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Decentralized Algorithm for Wind Power Regulation via TCLs in Smart Grids

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Abstract— **This paper studies the wind power regulation problem by controlling a population of thermostatically controlled loads (TCLs) in smart grids. With each TCL endowed with a cost function related to its room temperature, we formulate the wind power regulation problem into a quadratic optimization problem, for which we present a decentralized bisection algorithm relying on an aggregator. The algorithm converges fast and is decentralized in the sense that the TCLs conduct local computation and keep the parameters' privacy from the aggregator. The proposed algorithm also includes a Kalman filter based parameter identification technique to deal with the time-varying thermal characteristics of TCLs. Simulations are given to show the performance of our algorithm.**

I. INTRODUCTION

Distributed power generation feeding on renewable energy (e.g. wind and solar energy) has been intensively researched by scientific communities, with a growing penetration in future smart grids due to its environmental and economic benefits, including sustainable development, less power transmission loss due to relatively short transmission distance and better robustness than centralized power plants. Due to the volatile nature of the renewable energy, the integration of distributed generation with conventional generation remains a great challenge.

A possible way to counter the fluctuations of distributed power generation is to deploy real-time control of the load in the energy management system (EMS) such that the fluctuations in the power supply can be absorbed cooperatively by the variations in the loads. In this paper, we adopt the idea of manipulating a population of TCLs (e.g., air conditioning and refrigeration systems) to meet a supply and demand balance between the aggregate power consumption of TCLs and fluctuating distributed power generation. The idea of real-time power regulation using TCLs was, to the authors' knowledge, originated in [1]. In this paper, we focus on countering the fluctuations of wind power generators whose outputs can be reliably forecasted [2], [3].

Future smart grid, which will likely integrate advanced metering infrastructure (AMI), distribution automation (DA) devices and distributed generation and storage, is typically a large-scale system, where decentralized algorithms for control, estimation and optimization are preferred to centralized ones, due to the features including enhanced robustness, reduction in communication between agents, and uniform power consumption between agents [4], [5].

Continuing along the idea developed in [1], and in the framework of smart girds, we formulate the wind power regulation problem into a quadratic optimization problem, in which the individual cost function of each VFAC reflects its relative temperature deviation, i.e., the difference between the room temperature and the temperature set point scaled by its comfort tolerance set by its users, and propose a decentralized bisection method, which converges fast due to the nature of bisection [6], [7]. Furthermore, due to the time-varying nature of thermal characteristics, e.g., time-varying ambient temperature through the day, we further propose a Kalman filter based technique for online parameter identification. Compared with others' work mainly by [1], [8], [9], ours has the following novel features. We assume the TCLs to be *variable frequency* air conditioners (VFACs), which can run with any power between 0 and the rated power. Though requiring an aggregator, our algorithm is decentralized in the sense that the TCLs also conduct local computation, and hence the computation burden of the aggregator is reduced, and TCLs' privacy including parameters, operation states, is preserved, while the ones are centralized in [1], [8], [9]. Moreover, our algorithm can be applied to deterministic and heterogeneous models of VFACs, and thus can be applied to both large-scale systems and small-scale systems. In comparison, the work in [1] used a hybrid model based on probability distribution of on/off states as the aggregated model of TCLs, which actually requires a large amount of TCLs for sake of controlling accuracy. Finally, by formulating the wind power regulation problem into a quadratic optimization problem, we try to dispatch the power supply as fairly as possible, while previous work in [1], [8], [9] didn't involve fair dispatch.

In Section II, we introduce the model of single air conditioner, and present the problem formulation. In Section III, we propose the the decentralized bisection algorithm and the Kalman filter based parameter identification method. In Section IV, simulations are given to illustrate the algorithm. We conclude our paper in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this section, we first introduce the single air conditioner's model, and then formulate the power regulation problem.

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A. Air Conditioner Model

We assume that by changing the frequency of the compressor through the frequency converter, the power of a VFAC can vary from zero to its rated power continuously. As for a *constant frequency* air conditioner (CFAC), by adopting the idea of *PWM control*, or with the help of *direct load control programmes*, we can approximate the average power consumption of a CFAC over short periods of time to be any desirable value between 0 and the rated power, since air conditioners are such electrical appliances that can be turned off or cycled for relatively short periods of time [10]. Moreover, in a smart grid scenario, due to VFACs' advantages compared with CFACs, including energy saving, lower noise and high precision of temperature control, VFACs will eventually take place of CFACs. Therefore, we assume all the TCLs to be VFACs hereinafter.

Let n be the number of air conditioners under consideration. We consider the cooling process and use the following discrete time model, originally developed in [11]:

$$T_i[k+1] = a_i T_i[k] + (1-a_i)(T_{a,i} - \eta_i R_i x_i[k]) + w_i[k], \tag{1}$$

where $T_i[k]$ (°C), $T_{a,i}$ (°C) and $x_i[k]$ (kW), $i = 1, 2, \ldots, n$, are the room temperature, ambient temperature, and the power of the ith VFAC at time k, respectively. The sampling time interval is denoted by $\Delta\tau$, and the power consumption is assumed constant as $x_i[k]$ during the interval between time k and $k+1$. The parameters in (1) are as follows: $a_i = \exp(-\Delta\tau / C_i R_i)$ represents the thermal characteristics, where C_i (kWh/°C) is the thermal capacitance, and R_i (°C/kW) is the thermal resistance; η_i is the load efficiency, which equals to the rate of energy transfer between the thermal mass and its environment divided by the power consumption of the ith VFAC; $w_i[k]$ is Gaussian white noise with variance $\Delta\tau\sigma^2$.

In this paper, we take $\Delta\tau = 5$ minutes and assume that all the parameters including C_i, R_i, η_i, $T_{a,i}$ are constant. These parameters can be obtained by direct measurements or by system identification. The latter method is preferred, which does not require extra measuring apparatus. A system identification method will be introduced in a later section.

B. Problem Formulation

Consider a power grid consisting of n VFACs, denoted by the set $V = \{1, 2, ..., n\}$, where the wind power supply fluctuates over time. The wind power supply meant for VFACs only, denoted by P, is assumed to be predictable and with a reasonable forecasting period, is considered to be constant during each interval, i.e., the power supply is assumed constant as $P[k]$ between time k and $k+1$.

The power of the ith VFAC shall be non-negative and no larger than its rated power ($i = 1, \ldots, n$), i.e.,

$$0 \leqslant x_i[k] \leqslant x_i^{rated}, \ k = 1, 2, \ldots \tag{2}$$

where x_i^{rated} is the rated power of the ith VFAC. In order to compensate the wind power supply's fluctuations, the aggregate power consumption of n VFACs should satisfy the supply constraint:

$$\sum_{i=1}^{n} x_i[k] = P[k], \ k = 1, 2, \ldots \tag{3}$$

Combined with (2) and (3), it is assumed that:

A1: There exists an aggregator, and the forecast of the total power supply $P[k]$ is known by the aggregator and satisfies

$$0 \leqslant P[k] \leqslant \sum_{i=1}^{n} x_i^{rated}, \ k = 1, 2, \ldots \tag{4}$$

To make the decentralized algorithm in our paper work, we further assume that:

A2: The aggregator is computationally capable, and conducts bidirectional communication with all the VFACs, i.e., the communication topology between the aggregator and the VFACs is a star network, where the aggregator acts as the hub node and the VFACs are the leaf nodes.

Remark 1: Though in reality the power grid also consists of other inflexible electrical appliances which share a common power supply with VFACs, in a smart gird scenario assumption A1 and A2 can be technically met with the help of advanced metering infrastructures (AMIs) [12], i.e., an AMI can measure the power consumption of any load among all the loads in a house and communicate bidirectionally with the aggregator. Denoting the total power supply meant for all the loads in the grid between time k and $k+1$ by $P^\dagger[k]$, the aggregator receives and sums up the power consumption of any other loads except VFACs, and computes:

$$P[k] = P^\dagger[k] - Q[k],$$

where $Q[k]$ is the total power consumption of the other inflexible loads in the gird during time k and $k+1$.

We hope by using VFACs to counter wind power's fluctuations, the users' comfort won't be severely compromised. For this purpose, each VFAC is given a temperature set point $T_{s,i}$ (°C) and a thermal comfort tolerance ΔT_i (°C) by its users, i.e., the users expect their room temperature to stay within the comfort zone $[-\Delta T_i + T_{s,i}, \Delta T_i + T_{s,i}]$. Since the temperature $T_i[k+1]$ at time $k+1$ depends on $x_i[k]$, with the process noise ignored, for each VFAC we define its local comfort degree as the relative temperature deviation of VFAC i, given by

$$h_i(x_i[k]) = \frac{T_{s,i} - T_i[k+1]}{\Delta T_i}. \tag{5}$$

Along with (1), it follows

$$h_i(x_i[k]) = \alpha_i x_i[k] + \beta_i[k], \tag{6}$$

where

$$\alpha_i = (1-a_i)\eta_i R_i / \Delta T_i, \text{ and}$$

$$\beta_i[k] = (T_{s,i} - a_i T_i[k] - (1-a_i)T_{a,i}) / \Delta T_i.$$

We then endow each VFAC with its own cost function at time k, defined as

$$f_i(x_i[k]) = \frac{h_i^2(x_i[k])}{2\alpha_i} = \frac{(\alpha_i x_i[k] + \beta_i[k])^2}{2\alpha_i}. \tag{7}$$

Therefore the total cost function of all the VFACs is given by

$$f(x[k]) = \sum_{i=1}^{n} f_i(x_i[k]), \tag{8}$$

where $x[k] = [x_1[k], ..., x_n[k]]^T$ is the global vector. One can easily verify that $h_i(x_i[k]) = f_i^{'}(x_i[k]) = \mathrm{d}f_i(x_i[k])/\mathrm{d}x_i[k]$. In the physical sense, $h_i^2(x_i[k])$ is the square of relative temperature deviation, and the total cost function $f(x[k])$ is a weighted square sum of all the relative temperature deviations, where the weight for each VFAC is $1/2\alpha_i$.

From the above, we now formulate the wind power regulation problem at time $k = 1, 2, ...$ into the following quadratic optimization problem:

$$\begin{aligned} \text{minimize} \quad & f(x[k]) = \sum_{i=1}^{n} f_i(x_i[k]), \\ \text{subject to} \quad & \sum_{i=1}^{n} x_i[k] = P[k], \\ & 0 \leqslant x_i[k] \leqslant x_i^{rated}, \ i = 1, ..., n \end{aligned} \tag{9}$$

The centralized solution to problem (9) can be easily achieved by the Lagrange dual method [13]. Define

$$g_i(\lambda) = \begin{cases} 0 & \lambda < h_i(0) \\ (\beta_i[k] - \lambda)/\alpha_i & h_i(0) \leqslant \lambda \leqslant h_i(x_i^{rated}) \\ -x_i^{rated} & h_i(x_i^{rated}) < \lambda \end{cases} \tag{10}$$

If problem (9) is feasible, it has a unique *optimal Lagrange multiplier* $\lambda^*[k]$, satisfying

$$P[k] + \sum_{i=1}^{n} g_i(\lambda^*[k]) = 0. \tag{11}$$

Accordingly, problem (9) has a unique optimal solution given by $x_i^*[k] = -g_i(\lambda^*[k]), i = 1, 2, \ldots, n$, i,e.,

$$x_i^*[k] = \begin{cases} 0 & \lambda^*[k] < h_i(0) \\ (\lambda^*[k] - \beta_i[k])/\alpha_i & h_i(0) \leqslant \lambda^*[k] \leqslant h_i(x_i^{rated}) \\ x_i^{rated} & h_i(x_i^{rated}) < \lambda^*[k] \end{cases} \tag{12}$$

At any time $k \geqslant 0$, we solve the optimization problem (12) in a decentralized fashion and assign the optimal solution $x^*[k]$ to the VFACs as their power consumption during the interval between time k and $k+1$, and the equality between demand and supply is guaranteed by the equality constraint in (12).

Remark 2: The optimization problem (9) and its variations also find applications in economic dispatch problem (EDP) [14] and optimal resource allocation problem (ORAP) [15].

III. Main Results

In this section, we show some properties of the optimal power regulation (9), and then present a decentralized bisection method to solve the problem.

A. Decentralized Bisection Method

Since the total cost function (8) has a separable structure, i.e., it's the total sum of the local costs, it's possible to be solved in a decentralized fashion. With time index k omitted in this subsection, we now present a decentralized bisection method for the optimization problem (9).

Let $s = 0, 1, 2...$ denote the bisection steps. We establish 3 variables, $\lambda^+(s)$, $\lambda^-(s)$ and $\lambda(s)$, which are commonly shared by all the VFACs. For the problem with the utility function defined in (6), all the room temperatures are desired to be kept in the comfort zone $[-\Delta T_i + T_{s,i}, \Delta T_i + T_{s,i}]$, while the air conditioning systems are considered to absorb the power supply fluctuation. Thus the initialization at $s = 0$ is given by $\lambda^+(0) = 1$ and $\lambda^-(0) = -1$ as otherwise the problem does not have a feasible solution.

At each bisection step s, the update of $\lambda(s)$ performed by each VFAC follows

$$\lambda(s) = \frac{\lambda^+(s) + \lambda^-(s)}{2}. \tag{13}$$

For $i = 1, ..., n$, each VFAC then computes

$$x_i(\lambda(s)) = \begin{cases} 0 & \lambda(s) < h_i(0) \\ (\lambda(s) - \beta_i)/\alpha_i & h_i(0) \leqslant \lambda(s) \leqslant h_i(x_i^{rated}) \\ x_i^{rated} & h_i(x_i^{rated}) < \lambda(s) \end{cases} \tag{14}$$

Every VFAC then sends $x_i(\lambda(s))$ to the aggregator. The aggregator computes the sum of all the VFACs' $x_i(\lambda(s))$, denoted by $X(s)$:

$$X(s) = \sum_{i=1}^{n} x_i(\lambda(s)), \tag{15}$$

compares $X(s)$ with P, and broadcasts $\gamma(s)$ to all the VFACs, given by

$$\gamma(s) = \begin{cases} 1 & X(s) > P \\ 0 & X(s) \leqslant P \end{cases} \tag{16}$$

After receiving $\gamma(s)$, each VFAC then updates $\lambda^+(s+1)$ and $\lambda^-(s+1)$ by

$$\begin{cases} \lambda^+(s+1) = \lambda(s), \ \lambda^-(s+1) = \lambda^-(s) & \gamma(s) = 1 \\ \lambda^+(s+1) = \lambda^+(s), \ \lambda^-(s+1) = \lambda(s) & \gamma(s) = 0 \end{cases} \tag{17}$$

For practical use, Algorithm 1 can either run fixed S steps, e.g., $S = 20$ steps, and then stop, or stop when certain tolerable supply-demand gap is reached. The aggregator makes decision on whether to stop or to continue the algorithm and then broadcasts a signal δ given by:

$$\delta = \begin{cases} 0 & |X(s) - P| < \varepsilon \\ 1 & \text{otherwise} \end{cases}$$

where ε is a small positive number denoting the tolerable supply-demand gap. For each VFAC, it stops the algorithm if $\delta = 0$, and continues if $\delta = 1$.

One can easily verify that though all the VFACs update $\lambda(s)$, $\lambda^+(s)$ and $\lambda^-(s)$ locally, since they receive the same

$\gamma(s)$ from the aggregator, and obey the same updating rule (17), for $s = 0, 1, 2, ...$, they always get the same $\lambda^+(s+1)$, $\lambda^-(s+1)$ and $\lambda(s+1)$.

The inequality constraints (2) are guaranteed by (14), while the equality constraint (3) is gradually satisfied during the course of bisections. The complete process of the decentralized bisection method is described in Algorithm 1. Next we show in the following theorem that Algorithm 1 converges to the solution to our problem and also the optimal solution to optimization problem (9).

Algorithm 1 Decentralized Bisection Method

Input: $P[k]$: the forecast of the wind power supply at time k.
Output: $x_i[k]$: power assignment for each VFAC at time k, $i = 1, ..., n$.

```
Initialization: λ⁻(0) = −1; λ⁺(0) = 1;
for s = 0, 1, 2, ... do
    Each VFAC updates λ(s) = (λ⁻(s) + λ⁺(s))/2;
    Each VFAC computes x_i(λ(s)) using (14) and sends
    it to the aggregator;
    The aggregator computes X(s) and γ(s), and broad-
    casts γ(s) and δ to the VFACs;
    Each VFAC receives γ(s) and δ, and then updates
    λ⁻(s+1) and λ⁺(s+1) using (17);
    if δ = 0 then
        Break;
    end if
end for
```

Theorem 1: If problem (9) is feasible, Algorithm 1 converges to the unique optimal solution of problem (9) as $s \to \infty$.

Proof The individual cost function $f_i(x_i)$ is twice continuously differentiable with second derivative $f_i^{''}(x_i) = \alpha_i > 0$, thus the total cost function $f(x) = \sum_i^n f_i(x_i)$ is strictly convex, which means problem (9) has at most one optimal solution. Since $\alpha_i > 0$, $x_i(\lambda)$ is monotonically increasing with respect to $\lambda(s)$. Thus, if there is a feasible solution for the problem, then the sum $\sum_{i=1}^n x_i(\lambda)$ is strictly increasing with respect to λ. Therefore, combined with the nature of bisection, Algorithm 1 converges. Moreover, since the optimal solution of problem (9) is unique, Algorithm 1 converges to the unique one.

Remark 3: Due to the nature of bisection, within s steps, we have

$$|\lambda(s) - \lambda^*| \leq |\lambda^+(0) - \lambda^-(0)| 2^{-s},$$

where λ^* is the optimal Lagrange multiplier. Therefore our algorithm converges fast.

Remark 4: Advantages of our algorithm are as follows due to the decentralization. The VFACs do not need to know how large $P[k]$ is. They merely update $\lambda(s)$ and send their $x_i(\lambda(s))$ to the aggregator. On the other hand, the aggregator does not need to know the VFACs' parameters, including α_i, β_i and x_i^{rated}. Therefore, the computational burden of the aggregator is reduced and the VFACs' privacy is preserved.

B. Parameter Identification

In Section II-A, the temperature evolution of thermal mass is described by the discrete time model (1), where the parameters remain to be determined. In [1], the per square meter (of floorage) capacitance C and resistance R are approximately described by variables randomly distributed, e.g., capacitance C per square meter ranges from about 0.015 to 0.065 kWh°C. Thus a feasible way to determine the parameters of a room is to simply multiply the area the floor by the associate per square meter factor. However, it is crucial to get as precise estimation of the parameters as possible, because though small parameter errors may not make much difference for one single VFAC, they cause a large deviation for the population of VFACs, i.e., the parameter errors of each VFAC will accumulate. Besides, we cannot get the ambient temperature, unless the room is equipped with extra thermometers. A better method with higher accuracy is by system identification, which requires no extra equipment and guarantees more accurate estimation of parameters. We now present a Kalman filtering based method [16].

Let us assume all the parameters to be time-varying, i.e., every parameter is subject to some perturbation, which allows us to ignore the process $w_i[k]$. Then all the parameters are indexed by k. Assuming the measurements to be noisy, and omitting the index i, defining $I[k] \equiv 1$, the state equation (1) can be transformed into the following equation

$$T[k] + \alpha_1[k]T[k-1] = \alpha_2[k]I[k-1] + \alpha_3[k]P[k-1] + \nu[k] \tag{18}$$

where for $j = 1, 2, 3$,

$$\begin{aligned} \alpha_1[k] &= -a[k] \\ \alpha_2[k] &= (1 - a[k])T_a[k] \\ \alpha_3[k] &= -(1 - a[k])\eta[t]R[k] \\ \alpha_j[k+1] &= \alpha_j[k] + \mu_j[k] \end{aligned} \tag{19}$$

and $\mu_j[k]$ and $\nu[k]$ are zeros mean, white, gaussian random process and mutually independent. Define

$$\xi[k] = [\alpha_1[k], \alpha_2[k], \alpha_3[k]]^T, \ \mu[k] = [\mu_1[k], \mu_2[k], \mu_3[k]]^T.$$

From (19), we have

$$\xi[k+1] = \xi[k] + \mu[k] \tag{20}$$

Then define

$$H[k] = [-T[k-1], I[k-1], P[k-1]]^T. \tag{21}$$

From the above, we have

$$T[k] = H^T[k]\xi[k] + \nu[k] \tag{22}$$

Now the system identification problem has been converted into a Kalman filtering problem [16]. Denoting the state estimated at time $k+1$ from measurements at time k by $\hat{\xi}_k[k+1]$, the corresponding covariance as $\Sigma_k[k+1]$ and

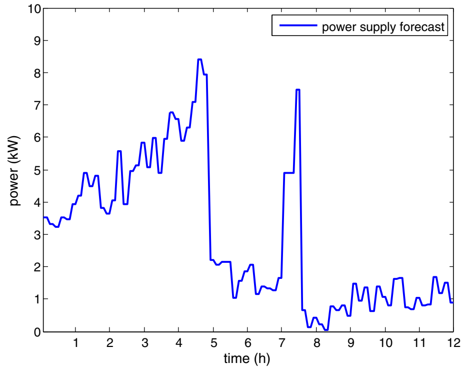

Fig. 1. Power supply forecast for 12 hours.

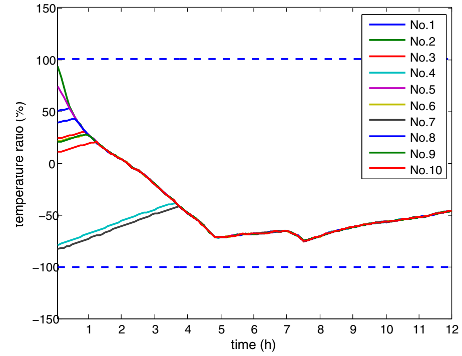

Fig. 2. Temperature ratios of the VFACs.

Kalman gain as $K[k]$, the filter is

$$\begin{aligned}\hat{\xi}_k[k+1] &= [I - K[k]H[k]]\hat{\xi}_{k-1}[k] + K[k]T[k],\\ \Sigma_k[k+1] &= \Sigma_{k-1}[k] - \Sigma_{k-1}[k]H^T[k] \times [H[k]\Sigma_{k-1}[k]H^T[k]\\ &\quad + S[k]]^{-1}H[k]\Sigma_{k-1}[k] + Q[k],\\ K[k] &= \Sigma_{k-1}[k]H^T[k][H[k]\Sigma_{k-1}[k]H^T[k] + S[k]]^{-1}.\end{aligned} \tag{23}$$

where $S[k] = E[\nu^2[k]]$ and $Q[k] = E[\mu[k]\mu^T[k]]$ are respectively the covariance of $\nu[k]$ and $\mu[k]$. Then through the method above, the parameters $\alpha_1[k]$, $\alpha_2[k]$, and $\alpha_3[k]$ are determined, and from (19), we can get $\eta[k]$, $R[k]$, $T_a[k]$ and $a[k]$ in an online fashion.

C. Decentralized Wind Power Regulation Scheme

In this subsection, we present the complete process for decentralized power regulation via VFACs by Algorithm 2.

Algorithm 2 Decentralized Power Regulation

1: **for** $k = 1, 2, \ldots$ **do**
2: The aggregator receives the forecast $P[k]$;
3: Each VFAC measures the current temperature $T_i[k]$ and estimates other necessary physical parameters by (19) - (23);
4: Each VFAC computes its own power assignment by the decentralized bisection method (Algorithm 1);
5: Each VFAC adjusts its working power.
6: **end for**

IV. Simulation Results

In this section we give several numerical examples to show the performance of our algorithm. We deal with wind power regulation problem with homogeneous VFACs in Case 1, while in Case 2 heterogenous VFACs are considered. We also show in Case 3 the demand-supply gap over time stays in an acceptable range.

Just for the demonstration purpose, we consider a small number of VFACs (10 VFACs and an aggregator in a star network). The forecast of fluctuating wind power is illustrated in Fig. 1.

A. Case 1: Dealing with Homogenous VFACs

In this case we use homogenous group of VFACs to regulate wind power, i.e., except for their initial temperatures $T_i[0]$, the 10 VFACs share the same parameters given by: the thermal capacity $C = 1.8$kWh/°C, thermal resistance $R = 12.5$°C/kW, ambient temperature $T_a = 30$°C, temperature set point $T_s = 25$°C, thermal efficiency $\eta = 2.5$, comfort tolerance $\Delta T_i = 2.5$°C and rated power $x^{rated} = 2.15$kW. The process noise $w_i[k]$ is considered with standard deviation $\sqrt{\Delta\tau\sigma^2} = 0.005$°C. Their initial temperatures take random values uniformly distributed in $[26, 30]$ °C.

At each time k, we perform 20 steps of bisection, and the results are shown in Fig. 2. The ratios of temperature deviation reach consensus at around 3.8h, after which all the VFACs stay in the consensus team, no matter how the power supply changes over time. Moreover, after consensus is reached, in this homogenous VFACs' case, the power assignment of every VFAC simply reduces to the average of the total power supply since then.

B. Case 2: Dealing with Heterogenous VFACs

In this case we consider heterogenous VFACs where all the parameters adopted from [17] are given as follows

$$\begin{aligned}C &= [1.5760\ 1.9222\ 1.8721\ 1.9661\ 1.7104\\ &\quad 1.9218\ 1.5826\ 1.5270\ 1.7037\ 1.6652],\\ R &= [13.091\ 12.177\ 12.834\ 12.586\ 12.252\\ &\quad 12.780\ 12.315\ 12.773\ 12.027\ 12.452],\\ T_a &= [29.824\ 30.324\ 30.1523\ 30.095\ 29.679\\ &\quad 30.107\ 30.436\ 29.620\ 29.777\ 30.284],\\ T_s &= [25\ 27\ 25\ 25\ 25\ 26\ 24\ 25\ 24\ 25],\\ \eta &= [2.3662\ 2.2277\ 2.2583\ 2.6941\ 2.6169\\ &\quad 2.3903\ 2.7701\ 2.2207\ 2.4632\ 2.4289],\\ \Delta T &= [2.0\ 2.0\ 2.0\ 2.5\ 5.0\ 3.0\ 2.5\ 2.0\ 1.5\ 2.5].\end{aligned}$$

The process noise is the same as the one in Case 1.

Fig. 3 shows the performance of our algorithm for heterogenous VFACs with 20 bisections performed at every time k as well. In this case, the ratios of temperature deviation still tend to reach consensus, and consensus is achieved at around

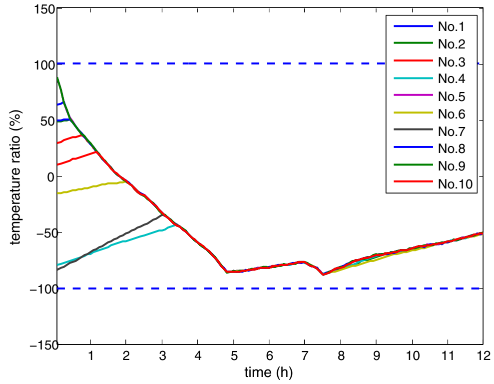

Fig. 3. Temperature ratios of the VFACs.

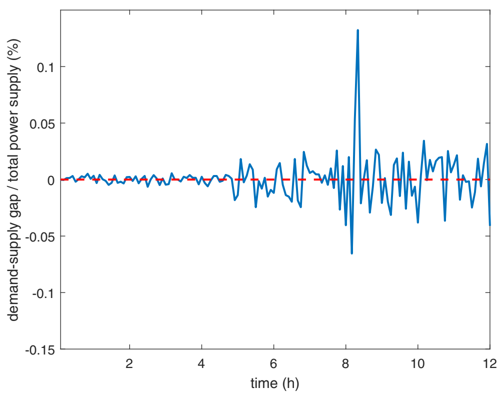

Fig. 4. The demand-supply gap for the example of 10 VFACs in a static network.

3.4h. However, after about 8h, some VFACs start to leave the consensus team, which is seen from the $6th$ VFAC's ratio curve. This is because the wind power supply is such small that the consensus takes on a trend of rising temperature and some of them may be assigned 0 power (turned off) by our algorithm. In particular, the temperature tolerance of the $6th$ VFAC is much larger than the others, which makes it unable to keep up with the others strictly. Nevertheless, the fair dispatch in general is still achieved.

C. Case 3: Demand-supply Gap

From the last 2 cases, we can see that both in homogenous and heterogenous cases, fair dispatch is achieved. Now we study to what extent the demand and supply constraint is met with in terms of Case 2.

The demand-supply gap (the difference between the total power supply and total power demand) is plotted in Fig. 4 for Case 2, in which the bisection algorithm runs 20 iterations at every step. From the simulation result we can observe that the demand-supply gap arrives below 0.15% of the total power supply.

V. Conclusions

In this paper, we study the problem of using TCLs to counter the fluctuations in wind power supply, which is formulated as a quadratic optimization problem. For this purpose, we present a decentralized bisection method. Through simulation we show that our algorithm is applicable to both homogeneous and heterogenous VFACs. We also comment that although we study the use of VFACs, the proposed algorithm can be applied to other types of loads and energy storage devices, such as refrigerators and plug-in electric vehicles.

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