2013 10th IEEE International Conference on Control and Automation (ICCA)
Hangzhou, China, June 12-14, 2013

Output Feedback Stabilization of Wireless Networked Control System with Packet Dropout *

JinFeng Gao[1,2] Minyue Fu[2] Jia Ren[1]

Abstract

The problem of modeling and stabilization of a wireless networked control system(NCS) with both packet dropout and time-varying delay is investigated in this paper, and the time-varying delay is more or less than one sampling period. The closed-loop wireless NCS is modeled as an asynchronous dynamic system(ADS) with three subsystems. By using the ADS approach, a sufficient condition for the closed-loop wireless NCS to be stable is presented. An illustrative example is provided to demonstrate the effectiveness of the proposed result.

1. Introduction

Sensors, controllers, and plants are often connected over a realtime network medium in modern control systems, which are called networked control systems (NCSs) [1]. It is widely used at almost all levels of operation and information processing in various areas, including manufacturing industry, remote operation and teleautonomy [2]-[7]. As an alternative for the wired network, wireless NCSs are becoming fundamental components of modern control systems due to their flexibility, ease of deployment and low cost [2], [8]-[11]. So wireless NCS is considered in this note. One of the main issues in a NCS is the effect of networked-induced delay that occurs when sensors, actuators and controllers exchange data across the shared network. This delay can degrade the performance of control systems designed without considering it and can even destabilize the system. On the other hand, data packet dropout results from network traffic congestion and limited network reliability. When a data packet is dropped, complete information of the NCS becomes unavailable. In this case, the controller or actuator has to decide, with incomplete information, what control signals to output.

Different from some separately considering the delay or the data packet dropout problem, this note intends to deal with the modelling, analysis and synthesis for the wireless NCS with both delay and data packet dropout as shown in Fig. 1. An asynchronous dynamic system(ADS) approach is presented to stabilize the wireless NCS. Firstly, a switched system with time-varying delay model is presented to describe the wireless NCS. Recently, some results are also presented about this problem. A new switched linear system model is proposed to describe NCS in [3] while the delay is assumed to be less than one sampling period with the state feedback controller. And stochastic optimal control method is used by an adaptive estimator (AE) and ideas from Q-learning to solve the infinite horizon optimal regulation of unknown wireless NCS with time-varying system matrices in [12]. Nevertheless, the computational complexity of the controller will increase when the delay bound is increased in NCS in [12]. Furthermore, the networked system identification problem and estimation problem are also studied in [13] and [14] based on the Matlab/Simulink simulator TrueTime and orthogonal projection principle, respectively, which aim at identifying mathematical models required in networked control/estimation/filtering systems while stabilization are not considered.

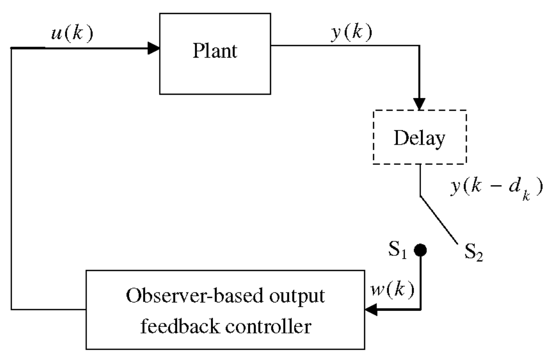

Figure 1: A graph of a networked control system with networked-induced delay and packet dropouts

Notations: Throughout this paper, R denotes the set of

*This work was supported by the National Natural Science Foundation of China under grant (61004034, 61203177), Zhejiang Provincial Natural Science Foundation of China (Y1090834, LQ12F03013). 1. J. Gao and J. Ren are with the institute of Automation, Zhejiang Sci-Tech University, Hangzhou, 310018, P.R.China. 2. M. Fu is with the School of Electrical Engineering and Computer Science, University of Newcastle, NSW, 2308, Australia

real numbers, R^n denotes the n-dimensional Euclidean space and $R^{n\times m}$ refers to the set of all $n\times m$ real matrices. A^T represents the transpose of the matrix A, while A^{-1} denotes the inverse of A. For real symmetric matrices X and Y, the notation $X\geq Y$ (respectively, $X>Y$) means that the matrix $X-Y$ is positive semi-definite, (respectively, positive-definite). I is the identity matrix with appropriate dimensions. $\|x\|$ refers to the Euclidean norm of the vector x, that is $\|x\|=\sqrt{x^Tx}$. For a symmetric matrix, $*$ denotes the matrix entries implied by symmetry.

2. Problem Formulation and NCS Modeling

The NCS with packet dropout and possible delay is illustrated in Fig.1, where the plant is described by the following model denoted Ω:

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the system state, $u(k)\in R^m$ is the control input, $y(k)\in R^r$ is the measured output. And the model of an observer-based output feedback controller is described as follows:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k)+Bu(k)+L[w(k)-\hat{y}(k)] \\ \hat{y}(k) &= C\hat{x}(k) \\ u(k) &= K\hat{x}(k), \end{aligned} \tag{2}$$

where $\hat{x}(k)\in R^n$ is the estimated state of the system (1) and $\hat{y}(k)\in R^r$ is the estimated output. $L\in R^{n\times r}$ and $K\in R^{m\times n}$ are the observer gain and controller gain matrices, respectively. It is also assumed that the pairs (A,B) are controllable and (C,A) observable. In fig.1, we can use a switch to denote the data packet loss of the states in the network channel. If the switch is closed, the data packet is successfully transmitted. And we have $w(k)=y(k)$ without networked delay or $w(k)=y(k-d_k)$ with networked delay. Whereas when the switch is open, the previous value of the switch output will be used in the controller (2) and a packet is lost. Then we have $w(k)=w(k-1)$ in this case.

For the NCS under consideration, we give the following assumptions without loss of generality, which will be useful in our main results.

Assumption 1:

1. The sensors and controllers are all time-driven and synchronized.
2. Time-stamping of measurements is necessary to reorder data packet at the observer side since they can arrive out of order. And the controller can get the delay of each data packet.
3. The maximum delay in the network is d_M that is a known integer.

Define the estimation error by $e(k)=x(k)-\hat{x}(k)$ and let

$$z(k)=[x^T(k)\quad e^T(k)\quad w^T(k-1)]^T \tag{3}$$

Then the dynamics of the closed-loop system can be described by the following three subsystems.

S1. There is data packet dropout, and the corresponding controller gain is K_1. Then the closed-loop NCS is described as:

$$\begin{aligned} &\Omega_1: z(k+1)=A_1z(k), \\ &A1=\begin{bmatrix} A+BK_1 & -BK_1 & 0 \\ LC & A-LC & -L \\ 0 & 0 & I \end{bmatrix} \end{aligned} \tag{4}$$

S2. The data packet is transmitted successfully without networked delay, and the corresponding controller gain is K_2 in this case. Then the closed-loop NCS can be described as:

$$\begin{aligned} &\Omega_2: z(k+1)=A_2z(k), \\ &A2=\begin{bmatrix} A+BK_2 & -BK_2 & 0 \\ 0 & A-LC & 0 \\ C & 0 & 0 \end{bmatrix} \end{aligned} \tag{5}$$

S3. The data packet is transmitted successfully with networked delay d_k, and the corresponding controller gain becomes K_{3,D_K} here. Then the closed-loop NCS is as follows:

$$\begin{aligned} &\Omega_3: z(k+1)=A_3z(k)+A_{d3}z(k-d_k), \\ &A3=\begin{bmatrix} A+BK_{3,d_k} & -BK_{3,d_K} & 0 \\ LC & A-LC & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ &A_{d3}=\begin{bmatrix} 0 & 0 & 0 \\ -LC & 0 & 0 \\ C & 0 & 0 \end{bmatrix} \end{aligned} \tag{6}$$

From the above analysis, we can conclude that there three different cases may appear during every sampling period. So the closed-loop NCS can be described as a discrete-time switched system within three subsystems Ω_1 to Ω_3. In subsystem Ω_3, when $d_k=0$, Ω_3 turns to Ω_2. And the system matrix Ω_1 and Ω_2 are similar. So subsystem Ω_3 in case 3 includes case 1 and 2 by appropriately choosing the value of the matrices. Then the wireless NCS can be represented by the following switched system with time-varying delay:

$$z(k+1)=A_iz(k)+A_{di}z(k-d_k),\ i=1,2,3, \tag{7}$$

where $A_{d1}=0, A_{d2}=0, d_k=1,2,..,d_M$.

To end this section, the following definition and lemma are introduced to obtain our main results.

Definition 1. *For any given initial conditions* $(k_0, \phi) \in \Re^+ \times C^n$*, (7) is globally exponentially stable if the solutions of* (8) *satisfy*

$$\| x(k) \| \leq a\lambda^{-(k-k_0)} \| x(k_0) \|, \forall k \geq k_0.$$

where $a > 0$ *is a constant and* $\lambda > 1$ *is the decay rate.*

Lemma 1. *[2] For any appropriately dimensioned matrices* $R = R^T > 0$*,* N*,* X*,* $\eta(l) \triangleq x(l+1) - x(l)$*, and two positive integer time-varying* $d(k_1)$*,* $d(k_2)$ *satisfying* $d(k_1) + 1 \leq d(k_2) \leq d_M$*, the following equality holds*

$$\begin{aligned} -\sum_{l=k-d(k_2)}^{k-d(k_1)-1} \eta^T(l)R\eta(l) &= 2\xi^T(k)N\big[x(k-d(k_1)) \\ &\quad - x(k-d(k_2))\big] + (d(k_2)-d(k_1))\xi^T(k)X\xi(k) \\ &\quad - \sum_{l=k-d(k_2)}^{k-d(k_1)-1} \begin{bmatrix} \xi(k) \\ \eta(l) \end{bmatrix}^T \begin{bmatrix} X & N \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \eta(l) \end{bmatrix} \end{aligned}$$

3. Stability Analysis of the Wireless NCS

More generally, we consider the following discrete-time switched system with time-varying delay:

$$\begin{aligned} z(k+1) = A_i z(k) + A_{di} z(k-d_k),\ i = 1, 2, ..., N, \\ d_k = 1, 2, ..., d_M \end{aligned} \tag{8}$$

where N is the number of the subsystems. Suppose that the event rates of the described subsystems S_i are defined as $r_1, r_2, ..., r_N$. The time interval $[0, kT]$ will be simplified $[0, k]$ in the following text. Let n_i, $i = 1, 2, ..., N$ denote the times that the subsystems S_i are activated on the interval $[0, k]$. Then we can obtain

$$k = \sum_{i=1}^{N} n_i;\ r_i = \frac{n_i}{k},\ i = 1, 2, ..., N;\ \sum_{i=1}^{N} r_i = 1 \tag{9}$$

The following theorem gives a criterion to guarantee the Lyapunov function $V(k)$ exponentially decays along state trajectory of system (8).

Theorem 1. *Given scalar* $\lambda > 0$ *and any delay satisfying* $0 \leq d_k \leq d_M$*, if there exist appropriate dimensional symmetric positive definite matrices* P*,* Q_1*,* Q_2*,* R*, symmetric matrix* $X = \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \geq 0$ *and matrices* G*,* $N = [N_1^T \quad N_2^T]^T$*,* $M = [M_1^T \quad M_2^T]^T$*, such that the following matrix inequalities hold*

$$\Phi(d_k) = \begin{bmatrix} \Phi_{11} & \Phi_{12} & -N_1 & P + (\hat{A}_i^T - I)G^T - G \\ * & \Phi_{22} & -N_2 & \hat{A}_{di}^T G^T \\ * & * & -Q_2 & 0 \\ * & * & * & P + d_M R - G - G^T \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} X & N \\ * & R \end{bmatrix} > 0,\ \begin{bmatrix} X & M \\ * & R \end{bmatrix} > 0, \tag{11}$$

where $d_k = 0, ..., d_M$*,* $\hat{A}_i = \lambda A_i$*,* $\hat{A}_{di} = \lambda^{1+d_k} A_{di}$*, and*

$$\begin{aligned} \Phi_{11} &= d_M Q_1 + Q_2 + d_M X_1 + Sym(M_1 + G(\hat{A}_i - I)), \\ \Phi_{12} &= d_M X_2 + N_1 - M_1 + M_2^T + G\hat{A}_{di}, \\ \Phi_{22} &= d_M X_3 - Q_1 + Sym(N_2 - M_2), \end{aligned}$$

then

$$V(k) < \lambda^{-2(k-k_0)} V(k_0).$$

Proof: The following expression of $V(k)$ is the Lyapunov function of system (8).

$$\begin{aligned} V_1(k) &= z^T(k)Pz(k) \\ V_2(k) &= \sum_{l=k-d_k}^{k-1} \lambda^{2(l-k)} z^T(l)Q_1 z(l) \\ &\quad + \sum_{m=-d_M+2}^{0} \sum_{l=k+m-1}^{k-1} \lambda^{2(l-k)} z^T(l)Q_1 z(l) \\ V_3(k) &= \sum_{l=k-d_M}^{k-1} \lambda^{2(l-k)} z^T(l)Q_2 z(l) \\ V_4(k) &= \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} (\lambda z(l+1) - z(l))^T \\ &\quad \times R(\lambda z(l+1) - z(l)) \end{aligned}$$

And defining $\xi(k) = \lambda^{k-k_0} z(k)$, $\lambda > 0$, $\hat{A}_i = \lambda A_i$, $\hat{A}_{di} = \lambda^{1+d_k} A_{di}$, combined with (8) we have

$$\begin{aligned} \xi(k+1) = \hat{A}_i \xi(k) + \hat{A}_{di} \xi(k-d_k), \\ i = 1, 2, ..., N,\ d_k = 1, 2, ..., d_M \end{aligned} \tag{12}$$

Choose the following Lyapunov function of system (12)

$$\begin{aligned} W(k) &= \xi^T(k)P\xi(k) + \sum_{l=k-d_k}^{k-1} \xi^T(l)Q_1\xi(l) \\ &\quad + \sum_{m=-d_M+2}^{0} \sum_{l=k+m-1}^{k-1} \xi^T(l)Q_1\xi(l) \\ &\quad + \sum_{l=k-d_M}^{k-1} \xi^T(l)Q_2\xi(l) \\ &\quad + \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \delta^T(l)R\delta(l) \end{aligned}$$

where $\delta(l) = \xi(l+1) - \xi(l)$. Then the forward difference for $W(k)$ along any trajectory of system (12) is given by

$$
\begin{aligned}
\Delta W(k) &= W(k+1) - W(k) \\
&= \xi^T(k+1)P\xi(k+1) - \xi^T(k)P\xi(k) \\
&\quad + d_M \xi^T(k)Q_1\xi(k) + \sum_{l=k+1-d_{k+1}}^{k-1} \xi^T(l)Q_1\xi(l) \\
&\quad - \sum_{l=k-d_M+1}^{k-1} \xi^T(l)Q_1\xi(l) - \sum_{l=k-d_k}^{k-1} \xi^T(l)Q_1\xi(l) \\
&\quad + \xi^T(k)Q_2\xi(k) - \xi^T(k-d_M)Q_2\xi(k-d_M) \\
&\quad + d_M \delta^T(k)R\delta(k) \\
&\quad + d_M \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T X \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix} \\
&\quad + 2\begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} (\xi(k-d_k) - \xi(k-d_M)) \\
&\quad + 2\begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} (\xi(k) - \xi(k-d_k)) \\
&\quad - \sum_{l=k-d_M}^{k-d_k-1} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix}^T \begin{bmatrix} X & N \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix} \\
&\quad - \sum_{l=k-d_k}^{k-1} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix}^T \begin{bmatrix} X & M \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix} \\
&\quad + 2(\xi^T(k) + \delta^T(k))G \\
&\quad \times \left[(A_i - I)\xi^T(k) + A_{di}\xi(k-d_k) - \delta(k)\right] \\
&\quad < \theta^T(k)\Phi\theta(k) < 0
\end{aligned}
$$

where $\theta^T(k) = [\xi^T(k) \quad \xi^T(k-d_k) \quad \xi^T(k-d_M) \quad \delta^T(k)]$, which means that $W(k) < W(k_0)$ for any $k \geq k_0$. Furthermore,

$$
\begin{aligned}
V_1(k) &= z^T(k)Pz(k) = \lambda^{-2(k-k_0)}\xi^T(k)P\xi(k) \\
V_2(k) &= \sum_{l=k-d_k}^{k-1} \lambda^{2(l-k)}\lambda^{-2(l-k_0)}\xi^T(l)Q_1\xi(l) \\
&\quad + \sum_{m=-d_M+2}^{0} \sum_{l=k+m-1}^{k-1} \lambda^{2(l-k)}\lambda^{-2(l-k_0)}\xi^T(l)Q_1\xi(l) \\
V_3(k) &= \sum_{l=k-d_M}^{k-1} \lambda^{2(l-k)}\lambda^{-2(l-k_0)}\xi^T(l)Q_2\xi(l) \\
V_4(k) &= \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)}\lambda^{-2(l-k_0)}\delta^T(l)R\delta(l) \\
V(k) &= V_1(k) + V_2(k) + V_3(k) + V_4(k) = \lambda^{-2(k-k_0)}W(k)
\end{aligned}
$$

As $W(k_0) = V(k_0)$, it is easy to verify that, $V(k) < \lambda^{-2(k-k_0)}V(k_0)$. This completes the proof.

The following theorem gives a sufficient condition for the closed-loop NCS (8) to be exponentially stable.

Theorem 2. *The discrete-time switched system* (8) *is stable if there exist Lyapunov function $V(k)$, r_i defined in* (9) *and some positive scalars μ_i, $i = 1, 2, , \ldots, N$ which correspond to each subsystem such that the following inequalities hold.*

$$
\begin{aligned}
V(k) &< \mu_i^{-2(k-k_0)}V(k_0), \\
\prod_{i=1}^{N} \mu_i^{r_i} &< 1
\end{aligned}
\tag{13}
$$

Proof: Defining the transition time of the subsystems to be $t_1 = 0$, $t_2, \ldots, t_k = k$, then

$$
\begin{aligned}
V(k) &< \mu_{t_k}^{t_k - t_{k-1}} V(t_{k-1}) < \mu_{t_k}^{t_k - t_{k-1}} \mu_{t_{k-1}}^{t_{k-1} - t_{k-2}} V(t_{k-2}) \\
&< \ldots < \prod_{i=1}^{N} \mu_i^{n_i} V(0) = \left(\prod_{i=1}^{N} \mu_i^{r_i}\right)^k V(0)
\end{aligned}
\tag{14}
$$

So the system is stable if $\prod_{i=1}^{N} \mu_i^{r_i} < 1$. This completes the proof.

4. output feedback controller design

An algorithm to design the observer-based output feedback controller of the wireless NCS is presented in this section. Since the data packets are time-stamped, the data packet loss rate and time delay are known to the controller, which is designed to depend on both the packet loss rate and time delay. Firstly, A_i can be rewritten as $A_i = A_{0i} + B_{00}D_iC_{0i}$, $i = 1, 2, 3$, where

$$
\begin{aligned}
A_{01} &= \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & I \end{bmatrix}, B_{00} = \begin{bmatrix} B & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \\
D_i &= \begin{bmatrix} K_i & 0 \\ 0 & L \end{bmatrix}, C_{01} = \begin{bmatrix} I & -I & 0 \\ C & -C & -I \end{bmatrix}
\end{aligned}
\tag{15}
$$

$$
A_{02} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ C & 0 & 0 \end{bmatrix}, C_{02} = \begin{bmatrix} I & -I & 0 \\ 0 & C & 0 \end{bmatrix}
\tag{16}
$$

$$
A_{03} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_{03} = \begin{bmatrix} I & -I & 0 \\ C & -C & 0 \end{bmatrix}
\tag{17}
$$

Theorem 3. *The system* (7) *is stable if there exist positive scalars μ_i, $i = 1, 2, 3$, satisfying $\prod_{i=1}^{3} \mu_i^{r_i} < 1$ and appropriate dimensional matrices P, Q_1, Q_2, R, X_1, X_3, matrices M_1, M_2, N_1, N_2, X_2 and matrices G_1, G_2, G_3, matrices K_1, K_2, K_{3,d_k} such that the following matrix inequality holds.*

$$
\Phi_0 + \Phi_1 < 0,
$$

$$
\begin{aligned}
&\Phi_0+\Phi_2<0,\\
&\Phi_0+\Phi_3(d_k)<0,\, d_k=1,\,2,\,...,\,d_M,,\\
&\begin{bmatrix} X & N \\ * & R \end{bmatrix}>0, \begin{bmatrix} X & M \\ * & R \end{bmatrix}>0.
\end{aligned}
\tag{18}
$$

where

$$
\Phi_0=\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & -N_1 & P-G^T-G \\ * & \Sigma_{22} & -N_2 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & P+d_MR-G-G^T \end{bmatrix},
$$

$$
\begin{aligned}
\Sigma_{11}&=d_MQ_1+Q_2+d_MX_1+Sym(M_1-G),\\
\Sigma_{12}&=d_MX_2+N_1-M_1+M_2^T,\\
\Sigma_{22}&=d_MX_3-Q_1+Sym(N_2-M_2),
\end{aligned}
$$

$$
G=\begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}, \Phi_1=\begin{bmatrix} \bar{\phi}_{11} & 0 & 0 & \bar{\phi}_{14} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},
$$

$$
\bar{\phi}_{11}=\begin{bmatrix} \bar{\psi}_{11} & \bar{\psi}_{12} & 0 \\ * & \bar{\psi}_{22} & -\mu_1G_2L \\ * & * & \mu_1G_3+\mu_1G_3^T \end{bmatrix},
$$

$$
\begin{aligned}
\bar{\psi}_{11}&=Sym(\mu_1G_1A+\mu_1G_1BK_1),\\
\bar{\psi}_{12}&=-\mu_1G_1BK_1+\mu_1C^TL^TG_2^T,\\
\bar{\psi}_{22}&=Sym(\mu_1G_2A-\mu_1G_2LC),
\end{aligned}
$$

$$
\bar{\phi}_{14}=\begin{bmatrix} \bar{\varphi}_{11} & -\mu_1C^TL^TG_2^T & 0 \\ -\mu_1K_1^TB^TG_1^T & \bar{\varphi}_{22} & 0 \\ 0 & -\mu_1L^TG_2^T & \mu_1G_3^T \end{bmatrix}
$$

$$
\begin{aligned}
\bar{\varphi}_{11}&=\mu_1A^TG_1^T+\mu_1K_1^TB^TG_1^T,\\
\bar{\varphi}_{22}&=\mu_1A^TG_2^T-\mu_1C^TL^TG_2^T.
\end{aligned}
$$

$$
\Phi_2=\begin{bmatrix} \hat{\phi}_{11} & 0 & 0 & \hat{\phi}_{14} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},
$$

$$
\hat{\phi}_{11}=\begin{bmatrix} \hat{\psi}_{11} & \hat{\psi}_{12} & 0 \\ * & \hat{\psi}_{22} & 0 \\ * & * & 0 \end{bmatrix},
$$

$$
\begin{aligned}
\hat{\psi}_{11}&=Sym(\mu_2G_1A+\mu_2G_1BK_2),\\
\hat{\psi}_{12}&=-\mu_2G_1BK_2,\\
\hat{\psi}_{22}&=Sym(\mu_2G_2A-\mu_2G_2LC),
\end{aligned}
$$

$$
\hat{\phi}_{14}=\begin{bmatrix} \hat{\varphi}_{11} & 0 & \mu_2C^TG_3^T \\ -\mu_2K_2^TB^TG_1^T & \hat{\varphi}_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{aligned}
\hat{\varphi}_{11}&=\mu_2A^TG_1^T+\mu_2K_2^TB^TG_1^T,\\
\hat{\varphi}_{22}&=\mu_2A^TG_2^T-\mu_2C^TL^TG_2^T.
\end{aligned}
$$

$$
\Phi_3=\begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & 0 & \tilde{\phi}_{14} \\ * & 0 & 0 & \tilde{\phi}_{24} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},
$$

$$
\tilde{\phi}_{11}=\begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} & 0 \\ * & \tilde{\psi}_{22} & 0 \\ * & * & 0 \end{bmatrix},
$$

$$
\tilde{\psi}_{11}=Sym(\mu_3G_1A+\mu_3G_1BK_{3,d_K}),
$$

$$
\begin{aligned}
\tilde{\psi}_{12}&=-\mu_3(G_1BK_{3,d_K}+C^TL^TG_2^T),\\
\tilde{\psi}_{22}&=Sym(\mu_3G_2A-\mu_3G_2LC),
\end{aligned}
$$

$$
\tilde{\phi}_{12}=\begin{bmatrix} 0 & 0 & 0 \\ -\mu^{1+d_k}G_2LC & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{\phi}_{24}=\tilde{\phi}_{12}^T,
$$

$$
\tilde{\phi}_{14}=\begin{bmatrix} \tilde{\varphi}_{11} & \mu_3C^TL^TG_2^T & 0 \\ -\mu_3K_{3,d_k}^TB^TG_1^T & \tilde{\varphi}_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix},
$$

$$
\begin{aligned}
\tilde{\varphi}_{11}&=\mu_3A^TG_1^T+\mu_3K_{3,d_k}^TB^TG_1^T,\\
\tilde{\varphi}_{22}&=\mu_3A^TG_2^T-\mu_3C^TL^TG_2^T.
\end{aligned}
$$

Replacing the system matrices of (10) by (7), it is easy to obtain the above results according to Theorem 1 and Theorem 2. Furthermore, let $G_2L=\bar{L}$, $G_1BK_i=\bar{K}_i$, $i=1,2$, $G_1BK_3=\bar{K}_{3,d_k}$, and the controllers gain matrices K_i, L can be gained by solving the corresponding linear matrix inequalities.

5. Simulation results

In this section, an numerical example of output feedback control of wireless NCS is evaluated.

Consider the following discrete-time system:

$$
A=\begin{bmatrix} 0.7852 & 0.2358 & 0.4832 \\ 0.9022 & 0.8086 & 0.5389 \\ 0.1506 & 0.5632 & 0.7528 \end{bmatrix}, B=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, C=\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^T.
$$

The eigenvalues of A are 1.7183, $0.3142\pm0.2595i$ and the system is unstable without control. Let the event rates of the data packet loss and time delay are $r_1=0.5$, and $r_3=0.2$ respectively, and the maximum delay is $d_M=3$. Choose $\mu_1=1.8$, $\mu_2=0.5$, $\mu_3=0.6$. So

$$
\mu_1^{r_1}\mu_2^{r_2}\mu_3^{r_3}=1.8^{0.5}\times0.5^{0.3}\times0.6^{0.2}=0.9839<1,
$$

By using the Theorem 3, a suitable controller gain matrix can be obtained. The simulation result is shown in Fig. 2. The above subgraph depicts the data packet loss and the time delay of the wireless NCS in Fig. 2. When the delay value is -1, it means this packet is lost. And the below graph is to describe the state trajectories of closed loop wireless NCS. From the Fig.2, the states of the system diverge at the case when the data packets are lost, but they converge to zero finally. So the example illustrates the effectiveness of the proposed method.

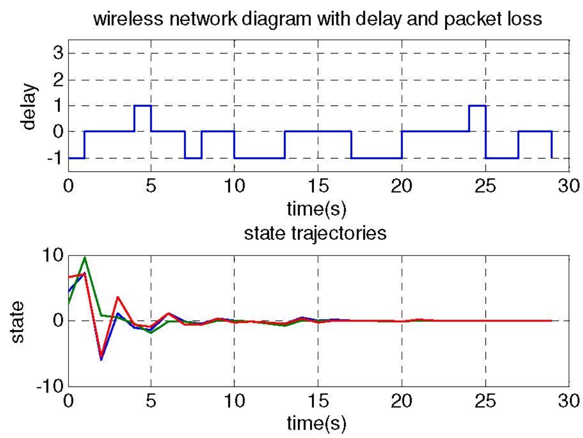

Figure 2: State trajectories of NCS with networked-induced delay and packet dropouts

6. Conclusions

This paper considers the problem of modeling and stabilization of wireless NCS with both packet dropout and time-varying delay. The closed-loop wireless NCS is modeled as an ADS with three subsystems. The output feedback controller is designed to stabilize the closed-loop wireless NCS by using ADS approach. And the illustrative example is provided to demonstrate the effectiveness of the proposed result.

References

[1] W. Zhang, M. S. Branicky, S. M. Phillips. Stability of networked control systems[J]. *IEEE Transactions on Control Systems Magazine*, 2001, 21(1): 84–99.

[2] K. Hu, J. Yuan, Finite sum equality aproach to H_∞ output feedback control for switched linear discrete-time systems with time-varying delay[J]. *IET Control Theory and Applications*, 2009, 3(8):1006–1016.

[3] W. Zhang, L. Yu. Modelling and control of networked control systems with both network-induced delay and packet-dropout[J]. *Automatica*, 2008, 44(12): 3206–3210.

[4] Y. Sun, S. Qin. Stability of networked control systems with packet dropout: an average dwell time approach[J]. *IET Control Theory and Applications*, 2011, 5(1):47–53

[5] G. C. Walsh, O. Beldiman, L. G. Bushnell. Asymptotic behavior of networked control systems[C]. *Proceedings of the International Conference on Control Applications*. Hawaii: IEEE Press, 1999, 2: 1448–1453.

[6] D. Nesic, A. R. Teel. Input-output stability properties of networked control systems[J]. *IEEE Transactions on Automatic Control*, 2004, 49(10): 1650–1667.

[7] L. A. Montestruque, P. J. Antsaklis. On the model-based control of networked systems[J]. *Automatica*, 2003, 39(10): 1837–1843.

[8] Y. Tipsuwan, M. Y. Chow. Control methodologies in networked control systems[J]. *IEEE Control Engineering and Practice*, 2003, 11(10):1099–1111.

[9] G. C. Walsh, H. Ye, L. G. Bushnell. Stability analysis of networked control systems[J]. *IEEE Transactions on Control Systems Technology*, 2002, 10(3): 438–446.

[10] L. Schenato, F. Fiorentin. Average TimeSynch: A consensus-based protocol for clock synchronization in wireless sensor networks[J]. *Automatica*, 2011, 47(9): 1878–1886.

[11] D. Bernardini, A. Bemporad. Energy-aware robust model predictive control based on noisy wireless sensors[J]. *Automatica*, 2012, 48(1):36–44

[12] H. Xu, S. Jagannathan, F. Lewis. Stochastic optimal control of unknown linear networked control system in the presence of random delays and packet losses[J]. *Automatica*, 2012, 48(6): 1017–1030.

[13] J. Wang, W. Zheng, T Chen. Identification of linear dynamic systems operating in a networked environment[J]. *Automatica*, 2009, 45(12): 2763–2772

[14] W. Zhang, L. Yu, G. Feng. Optimal linear estimation for networked systems with communication constraints[J]. *Automatica*, 2011, 47(9): 1992–2000.