still have a graph $\mathcal{G}'$ not globally rigid obtained from $\mathcal{G}$ if $\angle\alpha = \angle\beta$ in Fig. 3(b).

This kind of extreme cases in both [11] and the proof here will be excluded if we introduce a concept of *generic globally rigid*. The generic property leads a graph to avoid some specialized positions such as a co-linear three nodes position or $\angle\alpha = \angle\beta$ as in above cases.

V. Henneberg Operation-Based Localization Scheme

In this section, we will use Henneberg operations to explore the localizability of a sensor network. Since the global rigidity of a graph is guaranteed by Henneberg operations, we can detect the localizable nodes by using Henneberg operations in a sequential way.

The idea of our Henneberg operation-based localization scheme is as follows:

Starting with a set of localizable nodes, such as $\mathcal{S}_a$ shown in Fig.1(a), we can determine the localizability of one position-unknown node, say node i, in the set $\mathcal{S}_l$ to be localized by analyzing whether the connections between $\mathcal{S}_a$ and node i can

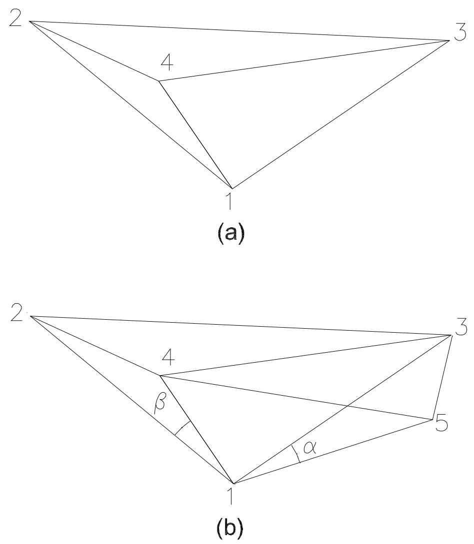


Fig. 3. Another form of 4-node fully connected graph.

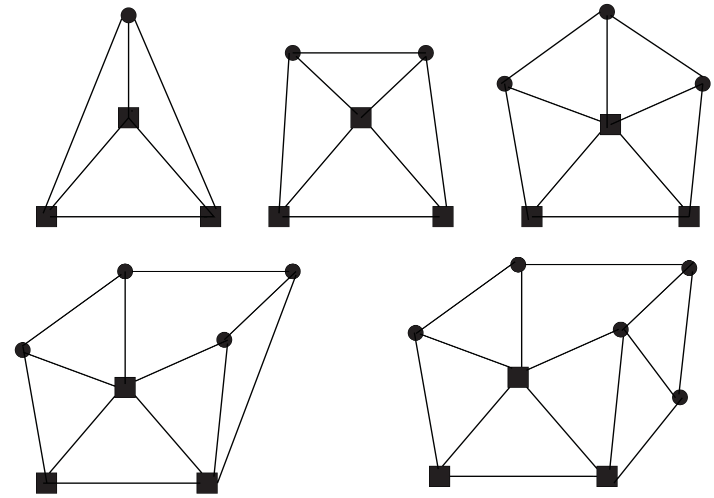


Fig. 4. An illustration of Henneberg operation based scheme.

Algorithm 1 Henneberg Operation-Based Localization Algorithm

A network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a set of m anchor nodes in $\mathcal{S}_a$, $m \geq 3$, and n sensor nodes in $\mathcal{S}_l$.

for any $i \in \mathcal{S}_l$ **do**
  if the neighbor set of i $\mathcal{N}_i < 3$ **then**
    Drop node i and continue
  else
    if $|\mathcal{N}_i \cap \mathcal{S}_a| \geq 3$ **then**
      i is localizable. Add i to $\mathcal{S}_a$.
    else if $|\mathcal{N}_i \cap \mathcal{S}_a| \geq 2$ and $|\mathcal{N}_j \cap \{\mathcal{S}_a \cup i\}| \geq 3$ and $j \in \mathcal{N}_i$ **then**
      i and j are localizable. Add i, j to $\mathcal{S}_a$.
    else if $|\mathcal{N}_i \cap \mathcal{S}_a| \geq 2$ and $|\mathcal{N}_j \cap \mathcal{S}_a| \geq 2$ and $|\mathcal{N}_k \cap \{\mathcal{S}_a \cup i \cup j\}| \geq 3$ and $i \in \mathcal{N}_k$ and $j \in \mathcal{N}_k$ **then**
      i, j and k are localizable. Add i, j, k to $\mathcal{S}_a$.
    end if
  end if
end for

be formed by a Henneberg operation starting from any three nodes in the joint graph of $\mathcal{S}_a$ and $\mathcal{S}_l$ shown in Fig. 1(a). If so, we add node i into the set of $\mathcal{S}_a$. Otherwise, we choose any one neighbor of i in $\mathcal{S}_l$, say node j, and test whether the connections between $\mathcal{S}_a$ and nodes i and j can be formed by a Henneberg operation starting from any three nodes in the joint graph. If so, we add both nodes i and j into $\mathcal{S}_a$. Otherwise, we consider one more neighbor of i in $\mathcal{S}_l$, say node k, which is also the neighbor of node j, and test the joint graph of i, j, k and $\mathcal{S}_a$. Although this process can continue, we only consider the three situations above due to complexity considerations.

After that, we repeat the same test on another node in $\mathcal{S}_l$. This process will be terminated until no more localizable node is detected.

Note that, when the number of position-unknown nodes equals to one, our scheme is identical with trilateration scheme. When considering three nodes at a time, our scheme here will cover all possible cases that can be detected by the WHEEL extension. When we consider more nodes in the set to be localized, it can find more nodes than the WHEEL extension scheme.

We give an intuitive comparison in Fig 4 among the three schemes, i.e., trilatration, WHEEL extension and Henneberg operation-based scheme. The nodes are marked in the same way as in Fig.1(a)-(d). The first and third subfigures in the upper row of Fig. 4 correspond to graphs that can be localized by the scheme of trilateration or WHEEL extension. The bottom row of the figure are two examples that cannot be detected by WHEEL extension, but can be handled by Henneberg operations.

VI. Performance Evaluation

As we mentioned earlier, so far, there is still not a simple necessary and sufficient condition on the localizability of one single node. To evaluate how many localizable nodes are missed during the exploration process of our Henneberg operations, we use the 3-path condition, which is necessary for a node to be localizable, to search through the whole network. The difference between these two numbers represents the gap between our proposed and the 3-path condition, not with the true localizable set of nodes. This is caused by the fact that the 3-path is necessary but not sufficient condition for localizability.

To do this comparison, we run a 100-round Monte Carlo simulation. In each round, we deploy a random network and then use the two schemes to detect localizable nodes. The number of detected localizable nodes is recorded. We also detect the nodes fitting the 3-path condition and record the number of these nodes. Then compare the gap between this 3-path condition and our proposed algorithm.

The result is shown in Fig. 5. The horizontal axis indicates the number of simulation rounds and the vertical axis indicates the proportion of the number of localizable nodes detected by our proposed algorithm together with that found by 3-path condition. The empty area between the solid area and the horizontal level at the level of 1 indicates the gap. From Fig. 5, we see that the gap between our proposed algorithm and the 3-path necessary condition is very small. The average gap of the 100 rounds simulation is 5.88%.

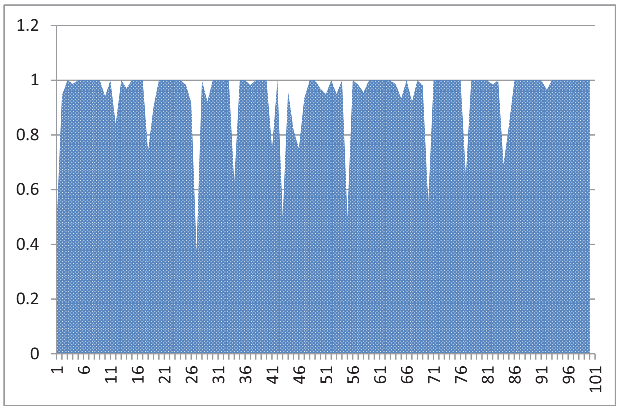


Fig. 5. Gap between the proposed algorithm and the 3-Path necessary condition

VII. Conclusion

In this paper, we have established a connection between the trilateration scheme and WHEEL extension. These two schemes are connected by a series of Henneberg operations. Towards a comment on WHEEL extension, we analyze the existence condition of the counter example in [11] and give a more precise conclusion on how to preserve global rigidity. We give a Henneberg operation-based algorithm to detect localizable nodes. The gap between our proposed algorithm and the 3-path necessary condition is shown to be small.

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