Optimal Design of Hybrid Filterbank Analog-to-Digital Converters using Input Statistics

Damián Marelli, Kaushik Mahata and Minyue Fu *Fellow, IEEE*
School of Electrical Engineering and Computer Science, University of Newcastle, N.S.W. 2308, Australia.

***Abstract*—The hybrid filterbank architecture permits implementing accurate, high speed analog-to-digital converters. However, its design is technically involved since perfect reconstruction of the desired samples cannot be achieved in general. In this paper we propose a design method which generalizes existing approaches by dropping the bandlimited assumption on the input signal. The design minimizes the power of the reconstruction error in the samples, for a given input signal power spectrum. We discuss the use of blind techniques to estimate the analysis filterbank parameters as well as the input power spectrum, and we present simulation results to demonstrate the clear advantage of the proposed design, even under input spectrum uncertainties.**

***Index Terms*—Analog digital conversion, hybrid filter banks, least mean square methods, multirate systems, multichannel sampling.**

I. Introduction

A high speed analog-to-digital converter (ADC) can be realized by using the so-called time-interleaved architecture [1], which consists of using a number of parallel ADCs having the same sampling rate but different sampling phases, as if they were a single ADC operating at a higher sampling rate. A drawback of this approach is its extreme sensitivity to converter mismatches and timing errors [2]. To overcome this limitation, the hybrid filterbank ADC (HFB-ADC) architecture was proposed in [3]. This technique uses a continuous-time analysis filterbank to split the input signal into different frequency bands, each of which is assigned to a different ADC. Then, a discrete-time synthesis filterbank is used to reconstruct the required samples.

A method for designing the discrete-time synthesis filterbank was proposed in [3], and improved in [4], and relies on the assumption that the input signal is bandlimited. Under this assumption, both methods are able to achieve perfect reconstruction if the impulse response of the synthesis filterbank can be arbitrarily large. An arguable point of this approach is that the bandlimited assumption might not be realistic in many applications. To address this issue, we propose in this paper a synthesis filterbank design method which uses the knowledge of the power spectrum of the input signal, to carry out a compensation in a statistically optimal (least-mean-squares (LMS)) sense. Also, in view of our non-bandlimited scenario, the proposed method permits designing the synthesis filterbank so that the reconstructed samples match those that would be obtained if the input signal was passed through a prescribed anti-alias filter before sampling. In addition, we show that the existing design method derived under a bandlimited assumption is a particular case of the proposed method.

This work was partially developed at the Faculty of Mathematics, NuHAG, University of Vienna, with financial support from the EU Marie Curie fellowship MEIF-CT-2006-023728.

The proposed method, as well as the methods in [3], [4], require the knowledge of the analysis filterbank parameters. Assuming this knowledge is unrealistic since the parameters of analog circuits are subject to imperfections, e.g., deviations from nominal values, aging, temperature drifts, etc. In view of this, a reference input was used in [5] to estimate the analog parameters. However, as pointed out in [4], a blind estimation technique (i.e., without the knowledge of the input signal) is preferred. In Section V we show how to use the blind estimation method in [6], to estimate not only the analysis filterbank parameters, but also the input power spectrum.

The rest of the paper is organized as follows. We give an overview of hybrid filterbank ADCs in Section II. In Section III we introduce the proposed synthesis filterbank method. In Section IV we show that the existing design method derived under a bandlimited assumption is a particular case of the proposed method. Finally, some simulation results are presented in Section VI and concluding remarks are given in Section VII.

II. Hybrid Filterbank Analog-to-Digital Converters

The HFB-ADC scheme is depicted in Figure 1. The continuous-time signal $x(t)$ is split into M signals using an array of continuous-time filters $h(s) = [h_1(s), \cdots, h_M(s)]^T$, whose outputs are then sampled at rate $1/MT$. In this way, the discrete-time signals $\mathbf{x}(k) = [\mathbf{x}_1(k), \cdots, \mathbf{x}_M(k)]^T$ are generated. The idea is to combine the signals $\mathbf{x}(k)$ to generate an estimate $\hat{\mathbf{y}}(k)$ of the samples $\mathbf{y}(k) = x(kT)$. This is typically done by upsampling the signals $\mathbf{x}(k)$ by a factor of M (i.e., $M-1$ zero valued samples are added between every two samples). Then filtering each component using the array of discrete-time filters $\mathbf{f}(z) = [\mathbf{f}_1(z), \cdots, \mathbf{f}_M(z)]^T$, and finally adding together all the resulting signals.

An approach for designing the synthesis filterbank $\mathbf{f}(z)$ was proposed in [3], and improved in [4]. This method assumes that the signal $x(t)$ is bandlimited to $1/2T$. Under

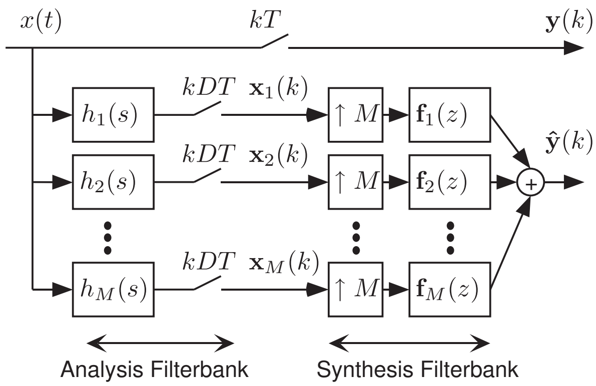

Figure 1. Hybrid filterbank analog-to-digital converter scheme.

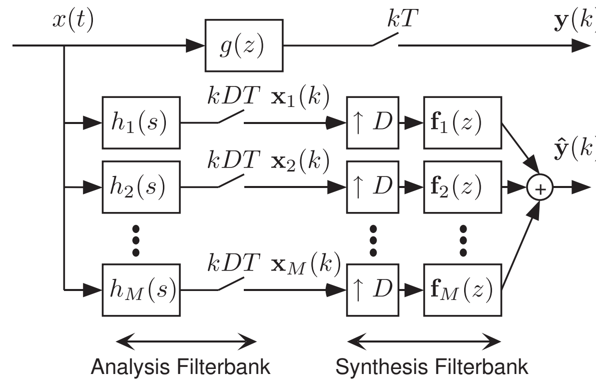

Figure 2. Slightly generalized scheme considered in this work.

this assumption, $\mathbf{f}(z)$ is designed as follows:

$$\mathbf{f}=\arg\min_{\tilde{\mathbf{f}}}\int_{-\pi}^{\pi}\left|\frac{1}{M}\sum_{m=0}^{M}\left(J_m-\mathbf{f}^T(e^{j\omega})\mathbf{h}\left(e^{j(\omega-\frac{2\pi m}{M})}\right)\right)\right|^2 \quad (1)$$

where $J_m = 1$ for $m = 1$ and 0 otherwise, and $\mathbf{h}(z)$ is a discrete-time equivalent of the analysis filterbank $h(s)$, whose frequency response is given by

$$\mathbf{h}(e^{j\omega}) = h(j\frac{\omega}{T}),\ \omega \in [-\pi,\pi]. \quad (2)$$

Moreover, perfect reconstruction can be achieved (i.e., $\hat{\mathbf{y}}(k) = \mathbf{y}(k)$) if the impulse response $\mathbf{f}(k)$ can be arbitrary large.

III. Proposed Method

In this section we propose an alternative to (1), for designing the synthesis filterbank. More precisely, we drop the bandlimited constraint on the input signal, and we assume instead that it has a known power spectrum $\phi_x(s)$. Then, we design the synthesis filterbank $\mathbf{f}(z)$ using a linear LMS criterion [7], i.e., aiming at minimizing the power of the reconstruction error

$$\mathbf{e}(k) = \mathbf{y}(k) - \hat{\mathbf{y}}(k). \quad (3)$$

We consider a slight generalization of the scheme in Figure. 1, which is depicted in Figure 2. This generalization permits the use of oversampling (i.e., $D \leq M$) as well as placing a prescribed anti-alias filter $g(s)$ before generating the samples $\mathbf{y}(k)$ to be reconstructed.

Using the polyphase representation [8], the scheme in Figure 2 can be transformed into that of Figure 3, where

$$\begin{aligned}\mathbf{y}_P(k) &= [\mathbf{y}(kD), \mathbf{y}(kD-1), \cdots, \mathbf{y}(kD-D+1)]^T\\ \hat{\mathbf{y}}_P(k) &= [\hat{\mathbf{y}}(kD), \hat{\mathbf{y}}(kD-1), \cdots, \hat{\mathbf{y}}(kD-D+1)]^T\end{aligned}$$

are the polyphase representations of $\mathbf{y}(k)$ and $\hat{\mathbf{y}}(k)$, respectively, and the $D \times M$ matrix $\mathbf{F}(z)$ is the polyphase representation of the synthesis filterbank, defined such that the impulse response $[\mathbf{F}(k)]_{d,m}$ of its (d,m)-entry is given by:

$$[\mathbf{F}(k)]_{m,d} = f_m(kD + d - 1).$$

In view of Figure 3, we can restate the problem as that of designing $\mathbf{F}(z)$ for estimating $\mathbf{y}_P(k)$ using $\mathbf{x}(k)$. If $x(t)$ is assumed to be a stationary random process, and the support of the impulse response $\mathbf{F}(k)$ of $\mathbf{F}(z)$ is constrained to the interval $[a,b]$, the LMS solution can be found by solving

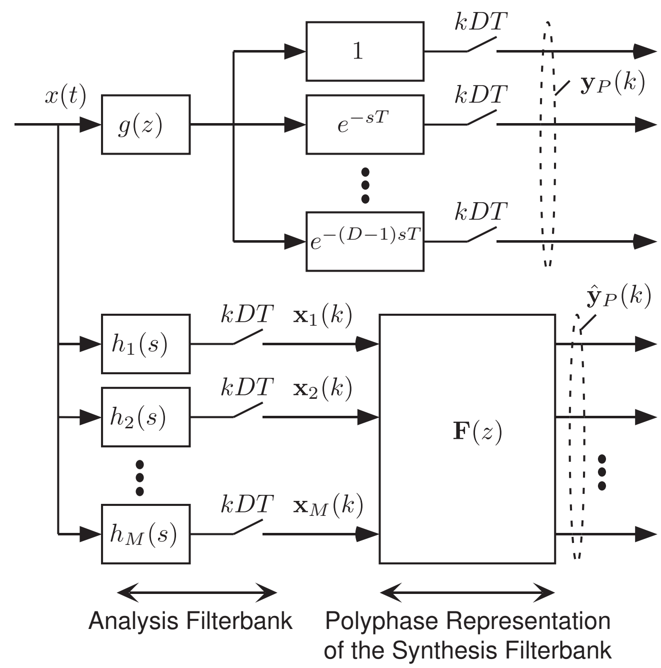

Figure 3. Transformed scheme using polyphase representation.

$$\mathbf{F} = \arg\min_{\tilde{\mathbf{F}}}\mathcal{E}\left\{\left\|\mathbf{y}_P(0) - \sum_{l=a}^{b}\tilde{\mathbf{F}}(l)\mathbf{x}(-l)\right\|_2^2\right\} \quad (4)$$

where $\mathcal{E}\{\cdot\}$ denotes expected value. Now, following the geometric argument in [7, Section 3.3], the solution of (4) satisfies

$$\mathbf{R}_{\mathbf{y}_P\mathbf{x}}(k) = \sum_{l=a}^{b}\mathbf{F}(l)\mathbf{R}_{\mathbf{x}}(k-l),\ \text{for all } k \in \{a,\cdots,b\}, \quad (5)$$

where $\mathbf{R}_{\mathbf{x}}(k)$ and $\mathbf{R}_{\mathbf{y}_P\mathbf{x}}(k)$ denote the correlation matrix of $\mathbf{x}(k)$ and the cross-correlation matrix between $\mathbf{y}_P(k)$ and $\mathbf{x}(k)$, respectively, i.e., $\mathbf{R}_{\mathbf{x}}(k) = \mathcal{E}\{\mathbf{x}(k)\mathbf{x}^T(0)\}$ and $\mathbf{R}_{\mathbf{y}_P\mathbf{x}}(k) = \mathcal{E}\{\mathbf{y}_P(k)\mathbf{x}^T(0)\}$. Hence, the impulse response

$\mathbf{F}(k)$ of the polyphase matrix $\mathbf{F}(z)$ can be obtained by solving the linear problem (5). To this end, we need the expressions of $\mathbf{R}_{\mathbf{x}}(k)$ and $\mathbf{R}_{\mathbf{y}_P\mathbf{x}}(k)$. It is straightforward to verify that

$$\mathbf{R}_{\mathbf{x}}(k) = \mathcal{L}^{-1}\left\{h(s)\phi_x(s)h^T(-s)\right\}(kDT) \quad (6)$$

$$\mathbf{R}_{\mathbf{y}_P\mathbf{x}}(k) = \mathcal{L}^{-1}\left\{\delta(s)g(s)\phi_x(s)h^T(-s)\right\}(kDT) \quad (7)$$

where $\delta(s) = [1, e^{-s}, \cdots, ^{-(M-1)s}]$ and $\mathcal{L}^{-1}\{\cdot\}$ denotes the inverse Laplace transform.

IV. Bandlimited Case

In this section we show that the synthesis filterbank design (1) is equivalent to our proposed design (5)-(7) when $M = D$, $g(z) = 1$ and the input power spectrum is given by

$$\phi_x(j\omega) = \begin{cases} 1, & |\omega| < \frac{\pi}{T} \\ 0, & \text{otherwise} \end{cases}. \quad (8)$$

Under these assumptions, it holds that $x(t) = \sum_{k=-\infty}^{\infty} \mathbf{y}(k)\,\mathrm{sinc}\left(\frac{t}{T} - k\right)$, and therefore

$$\mathbf{x}(k) = \sum_{l=-\infty}^{\infty} \mathbf{h}(l)\mathbf{y}(k-l), \quad (9)$$

where $\mathbf{h}(k)$ is the impulse response the discrete-time equivalent analysis filterbank (2). Let $\underline{\mathbf{y}}(k)$, $\underline{\hat{\mathbf{y}}}(k)$ and $\underline{\mathbf{e}}(k)$ denote truncated realizations of $\mathbf{y}(k)$, $\hat{\mathbf{y}}(k)$ and $\mathbf{e}(k)$, respectively, so that their z-transform $\underline{\mathbf{y}}(z)$, $\underline{\hat{\mathbf{y}}}(z)$ and $\underline{\mathbf{e}}(z)$ are well defined on the unit circle. Then, using the alias representation [8], we can write

$$\underline{\hat{\mathbf{y}}}(z) = \frac{1}{M}\mathbf{f}^T(z)\mathbf{H}_A(z)\underline{\mathbf{y}_A}(z)$$

where $\mathbf{H}_A(z)$ and $\underline{\mathbf{y}_A}(z)$ denote the alias representations of $\mathbf{h}(z)$ and $\underline{\mathbf{y}}(z)$, respectively, which are given by

$$\mathbf{H}_A(z) = [\mathbf{h}(z), \mathbf{h}(e^{j\frac{2\pi}{D}}z), \cdots, \mathbf{h}(e^{j\frac{2\pi(D-1)}{D}}z)] \quad (10)$$

$$\underline{\mathbf{y}_A}(z) = [\underline{\mathbf{y}}(z), \underline{\mathbf{y}}(e^{j\frac{2\pi}{D}}z), \cdots, \underline{\mathbf{y}}(e^{j\frac{2\pi(D-1)}{D}}z)]^T \quad (11)$$

Now, letting $\mathbf{J} = [1, 0, \cdots, 0]$, we can write $\underline{\mathbf{y}}(z) = \mathbf{J}\underline{\mathbf{y}_A}(z)$ and $\underline{\mathbf{e}}(z) = \left(\frac{1}{M}\mathbf{f}^T(z)\mathbf{H}_A(z) - \mathbf{J}\right)\underline{\mathbf{y}_A}(z)$. Now, (8) implies that $\mathbf{y}_A(k)$ is a white vector random process, then the LMS criterion for designing $\mathbf{f}(z)$ becomes

$$\mathbf{f} = \arg\min_{\tilde{\mathbf{f}}} \int_{-\pi}^{\pi} \left\| \frac{1}{M}\tilde{\mathbf{f}}^T(e^{j\omega})\mathbf{H}_A(e^{j\omega}) - \mathbf{J} \right\|_2^2 d\omega$$

which, in view of (10), is equivalent to (1).

V. Blind Estimation of the Input Spectrum and the Analysis Filterbank

A blind estimation method for rational models was proposed in [6]. This method can be used for estimating the analysis filterbank parameters as well as the input power spectrum, following the idea sketched below. For more details see [6].

We assume that the input power spectrum can be factorized as follows $\phi_x(s) = |l_x(s)|^2$, with $l_x(s)$ being a rational model whose numerator and denominator have a known order. If the orders of the analysis filterbank filters $h_m(s)$, $m = 1, \cdots, M$ are also known, we can write parametric versions $h_m(s, \alpha)$ and $l_x(s, \beta)$, where α and β denote the vectors of numerator and denominator coefficients of $h_m(s)$ and $l_x(s)$, respectively. Let $\overline{\mathbf{R}_{\mathbf{x}}}(k)$ be an estimate of the input correlation matrix (6) obtained from the available samples. Then, using (6), α and β can be estimated by solving the following minimization problem:

$$(\alpha, \beta) = \arg\min_{\alpha,\beta} \sum_{k=0}^{K} \left\| \overline{\mathbf{R}_{\mathbf{x}}}(k) - \mathbf{R}_{\mathbf{x}}(k, \alpha, \beta) \right\|_2^2$$

$$\mathbf{R}_{\mathbf{x}}(k, \alpha, \beta) = \mathcal{L}^{-1}\left\{h(s,\alpha)|l_x(s,\beta)|^2\overline{h^T(s,\alpha)}\right\}(kDT)$$

VI. Simulation

In order to evaluate the proposed LMS design method, we compare its performance with that of the design (1), derived under a bandlimited assumption on the input signal, which we denote by (BL). The comparison is made in terms of the signal-to-distortion ratio (SDR) of the reconstructed samples, defined by

$$SDR = 10\log_{10}\left(\frac{\sum_{k=1}^{N}|\mathbf{y}(k)|^2}{\sum_{k=1}^{N}|\mathbf{y}(k) - \hat{\mathbf{y}}(k)|^2}\right)$$

Following [4], we consider an eight-channel HFB-ADC, where for simplicity, we use the sampling period $T = 1$. The analysis filterbank is composed of Butterworth second-order bandpass filters of bandwidth $1/16$ Hz, except for the first one which is a first-order lowpass filter of the same bandwidth. The bandwidths are contiguously allocated so that they cover the whole frequency range from 0 Hz to 0.5 Hz. The output of each filter is sampled at $1/8$ Hz (i.e., the upsampling factor D in Figure 2 equals the number M of channels). Finally, we take $a = -3$ and $b = 4$ in (4), which results in the discrete-time filters $\mathbf{f}_m(z)$, $m = 1, \cdots, M$ having 64 taps with 31 non-causal taps.

In the first simulation we use $g(s) = 1$, and we generate the input signal as filtered white noise using a Butterworth lowpass filter (*input filter*) of 20-th order and varying cutoff frequency f_c. The frequency response of a family of such filters with different values of f_c is shown in Figure 4. We compare the performances of the BL and the proposed LMS method for several values of f_c. The result is shown in Fig. 5. We see how the LMS method clearly outperforms the BL method, especially for low cutoff frequency values.

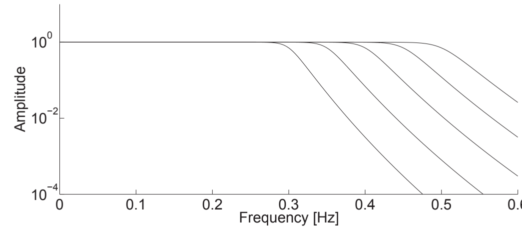

Figure 4. Frequency response of a family of input filters with different cutoff frequency values.

As already mentioned, the proposed LMS method requires the knowledge of the input power spectrum to optimally design

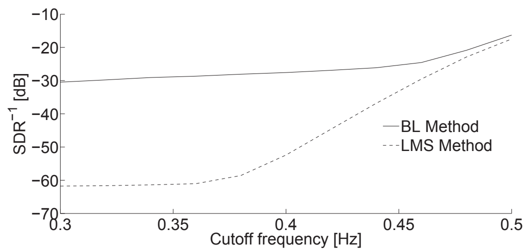

Figure 5. Performance comparison of the BL and LMS methods for different values of the input filter cutoff frequency.

the synthesis filterbank. In the second simulation we evaluate its performance degradation when there is a mismatch between the actual input signal power spectrum, determined by the *actual input filter*, and the nominal input power spectrum used to design the LMS compensator, determined by a *nominal filter*. We design the nominal input filter as a Butterworth lowpass filter of 20-th order and varying cutoff frequency f_c. For the actual input filter we use a Butterworth lowpass filter of 10-th order and varying cutoff frequency f_c in cascade with a second order filter with poles in $-0.0782\, f_c \pm j0.4938 f_c$. The frequency responses of the nominal and the actual input filters are shown in Fig. 6, and the simulation result is shown in Fig. 7. We see that, while not being optimal, the performance of the LMS method does not deteriorate significantly, and it is still clearly superior to that of the BL method.

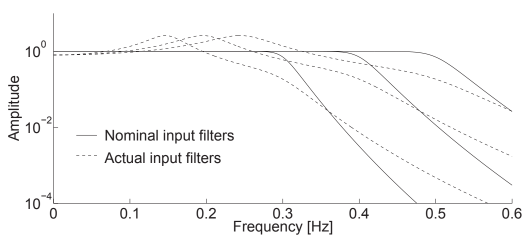

Figure 6. Frequency responses of nominal and actual input filters for different cutoff frequency values.

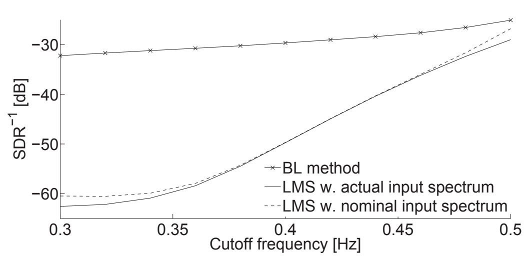

Figure 7. Performance degradation of the LMS method in the presence of input power spectrum mismatch.

In the final simulation we evaluate the performance of the proposed method when reconstructing the samples that would be obtained after filtering the input signal using a prescribed anti-alias filter $g(s)$. We use a Butterworth lowpass filter of 5-th order and varying cutoff frequency as input filter, and a Butterworth lowpass filter of 20-th order and fixed cutoff frequency $0.4Hz$ as anti-alias filter. The obtained SDR values for different cutoff frequencies are shown in Figure 8. As expected, the performance of the LMS method improves with the use of the anti-alias filter.

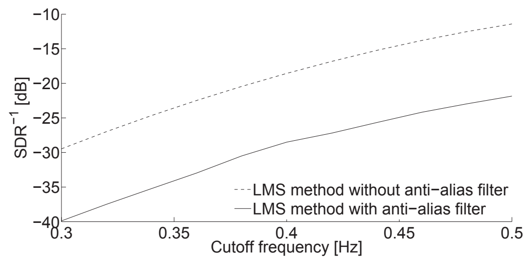

Figure 8. Performance of the LMS method, with and without anti-alias filter using a 5-th order Butterworth lowpass input filter.

VII. Conclusion

We proposed a synthesis filterbank design method for hybrid filterbank analog-to-digital converters, which generalizes existing approaches by dropping the bandlimited assumption on the input signal. The design is done in a statistically optimal sense, by minimizing the power of the sample reconstruction error, for a given input power spectrum. We discussed the use of blind techniques to estimate the analysis filterbank parameters and the input power spectrum, which are required by the proposed method, and we presented simulation results which demonstrate its clear advantage, even under uncertainties on the input power spectrum.

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