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PERFORMANCE CONTROL OF LINEAR SYSTEMS USING QUANTIZED FEEDBACK

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Abstract. This paper studies two control problems for linear systems using quantized feedback, namely quadratic performance control and H_∞ control. We first revisit the work by Elia and Mitter on quadratic stabilization of linear systems using quantized state feedback with a logarithmic quantizer. We reinterpret their result on coarsest quantization density using a sector bounded method. This allows us to generalize their work to the aforementioned performance control problems. We give conditions under which a given quantization density yields a linear controller that guarantees a required level of performance.

1 Introduction

There is a new line of research on quantized feedback control where an quantizer is regarded as an information coder. The fundamental question of interest is how much information needs to be communicated by the quantizer in order to achieve a certain control objective. Noticeable works include [1, 2, 3, 4]. In [4], the problem of quadratic stabilization of discrete-time single-input-single-output (SISO) linear time-invariant (LTI) systems using quantized feedback is studied. The quantizer is assumed to be static and time-invariant (i.e., memoryless and with fixed quantization levels). It is proved in [4] that for a quadratically stabilizable system, the quantizer is the so-called *logarithmic* (i.e., the quantization levels are linear in logarithmic scale). Further, the coarsest quantization density is given explicitly in terms of the system's unstable poles. The work of [4] is also generalized to some extent to guaranteed performance control [5], stabilization of two-input systems [6], and multi-input systems [7].

The most pertinent work to this paper is [4] which no doubt has a significant value in this line of research. However, the work in [4] is about stabilization only, and it seems non-trivial to generalize their approach to performance control. In a companion paper [8], we have revisited the key result in [4] which is on quadratic stabilization of SISO linear systems using quantized state feedback. We have shown that the coarsest quantization density for a logarithmic quantizer can be simply obtained using the sector bound method. This not only gives a simpler interpretation of the result, but also provides the basis for generalization of the result. Indeed, we have generalized the work of [4] to robust stabilization using quantized output feedback and to robust stabilization of multi-input-multi-output systems using quantized state or output feedback.

In this paper, we generalize our results in [8] further to performance control using quantized feedback. Both linear quadratic performance and H_∞ performance problems are studied. We give conditions under which a given quantization density yields a linear controller that guarantees a required level of performance.

2 Stabilization using Quantized State Feedback

This section is basically duplicated from [8]. The purpose is to introduce the main tool for our approach, i.e., the sector bound method for analysis of quantization error. More specifically, we revisit the work of Elia and Mitter [4] on stabilization using quantized state feedback and show how to reinterpret their result.

Consider the following system:

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times 1}$, x is the state and u is a quantized state feedback in the following form:

$$u(k) = f(v(k)) \tag{2}$$

$$v(k) = Kx(k) \tag{3}$$

In the above, $K \in \mathbf{R}^{1\times n}$ is the feedback gain, and $f(\cdot)$ is a quantizer assumed to be symmetric, i.e., $f(-v) = -f(v)$. The set of quantized levels is denoted by

$$\mathcal{U} = \{\pm u_i, i = 0, \pm 1, \pm 2, \cdots\} \cup \{0\} \tag{4}$$

Throughout this paper, we consider the so-called logarithmic quantizer below:

$$\begin{aligned}\mathcal{U} \;=\; & \{\pm u^{(i)} : u^{(i)} = \rho^i u^{(0)}, i = \pm 1, \pm 2, \cdots\} \\ & \cup\{\pm u^{(0)}\} \cup \{0\}, \quad 0 < \rho < 1, u^{(0)} > 0\end{aligned} \tag{5}$$

The parameter ρ will be called the *quantization density.*

For the quadratic stabilization problem, a quadratic Lyapunov function $V(x) = x^T Px, P = P^T > 0$, is used to assess the stability of the feedback system. That is, the quantizer must satisfy

$$\nabla V(x) = V(Ax + Bf(Kx)) - V(x) < 0, \ \forall x \neq 0 \quad (6)$$

The coarsest quantizer is the one which minimizes ρ subject to (6), and the result is given below as in [4], but we provide an alternative proof using the sector bound method.

Theorem 1 *Consider the linear system in (1). The coarsest quantization density is given by*

$$\rho = \frac{1-\delta}{1+\delta} \quad (7)$$

with

$$\delta^{-1} = \prod_i |\lambda_i^u| \quad (8)$$

where λ_i^u are the unstable eigenvalues of A.

Proof. Define the quantization error by

$$e = u - v = f(v) - v \quad (9)$$

Let the quantization levels be given by (5) for any $0 \leq \rho < 1$. It is straightforward to check that e is bounded by the following sector:

$$e = \Delta(v)v, \quad \|\Delta(v)\| \leq \delta \quad (10)$$

where δ is obtained from (7). Therefore, we can model the quantized feedback system as the following uncertain system:

$$x(k+1) = Ax(k) + B(1 + \Delta(Kx))Kx(k) \quad (11)$$

and the quadratic stabilization condition becomes

$$\nabla V(x) = V((A+B(1+\Delta(Kx))K)x) - V(x) < 0, \forall x \neq 0 \quad (12)$$

Let P and K be fixed for the moment. It is trivial to see that the above holds if the following holds:

$$\begin{aligned} \nabla P(\Delta) &= (A + B(1+\Delta)K)^T P(A + B(1+\Delta)K) - P \\ &< 0, \ \forall |\Delta| \leq \delta \end{aligned} \quad (13)$$

where Δ is independent of the state. Next, we show below that the converse is also true, i.e., (12) implies (13). Indeed, suppose (12) holds but (13) is violated for some $|\Delta_0| \leq \delta$. Let x_0 be the eigenvector of $\nabla P(\Delta_0)$ corresponding to a non-negative eigenvalue, i.e., $x_0^T \nabla P(\Delta_0) x_0 \geq 0$. Note that $Kx_0 \neq 0$ because of (12). Now choose any $x_1 = \alpha x_0$ for some scalar $\alpha \neq 0$ such that $\Delta(Kx_1) = \Delta_0$, which is possible because $\Delta(\cdot)$ swings between $-\delta$ and δ. We have

$$\nabla V(x_1) = x_1^T \nabla P(\Delta(Kx_1)) x_1 \geq 0 \quad (14)$$

This contradicts the assumption that (12) holds. Hence, the converse is proved.

The result above means that the problem of coarsest quantization is equivalent to finding the maximum δ for the following system

$$x(k+1) = Ax(k) + B(1+\Delta)u(k), \quad |\Delta| \leq \delta \quad (15)$$

to be quadratically stabilizable. It is well-known [9] that this is equivalent to minimizing the H_∞-norm of the transfer function

$$G_c(z) = K(zI - A - BK)^{-1}B \quad (16)$$

More specifically,

$$\sup_{P,K} \delta = \frac{1}{\inf_K \|G_c(z)\|_\infty} \quad (17)$$

Hence, it remains to show that the solution to (17) leads to (8). This is a standard H_∞ problem. The details can be found in [8]. □

3 Stabilization of MIMO Systems

In this section, we summarize a result from [8] for quantization stabilization of MIMO systems. Both quantized state feedback and quantized output feedback are considered. For the latter, we consider the case where the measured output $y(k)$ is quantized.

Qnantized State Feedback. The system is as in (1) except that $u \in \mathbf{R}^m$, $y \in \mathbf{R}^r$. Suppose quantized state feedback (2)-(3) is used, where $K \in \mathbf{R}^{m \times n}$ and

$$f(v) = \mathrm{diag}\{f_1(v_1), f_2(v_2), \cdots, f_m(v_m)\} \quad (18)$$

where v_j is the jth component of v and $f_j(\cdot)$ is a quantizer (5) with $0 < \rho_j < 1$. Because we have more than one quantizer, the notion of coarsest quantization is not well-defined. Instead, we ask the following question: Given a vector of quantization levels $\rho = [\rho_1 \ \rho_2 \ \cdots \ \rho_m]$, does there exist an quantized feedback controller that quadratically stabilizes the system (1)?

Theorem 2 *Given the system in (1) and a quantization level vector ρ, consider the auxiliary system:*

$$x(k+1) = Ax(k) + B(I + \Delta(k))v(k) \quad (19)$$

where $|\Delta_j(k)| \leq \delta_j$ for all $j = 1, 2, \cdots, m$ and k, and δ_j are converted from ρ_j using (7), and $v(k)$ is a control input. Suppose the auxiliary system is quadratically stablizable via state feedback (3), then (1) is quadratically stabilizable via quantized state feedback. Conversely, suppose the system (1) is quadratically stabilizable via quantized state feedback and, in addition, suppose $\ln \rho_i / \ln \rho_j$ are irrational numbers for all $i \neq j$

when $m > 1$. Then, for any (arbitrarily small) $\epsilon > 0$, the auxiliary system (19) with $|\Delta_j(k)| \le \delta_j - \epsilon$ is quadratically stabilizable via state feedback (3). Further, the auxiliary system is quadratically stabilizable via state feedback (3) if the following state feedback H_∞ control has a solution K for some diagonal scaling matrix $\Gamma > 0$:

$$\|\Lambda\Gamma K(zI - A - BK)^{-1}B\Gamma^{-1}\|_\infty < 1 \qquad (20)$$

where

$$\Lambda = \mathrm{diag}\{\delta_1, \cdots, \delta_m\} \qquad (21)$$

In particular, any K that renders (20) is a solution to either quadratic stabilization problem.

Remark 1 It is easy to see that if a given set of $\rho_j, j = 1, 2, \cdots, m$ do not satisfy the condition that $\ln\rho_i/\ln\rho_j$ are irrational for $i \neq j$, we can make it so by perturbing the ρ_j arbitrarily slightly. That is, the condition above holds generically.

Quantized Output Feedback. When quantized measurements are used, the system becomes

$$\begin{array}{rcl} x(k+1) & = & Ax(k) + Bu(k) \\ y(k) & = & Cx(k) \end{array} \qquad (22)$$

where $C \in \mathbf{R}^{r\times n}$, and $y(k)$ is to be quantized. The controller is in the form

$$\begin{array}{rcl} x_c(k+1) & = & A_c x_c(k) + B_c f(y(k)) \\ u(k) & = & C_c x_c(k) + D_c f(y(k)) \end{array} \qquad (23)$$

It is verified that the closed-loop system is given by

$$\bar{x}(k+1) = \mathcal{A}(\Delta(y(k))\bar{x}(k) \qquad (24)$$

where $\Delta(\cdot)$ is the same as in (10) and

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 & B \\ I & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & I \\ C & 0 \end{bmatrix}$$
$$\hat{I} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \quad \bar{K} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \qquad (25)$$

$$\mathcal{A}(\Delta) = \bar{A} + \bar{B}\bar{K}(\bar{C} + \hat{I}\Delta\hat{C}) \qquad (26)$$

Theorem 3 *Given the system in (22) and a quantization level vector ρ, consider the auxiliary system:*

$$\begin{array}{rcl} x(k+1) & = & Ax(k) + Bu(k) \\ y(k) & = & Cx(k) \\ v(k) & = & (I + \Delta(k))y(k) \end{array} \qquad (27)$$

where $|\Delta_j(k)| \le \delta_j$ for all $j = 1, 2, \cdots, m$ and k, and δ_j are converted from ρ_j using (7), and $v(k)$ is the output available for feedback. Suppose the auxiliary system is quadratically stablizable, then (22) is quadratically stabilizable via (23). Conversely, suppose the system (22) is quadratically stabilizable via (23) and, in addition, suppose $\ln\rho_i/\ln\rho_j$ are irrational numbers for all $i \neq j$ when $m > 1$. Then, for any (arbitrarily small) $\epsilon > 0$, the auxiliary system (27) with $|\Delta_j(k)| \le \delta_j - \epsilon$ is quadratically stabilizable. Further, the auxiliary system is quadratically stabilizable if the following state feedback H_∞ control has a solution $H(z)$ for some diagonal scaling matrix $\Gamma > 0$:

$$\|\Lambda\Gamma(I - G(z)H(z))^{-1}G(z)H(z)\Gamma^{-1}\|_\infty < 1 \qquad (28)$$

where Λ is given in (21). In particular, any $H(z)$ that renders (20) is a solution to either quadratic stabilization problem.

4 Quantized Quadratic Performance Control

Now we extend the results in the previous sections to including a quadratic performance objective. Consider the system in (22). Suppose the output $y(k)$ needs to be quantized. We now want to design a controller in (23) such that the following performance cost function

$$\begin{array}{rcl} J(x(0)) & = & \displaystyle\sum_{k=0}^{\infty} x^T(k)Qx(k) + u^T(k)Ru(k), \\ & & Q = Q^T \ge 0, \ R = R^T > 0 \end{array} \qquad (29)$$

is minimized in the sense below:

$$\min EJ(x_0) \qquad (30)$$

In the above, $x(0)$ is assumed to be a white noise with covariance $Ex(0)x^T(0) = \sigma^2 I$ for some $\sigma > 0$.

Because the state of the closed-loop system is $\bar{x}(k)$, we may rewrite the performance cost as

$$J(\bar{x}(0)) = \sum_{k=0}^{\infty} \bar{x}^T(k)\bar{Q}\bar{x}(k) + u(k)^T Ru(k) \qquad (31)$$

where

$$\bar{x}(0) = \begin{bmatrix} x(0) \\ 0 \end{bmatrix}; \quad \bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} \qquad (32)$$

Suppose we want the closed-loop system to be quadratically stable. Let $V(\bar{x}) = \bar{x}^T\bar{P}\bar{x}, \bar{P} = \bar{P}^T > 0$, be the associated Lyapunov function. Define

$$\nabla V(\bar{x}(k)) = V(\bar{x}(k+1)) - V(\bar{x}(k)) \qquad (33)$$

Then, using (24), the performance cost is given by

$$\begin{array}{rl} & J(\bar{x}(0)) \\ = & \bar{x}^T(0)\bar{P}\bar{x}(0) \\ + & \displaystyle\sum_{k=0}^{\infty} \nabla V(\bar{x}(k)) + \bar{x}^T(k)\bar{Q}\bar{x}(k) + u(k)^T Ru(k) \\ = & \bar{x}^T(0)\bar{P}\bar{x}(0) + \displaystyle\sum_{k=0}^{\infty} \bar{x}^T(k)\bar{\Omega}(\Delta(y(k)))\bar{x}(k) \end{array} \qquad (34)$$

where

$$\bar{\Omega}(\Delta) = \mathcal{A}(\Delta)^T\bar{P}\mathcal{A}(\Delta) - \bar{P} + \bar{Q} + (\bar{C}+\hat{I}\Delta)\hat{C})^T\bar{K}^T\hat{I}R\hat{I}^T\bar{K}(\bar{C}+\hat{I}\Delta)\hat{C}) \quad (35)$$

For the case without quantization, i.e. $\Delta(\cdot) = 0$, it is well-known (and easy to see from above) that the optimal solution for $\bar{K}$ is such that $\bar{x}^T(k)\bar{\Omega}(0)\bar{x}(k) = 0$ for all k, which leads to $J(\bar{x}(0)) = \bar{x}^T(0)\bar{P}\bar{x}(0)$ and minimization of tr$\bar{P}$. In the presence of the quantizer, we can formulate the performance control problem as follows: Given a performance bound $\gamma > 0$ and $\rho > 0$, find $\bar{P}$, $\bar{K}$, if exist, such that tr$(\bar{P}) < \gamma$ subject to

$$\bar{x}^T\bar{\Omega}(\Delta(\hat{C}\bar{x}))\bar{x} < 0, \quad \forall \bar{x} \neq 0 \quad (36)$$

This problem will be called *Quantized Quadratic Performance Control (QQPC)* problem. Its solution is related to the so-called *guaranteed-cost control (GCC)* for the auxiliary system (22) and (27), i.e., we want to find $\bar{P}$, $\bar{K}$ such that tr$(\bar{P}) < \gamma$ subject to

$$\bar{\Omega}(\Delta) < 0, \quad \forall\ |\Delta_j| \le \delta_j \quad (37)$$

where δ_j and ρ_j are related by (7).

Theorem 4 *Consider the system in (22), the performance cost in (29), the controller structure in (23), some performance bound $\gamma > 0$ and quantization level vector $0 < \rho < 1$. Suppose the GCC problem has a solution. Then, there exists a solution to the QQPC problem. Conversely, if the QQPC problem has a solution and in addition (when $m > 1$), $\ln\rho_i/\ln\rho_j$ are irrational numbers for all $i \neq j$, then, given any (arbitrarily small $\epsilon > 0$), the GCC problem for (37) has a solution for $|\Delta_j(k)| \le \delta_j - \epsilon$.*

Proof. The key is to show the relationship between (36) and (37). Obviously, (37) implies (36). The fact that (36) implies (37) but with $|\Delta_j| \le \delta_j - \epsilon$ is proved following Theorem 2. The details are omitted. □

When quantized state feedback is used instead, we have the following result:

Theorem 5 *Consider the system (1) with $B \in \mathbf{R}^{n\times m}$ and quantized state feedback as in (2)-(3), where $f(\cdot) = [f_1(\cdot), \cdots, f_m(\cdot)]^T$ with given quantization levels $0 < \rho_1, \cdots, \rho_m < 1$. Given the performance cost function in (29) and a performance bound $\gamma > 0$, the QQPC problem becomes to finding $P = P^T > 0$ and K, if exist, such that tr$P < \gamma$ subject to*

$$x^T\Omega(\Delta(v))x < 0, \ \forall x \neq 0 \quad (38)$$

where $v = Kx$ and

$$\Omega(\Delta) = (A + B(I+\Delta)K)^T P(A + B(I+\Delta)K) - P + Q + K^T(I+\Delta)R(I+\Delta)K \quad (39)$$

The related GCC problem becomes to finding $P = P^T > 0$ and K, if exist, such that tr$P < \gamma$ subject to

$$\Omega(\Delta) < 0, \quad \forall|\Delta_j| \le \delta_j \quad (40)$$

Further, the GCC problem has a solution if the following linear matrix inequalities

$$\text{tr } \tilde{P} < \gamma, \quad \begin{bmatrix} -\tilde{P} & I \\ I & -S \end{bmatrix} \le 0 \quad (41)$$

$$\begin{bmatrix} -S & * & * & * & * \\ AS+BW & -S+B\Lambda\Gamma\Lambda B^T & * & * & * \\ W & \Lambda\Gamma\Lambda B^T & -\tilde{R} & * & * \\ W & 0 & 0 & -\Gamma & * \\ Q^{1/2}S & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (42)$$

*have a solution for some $\tilde{P} = \tilde{P}^T, S = S^T$, W and a diagonal scaling matrix Γ, where $\tilde{R} = R^{-1} - \Lambda S\Lambda$, Λ is given in (21), and * denotes the symmetric part. Also, P and K are related to S and W as follows:*

$$P = S^{-1}, \ K = WP \quad (43)$$

Proof. The simplification of the QQPC and GCC problems is easy to check. We proceed to verify (42) as a sufficient condition for the GCC problem. Indeed, (40) holds if and only if

$$\begin{bmatrix} -P+Q & * & * \\ A+B(I+\Delta)K & -P^{-1} & * \\ (I+\Delta)K & 0 & -R^{-1} \end{bmatrix} < 0 \quad (44)$$

for all $|\Delta_j| \le \delta_j$. Using (43), the above becomes

$$\begin{bmatrix} -S+SQS & * & * \\ AS+B(I+\Delta)W & -S & * \\ (I+\Delta)W & 0 & -R^{-1} \end{bmatrix} < 0 \quad (45)$$

which is equivalent to

$$\begin{bmatrix} -S+SQS & * & * \\ AS+BW & -S & * \\ W & 0 & -R^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ B \\ I \end{bmatrix} \Delta[W\ 0\ 0] + \begin{bmatrix} W^T \\ 0 \\ 0 \end{bmatrix} \Delta[0\ B^T\ I] < 0 \quad (46)$$

Taking $\Gamma > 0$ to be a diagonal matrix, (46) holds if

$$\begin{bmatrix} -S+SQS & * & * \\ AS+BW & -S & * \\ W & 0 & -R^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ B\Lambda \\ \Lambda \end{bmatrix} \Gamma[0\ \Lambda B^T\ \Lambda] + \begin{bmatrix} W^T \\ 0 \\ 0 \end{bmatrix} \Gamma^{-1}[W\ 0\ 0] < 0 \quad (47)$$

which is equivalent to (42) by Schur complement. □

5 Quantized H_∞ Control

Next, we extend the quantization results to H_∞ control. For simplicity, only quantized state feedback is considered. The system of interest is as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_1 w(k) \\ z(k) &= Cx(k) + Du(k) + D_1 w(k) \end{aligned} \quad (48)$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$, $w \in \mathbf{R}^{m_1}$, $z \in \mathbf{R}^\ell$ and the control signal is in the form of (2)-(3). Given a quantization level vector ρ and H_∞ performance bound $\gamma > 0$, the design objective is to find K such that the induced L_2-gain from w to z is less than γ.

Then, the closed-loop system is given by

$$\begin{aligned} x(k+1) &= [A + B(I + \Delta(v))K]x(k) + B_1 w(k) \\ z(k) &= [C + D(I + \Delta(v))K]x(k) + D_1 w(k) \end{aligned} \quad (49)$$

As in the quadratic performance control problem, we consider the following relaxed H_∞ control problem: Find $P = P^T > 0$ and K such that

$$x^T \Pi(\Delta(Kx))x < 0, \quad \forall x \neq 0 \quad (50)$$

where

$$\begin{aligned} \Pi(\Delta) = \; & A_\Delta^T P A_\Delta^T - P + \gamma^{-2}(A_\Delta^T P B_1 + C_\Delta^T D_1) \\ & \times [I - \gamma^{-2}(D_1^T D_1 + B_1^T P B_1)]^{-1} \\ & \times (B^T P A_\Delta + D_1^T C_\Delta) + C_\Delta^T C_\Delta < 0 \end{aligned} \quad (51)$$

$$A_\Delta = A + B(I + \Delta(v))K, \quad C_\Delta = C + D(I + \Delta(v))K \quad (52)$$

Theorem 6 *Consider the given system (48), controller structure (2)-(3), quantization level vector ρ and a H_∞ performance bound $\gamma > 0$. Suppose there exist $P = P^T > 0$ and K such that (50) holds, then the induced L_2-norm from w to z is less than γ.*

Further, for any $P = P^T > 0$ and K, (50) holds if $\Pi(\Delta) < 0$ for all $|\Delta_j| \leq \delta_j$, where δ_j are related to ρ_j by (5). Conversely, if (50) holds, $\Pi(\Delta) < 0$ for all $|\Delta_j| \leq \delta_j - \epsilon$, where $\epsilon > 0$ is arbitrarily small.

In addition, there exist $P = P^T > 0$ and K such that $\Pi(\Delta) < 0$ for all $|\Delta_j| \leq \delta_j$ if the following linear matrix inequality

$$\begin{bmatrix} -S + B\Lambda\Gamma\Lambda B^T & * & * & * & * \\ (AS + BW)^T & -S & * & * & * \\ B_1^T & 0 & -\gamma I & * & * \\ D\Lambda\Gamma\Lambda B^T & CS + D_1 W & D_1 & -\gamma I & * \\ 0 & W & 0 & 0 & -\Gamma \end{bmatrix} < 0 \quad (53)$$

has a solution for $S = S^T, W$ and diagonal scaling matrix Γ, where Λ and the relationship between (S, W) and (P, K) are the same as in Theorem 5.

Proof. The proof is similar to that of Theorem 5. The details are omitted. □

6 Conclusions

We have generalized the work of [4] to performance control problems. We have given conditions, in terms of linear matrix inequalities, under which a linear controller exists to provide a guaranteed level of performance for a given logarithmic quantization density. Furthermore, the use of the sector bound method allows us to see the connection between quantized feedback control and robust control with sector-bounded uncertainties. This insight is important because it allows many robust control results to be applied to quantized feedback control. For example, we now understand why it is difficult to find the coarsest quantization level for performance control. The reason is that this problem is similar to finding the maximum sector bound for an uncertainty block in a linear system which yields a guaranteed level of performance, a difficult problem well-known in the robust control area.

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