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A linear approach to formation control under directed and switching topologies

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Abstract— **The paper studies the formation control problem for distributed robot systems. It is assumed each robot only has access to local sensing information (i.e. the relative positions and IDs of its neighbors). Taking into consideration physical sensing constraints (e.g. limited sensing range) and the motion of the robots over time, it may be noted that the sensing graph for the system is directed and time-varying. This presents a challenging situation for formation control. As an initiative attempt to study this challenging situation, we suppose the sensing graph switches among a family of graphs with certain connectivity properties, under which a switching linear control law is then proposed. We show that for arbitrary dwell times or average dwell times, the proposed control law with properly designed control parameters can ensure global convergence to a desired formation shape. The proposed formation control law can be implemented in a distributed manner while the design of certain control parameters requires some global information.**

I. INTRODUCTION

Distributed robot systems including autonomous underwater, ground or aerial vehicles, which are intended to perform coordinated tasks, have a broad range of applications ([16], [19]) such as search and rescue operations in hazardous environments, ocean data retrieval and sampling, and surveillance/combat tasks. Among all the coordinated missions, formation control has been considered as one of the fundamental issues, with the goal of controlling a team of autonomous robots to form a rigid or flexible structure. The research on formation control of distributed robot systems has experienced rapid growing since the 1990s. However, several challenges still exist and have not been fully overcome in the situations that only local information (GPS-free) is accessible and the sensing graph may change over time.

For realizability of rigid formations, graph rigidity is used to characterize information architecture among robots ([2], [8], [17]), for which relative distances between pairs of robots are concerned. With this idea, nonlinear gradient control strategies are developed to stabilize a group of mobile robots into a desired rigid formation ([4], [12]) and maneuver a robot formation while maintaining a formation shape ([1], [10]). A variation is the use of bearing angle information for the objective of formation shape control ([3], [11]). Moreover, in responding to possible environment changes, the problem of scaling the size of a formation is addressed without a re-design of formation control laws [5]. Our previous works ([14], [20]–[22]) also establish a new framework for formation shape control with a flexible formation scale, which is adaptable to environment changes. However, unlike consensus control [13], [18], there are few works in formation control dealing with switching topologies, which are practically significant due to unreliable sensing and communications.

This paper concentrates on the switching topology case by extending our previous work on the fixed topology case ([14], [20]–[22]). The goal is to achieve a formation shape for a team of autonomous robots when the sensing graph may switch over time. The scale of an achieved formation is not a concern in the paper. This is because as shown in [15], if a formation shape can be achieved, then the scale of the entire formation can be controlled by only a small portion of robots (e.g., two robots in the team). For formation shape control under a fixed sensing graph, we show in our earlier work [21] that a formation shape is realizable if and only if the sensing graph is 2-rooted, a kind of connectedness in graph theory. Thus, in the paper, we assume that the sensing graph switches over a family of 2-rooted graphs to make the formation control problem feasible. Under this assumption, a switching linear control law is proposed depending on the sensing graph. It is then shown that for any switching signal satisfying a dwell time or average dwell time condition, which is often met for practical systems, the proposed linear control law with properly designed parameters ensures globally exponential convergence of mobile robots to a desired formation shape. The approach can also be applied for formation maneuvering provided that their maneuvering velocities are synchronized. Simulations and experiments using Rovio omni-directional mobile robots are conducted to validate our theoretic results.

Notation: $\mathbb{C}$ and $\mathbb{R}$ denote the set of complex and real numbers, respectively. $\iota = \sqrt{-1}$ denotes the imaginary unit and $\mathbf{1}_n$ represents the n-dimensional vector of ones.

II. PRELIMINARY AND PROBLEM SETUP

A. Graphical notions and preliminary results

A *directed graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a non-empty node set $\mathcal{V} = \{1, 2, \cdots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The neighbor set of node i is denoted by $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$. In the paper, we assume that directed graphs do not have self-loops, i.e., $i \notin \mathcal{N}_i$ for any node i. Next, we introduce two concepts from [20]. For a directed graph $\mathcal{G}$, a node v is said to be *2-reachable* from a non-singleton set $\mathcal{U}$ of nodes if there exists a path from a node in $\mathcal{U}$ to v after removing

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any one node except node v. Moreover, a directed graph $\mathcal{G}$ is said to be *2-rooted* if there exists a subset of two nodes, from which every other node is 2-reachable. These two nodes are called *roots* in the graph.

Examples are given in Fig 1-3 to explain these notions. In Fig. 1, let $\mathcal{U} = \{u_1, u_2, u_3\}$. Node v is 2-reachable from $\mathcal{U}$ as after removing any one other node there is still a path from a node in $\mathcal{U}$ to node v. However, in Fig. 2, v is not 2-reachable from $\mathcal{U}$ because when node u_2 is removed, there does not exist a path from a node in $\mathcal{U}$ to node v. The graph in Fig. 3 is 2-rooted, for which nodes u_1 and u_2 are the two roots, because nodes v_1, v_2, and v_3 are all 2-reachable from $\mathcal{U} = \{u_1, u_2\}$.

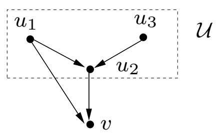


Fig. 1. 2-reachable.

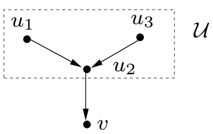


Fig. 2. Not-2-reachable.

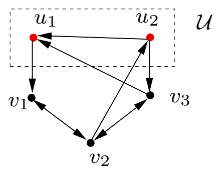


Fig. 3. 2-rooted.

For a directed graph $\mathcal{G}$, we define a corresponding complex Laplacian L as follows: The ijth entry

$$L(i,j) = \begin{cases} -w_{ij} & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i \\ 0 & \text{if } i \neq j \text{ and } j \notin \mathcal{N}_i \\ \sum\limits_{j \in \mathcal{N}_i} w_{ij} & \text{if } i = j \end{cases}$$

where $w_{ij} \in \mathbb{C}$ is called the *complex weight* on edge (j, i). From the definition, it is true that a complex Laplacian has at least one zero-eigenvalue whose associated eigenvector is $\mathbf{1}_n$. Finally, we recall two preliminary results from [21] about a complex Laplacian with minor modifications.

Lemma 2.1: Consider a directed graph $\mathcal{G}$ of n nodes and a generic vector $\xi \in \mathbb{C}^n$. Then $\text{rank}(L) = n - 2$ for almost all complex Laplacians L associated to $\mathcal{G}$ and satisfying $L\xi = 0$ if and only if $\mathcal{G}$ is 2-rooted.

Lemma 2.2: Consider a directed graph $\mathcal{G}$ of n nodes and a generic vector $\xi \in \mathbb{C}^n$. If $\mathcal{G}$ is 2-rooted, then for almost all complex Laplacians L satisfying $L\xi = 0$, there exists a diagonal matrix D such that the eigenvalues of DL can be assigned at any locations in addition to two fixed eigenvalues at the origin.

B. Problem setup

We consider a group of n robots in the plane. The positions of n robots are denoted by complex numbers $z_1, \cdots, z_n \in \mathbb{C}$. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of n nodes represents the *sensing graph* in which an edge (j, i) indicates that robot i can measure the relative position of robot j in robot i's local frame and also its ID. Our goal is to make the n robots reach and maintain a desired formation shape.

We use a complex number $\xi_i \in \mathbb{C}$ $(i = 1, \cdots, n)$ to represent a point in a reference frame Σ. Let the n-dimensional composite complex vector $\xi = [\xi_1, \xi_2, \cdots, \xi_n]^T \in \mathbb{C}^n$ be the formation vector for the n robots in the reference frame Σ. In the paper, we consider point robots and assume that $\xi_i \neq \xi_j$ if $i \neq j$, meaning that no two robots overlap each other. For practical applications, pairs of robots should be separated by at least certain distance away in the target formation.

Next we introduce the concept of *similar formation*. For a directed graph $\mathcal{G}$ and an associated complex Laplacian L, if a configuration $\xi \in \mathbb{C}^n$ satisfies a constraint $\mathcal{L} : L\xi = 0$, we call ξ a realization of the graph $\mathcal{G}$ and the linear constraint $\mathcal{L}$. A *framework* is a graph together with a realization, denoted as $(\mathcal{G}, \xi, \mathcal{L})$ where $\mathcal{G}$ is a directed graph, ξ is a configuration and $\mathcal{L}$ represents the linear constraint $L\xi = 0$. In this paper, a framework is defined in terms of a linear constraint rather than the distance constraints on edges as in [6], [7].

Definition 2.1: A framework $(\mathcal{G}, \xi, \mathcal{L})$ is said *similar* if

$$\ker(L) = \{c_1 \mathbf{1}_n + c_2 \xi : \; c_1, c_2 \in \mathbb{C}\}.$$

Remark 2.1: Note that a complex number c_2 can be written in the polar coordinate form (namely, $c_2 = \rho e^{\iota\theta}$). So the solutions to the linear constraint $L\xi = 0$ consist of points related by translations c_1, rotations θ, and scaling ρ (four degrees of freedom). That is, the formations subject to the linear constraint $L\xi = 0$ are scalable from the formation ξ in addition to rigid body motions (translations and rotations). Moreover, it is worth to point out that if there is additional distance constraints on at least one edge, the framework then becomes globally rigid with a determined size.

In our previous work [20], [21], we have derived the necessary and sufficient graphical condition for a framework to be similar, which is stated below.

Theorem 2.1 ([20], [21]): A framework $(\mathcal{G}, \xi, \mathcal{L})$ is similar for almost all L and ξ if and only if $\mathcal{G}$ is 2-rooted.

The theorem says that in order to reach a formation shape for a group of n robots, the sensing graph $\mathcal{G}$ should be 2-rooted. Otherwise, with insufficient number of information links, a group of n robots is never able to reach a desired formation shape. In the paper, we consider a sensing graph that may switch over time as the robots evolve. Denote by $\{\mathcal{G}_p : p \in \mathcal{P}\}$ the family of possible sensing graphs that the robots may take. But in order to make the formation shape control problem feasible, we assume in the following that

A1: $\mathcal{G}_p$ is 2-rooted for all $p \in \mathcal{P}$.

For robots modeled by continuous time dynamics

$$\dot{z}_i(t) = u_i(t), \; i = 1, \ldots, n, \tag{1}$$

the aim of the paper is to design a distributed control law such that a network of robots can globally exponentially reach and maintain a formation shape (i.e., similar to a desired formation ξ) under switching sensing topologies.

III. DISTRIBUTED FORMATION CONTROL AND STABILITY

In this section, we first propose a distributed formation control law under switching topologies. Next, we show that under dwell time (average dwell time) conditions with arbitrarily small dwell time (average dwell time), our proposed control law with properly designed control parameters ensures globally exponential convergence of multi-robot formations. Finally, we discuss the design, distributed implementation, and extension to formation maneuvering of our control law.

A. A switching formation control law

We denote by $\mathcal{G}_{\sigma(t)}$ a switching sensing graph with σ: $[0,\infty) \to \mathcal{P}$ being a piecewise constant switching signal, and denote by $\mathcal{N}_i(t)$ the set of neighbors at t. For a generic formation $\xi \in \mathbb{C}^n$ and for any $p \in \mathcal{P}$, we select complex weights $w_{ij}(p)$'s associated to edges (j,i)'s on graph $\mathcal{G}_p$ such that the complex Laplacian L_p associated with $\mathcal{G}_p$ and complex weights $w_{ij}(p)$ satisfies $L_p\xi = 0$. Detailed explanation on how to pre-calculate complex weights will be given in Subsection III-C. Without loss of generality, we assume that $\text{rank}(L_p) = n-2$ as it can always be assured for 2-rooted graph $\mathcal{G}_p$'s by Lemma 2.1.

The following linear switching control law is proposed for each robot $i = 1,\dots,n$:

$$u_i = d_i(\sigma(t)) \sum_{j\in\mathcal{N}_i(t)} w_{ij}(\sigma(t))(z_j - z_i), \tag{2}$$

where $d_i(\sigma(t)) \in \mathbb{C}$ is a control parameter to be designed.

Under the distributed control law (2), the overall closed-loop dynamics of n continuous-time robots becomes

$$\dot{z} = -D_{\sigma(t)}L_{\sigma(t)}z \tag{3}$$

where $z \in \mathbb{C}^n$ is the aggregate state vector of n robots, $D_{\sigma(t)} = \text{diag}\{d_1(\sigma(t)),\cdots,d_n(\sigma(t))\}$ is an n-by-n diagonal complex matrix, and $L_{\sigma(t)}$ is a complex Laplacian matrix associated to $\mathcal{G}_{\sigma(t)}$.

It is clear that the switched system (3) switches its dynamics from a family of subsystems

$$\dot{z} = -D_pL_pz,\ p \in \mathcal{P}. \tag{4}$$

However, the stability of all subsystems does not ensure the stability of the switched system (3). But notice that in practical applications, the sensing graph usually does not switch arbitrarily fast. In other words, it must satisfy a dwell time constraint or an average dwell time constraint. In the following, we are going to show that under a dwell time or average dwell time condition, globally exponential stability of multi-robot formations can be guaranteed by a properly designed control gain D_p for every subsystem.

B. Stability analysis

First, we introduce the concept of dwell time. Suppose the switching signal $\sigma(t)$ switches its value at time instants t_1, t_2, ... We say the switching signal has *dwell time* τ_D if $t_{i+1} - t_i \ge \tau_D$ for all i's. We denote by $\mathcal{S}(\tau_D)$ the set of all switching signals with dwell time τ_D, i.e.,

$$\mathcal{S}(\tau_D) = \{\sigma(t) : t_{i+1} - t_i \ge \tau_D \text{ for all } i\}.$$

We then present a main result to show that the switched system (3) can be made globally exponentially convergent under a dwell time condition.

Theorem 3.1: For any $\tau_D > 0$ and any switching signal $\sigma(t) \in \mathcal{S}(\tau_D)$, there exist D_p's, $p \in \mathcal{P}$, such that a network of n robots globally exponentially reaches a formation (similar to ξ) under the distributed control law (2).

Proof: Let e_i be the n-dimensional vector whose ith entry is 1 and all others are 0. Then we define an $n \times n$ matrix

$$Q = \begin{bmatrix} e_1 & \cdots & e_{n-2} & \mathbf{1}_n & \xi \end{bmatrix}.$$

For a nonsingular matrix D_p, D_pL_p has rank $n-2$ and moreover $D_pL_p\mathbf{1} = D_pL_p\xi = 0$. So $-Q^{-1}D_pL_pQ$ must be of the following form

$$-Q^{-1}D_pL_pQ = \left[\begin{array}{c|c} M_p & 0 \\ \hline * & 0 \end{array}\right] \tag{5}$$

where $M_p \in \mathbb{C}^{(n-2)\times(n-2)}$ satisfying $\text{rank}(M_p) = n-2$. Furthermore, M_p has the same eigenvalues as $-D_pL_p$ except the two zero eigenvalues.

Consider the coordinate transformation $y = Q^{-1}z$ for the switched system (3). Then it can be obtained that

$$\dot{y} = \left[\begin{array}{c|c} M_{\sigma(t)} & 0 \\ \hline * & 0 \end{array}\right] y.$$

Let x be a sub-vector of y consisting of the first $n-2$ components of z. Then it is clear that

$$\dot{x} = M_{\sigma(t)}x. \tag{6}$$

It is true that globally exponential stability of (6) is equivalent to globally exponential convergence of n robots to a formation similar to ξ.

Suppose temporarily that for any $p \in \mathcal{P}$, $\dot{x} = M_px$ is exponentially stable. That is, there exist positive constants c and α_0 such that $\|e^{M_pt}\| \le ce^{-\alpha_0 t}$ for all $p \in \mathcal{P}$ and all $t \ge 0$. Now consider a switching signal $\sigma(t) \in \mathcal{S}(\tau_D)$ for a constant τ_D. Denote by $t_1, t_2, \dots$ the time instants, at which the switching occurs and suppose $\sigma(t) = p_i$ for $t \in [t_{i-1}, t_i)$. We choose a constant $\alpha_c \in (0, \alpha_0)$ and show in the following by induction that if $\tau_D \ge \frac{\ln c}{\alpha_0 - \alpha_c}$, then $\|x(t_k)\| \le c\|x(0)\|e^{-\alpha_c t_k}$.

At t_1, we have

$$\|x(t_1)\| = \|e^{M_{p_1}t_1}x(0)\| \le c\|x(0)\|e^{-\alpha_0 t_1} \le c\|x(0)\|e^{-\alpha_c t_1}.$$

Suppose at t_k, $\|x(t_k)\| \le c\|x(0)\|e^{-\alpha_c t_k}$. Then at t_{k+1},

$$\begin{aligned} \|x(t_{k+1})\| &= \|e^{M_{p_{k+1}}(t_{k+1}-t_k)}x(t_k)\| \\ &\le c\|x(t_k)\|e^{-\alpha_0(t_{k+1}-t_k)} \\ &\le c^2\|x(0)\|e^{-\alpha_c t_k}e^{-\alpha_0(t_{k+1}-t_k)}. \end{aligned}$$

Note that $t_{k+1} - t_k \ge \tau_D \ge \frac{\ln c}{\alpha_0-\alpha_c}$. So it follows that $ce^{-\alpha_0(t_{k+1}-t_k)} \le e^{-\alpha_c(t_{k+1}-t_k)}$ and thus

$$\|x(t_{k+1})\| \le c\|x(0)\|e^{-\alpha_c t_{k+1}}.$$

Also, notice that for t in any interval $[t_k, t_{k+1})$, it holds that $\|x(t)\| \le c\|x(t_k)\|e^{-\alpha_0(t-t_k)}$. Therefore, if

$$\tau_D \ge \frac{\ln c}{\alpha_0 - \alpha_c}, \tag{7}$$

then for any switching signal $\sigma(t) \in \mathcal{S}(\tau_D)$, the switched system (6) is globally exponentially stable, implying that the

n robots globally exponentially reach the desired formation shape.

Finally, we show for any given τ_D how $\tau_D \geq \frac{\ln c}{\alpha_0 - \alpha_c}$ can be satisfied by properly designing D_p's. Suppose without loss of generality that D_p is of the form $D_p = \gamma_p D_p'$ where γ_p is a positive real constant and D_p' is a complex diagonal matrix. Since every $\mathcal{G}_p$ is 2-rooted, then by Lemma 2.2 it follows that the eigenvalues of $D_p' L_p$, $p \in \mathcal{P}$, can be assigned in the right complex plane in addition to two fixed eigenvalues at the origin by designing a proper D_p'. Moreover, notice that c in (7) is a parameter satisfying $\|e^{M_p t}\| \leq ce^{-\alpha_0 t}$ for all $p \in \mathcal{P}$, which relates to the eigenvectors of M_p (or equivalently $D_p' L_p$), $p \in \mathcal{P}$, as γ_p does not change the eigenvectors. However, the choice of γ_p can change the locations of non-zero eigenvalues and thus can change α_0 to any value. Therefore, the condition $\tau_D \geq \frac{\ln c}{\alpha_0 - \alpha_c}$ can be met by selecting a proper D_p (namely, D_p' and γ_p). Thus, the conclusion follows. ∎

Theorem 3.1 shows that if a switching signal has dwell time τ_D, then the n robots can reach a desired formation shape under the proposed control law. However, in certain situations, the switching signals may occasionally have consecutive discontinuities separated by less than τ_D, but for which the average interval between consecutive discontinuities is no less than τ_D. This leads to the concept of average dwell time. For a switching signal $\sigma(t)$, we let $N_\sigma(t_0, t)$ denote the number of discontinuities of $\sigma(t)$ in the interval $[t_0, t)$. Then the set of all switch signals with *average dwell time* τ_D and *chatter bound* N_0 is denoted as

$$\mathcal{S}_{ave}(\tau_D, N_0) = \{\sigma(t) : N_\sigma(t_0, t) \leq N_0 + \frac{t - t_0}{\tau_D}\}.$$

Roughly speaking, as the time interval is long enough, the average dwell time is approximately τ_D considering the upper-bound of the number of discontinuities in the interval. Then we have the following result in terms of average dwell time.

Theorem 3.2: For any switching signal $\sigma(t) \in \mathcal{S}_{ave}(\tau_D, N_0)$ with any average dwell time $\tau_D > 0$ and arbitrary chatter bound N_0, there exist D_p's, $p \in \mathcal{P}$, such that a network of n robots globally exponentially reaches a formation (similar to ξ) under the distributed control law (2).

Proof: From the proof of Theorem 3.1, we know that there exist D_p's, $p \in \mathcal{P}$, such that $\|e^{M_p t}\| \leq ce^{-\alpha_0 t}$ where c is a constant and α_0 can be made arbitrarily large. We choose a constant $\alpha_c \in (0, \alpha_0)$ and by designing D_p's so that $\tau_D \geq \frac{\ln c}{\alpha_0 - \alpha_c}$ holds.

Denote by $t_1, t_2, \ldots$ the time instants, at which the switching occurs and suppose $\sigma(t) = p_i$ for $t \in [t_{i-1}, t_i)$. For any $t \in [t_i, t_{i+1})$, we have

$$x(t) = e^{M_{p_{i+1}}(t - t_i)} e^{M_{p_i}(t_i - t_{i-1})} \cdots e^{M_{p_1} t_1} x(0).$$

As $\|e^{M_p t}\| \leq ce^{-\alpha_0 t}$, we have

$$\|x(t)\| \leq c^{N_\sigma(0,t)} e^{-\alpha_0 t} \|x(0)\|$$

where $N_\sigma(0, t)$ is the number of switchings of $\sigma(t)$ in the interval $[0, t)$. Note that $c \geq 1$ and $N_\sigma(0, t) \leq N_0 + \frac{t}{\tau_D}$ for any switching signal $\sigma(t) \in \mathcal{S}_{ave}(\tau_D, N_0)$, so it follows that

$$\|x(t)\| \leq c^{N_0} c^{\frac{t}{\tau_D}} e^{-\alpha_0 t} \|x(0)\|.$$

Moreover, since $\tau_D \geq \frac{\ln c}{\alpha_0 - \alpha_c}$, then

$$\|x(t)\| \leq c^{N_0} \|x(0)\| e^{-\alpha_c t},$$

meaning that the switched system (6) is globally exponentially stable and the n robots globally exponentially reach a formation similar to ξ. ∎

C. Design of control parameters and distributed implementation

In order to run the control law (2) for the purpose of formation shape control, the parameters should be pre-computed. It consists of two main steps. The first is the design of $w_{ij}(p)$ for each robot i and for $j \in \mathcal{N}_i$ in every possible sensing graph $\mathcal{G}_p$. This can be done in a distributed manner. That is, the complex weights $w_{ij}(p)$ can be calculated by robot i from the following equation

$$\sum_{j \in \mathcal{N}_i(\mathcal{G}_p)} w_{ij}(p)(\xi_j - \xi_i) = 0$$

as ξ_j's for $j \in \mathcal{N}_i(\mathcal{G}_p)$ are available to robot i. It should be pointed out that the solution may not be unique. The second is the design of D_p. A distributed approach for the design of D_p is challenging and is still unknown. Thus, a centralized way is considered in the paper. First, a diagonal matrix D_p' (introduced in the proof of Theorem 3.1) is designed using the homotopy Newton iteration method ([21]) to assign the eigenvalues of $D_p' L_p$ in the right complex plane in addition to two fixed eigenvalues at the origin. Second, a sufficiently large γ_p (introduced in the proof of Theorem 3.1) is chosen to satisfy the condition $\tau_D \geq \frac{\ln c}{\alpha_0 - \alpha_c}$ for any given dwell time or average dwell time τ_D. Thus, $D_p = \gamma_p D_p'$ is obtained.

Next, we discuss distributed and local implantation of the switching control law (2). First of all, the linear control law (2) uses only the relative positions (i.e., $(z_j - z_i)$, $j \in \mathcal{N}_i(t)$) of its neighbors at t. It is locally implementable by onboard sensors without requiring all the robots to have a common sense of direction and scale unit. However, a common notion of clockwise rotation should be shared by all the robots. Consider for example that robot i has two neighbors j and k. With an onboard sensor (e.g., camera) on robot i, it can measure the relative states $z_j - z_i$ and $z_k - z_i$ in its own local frame with the x-axis coincident to the optical axis. Then the control input is obtained by a linear combination of the two relative positions using complex weights also defined in its own local frame. A more detailed discussion on how to locally implement a control law on a point-mass robot using relative position measurements refers to [13] (pages 141-143). Second, as evident from (2), implementing the local control law requires every robot to know the switching sensing graph under which the robots are operating. The entire sensing graph is a piece of global information, but it can be known by all the robots in a distributed way. At time t, each robot i knows its neighbor set $\mathcal{N}_i(t)$. By knowing this, robot i then knows a subset of graphs, which the current entire sensing graph belongs to, denoted by $\mathcal{P}_i(0) = \{p \in \mathcal{P} : \mathcal{N}_i(\mathcal{G}_p) = \mathcal{N}_i(t)\}$. Every robot exchanges $\mathcal{P}_i(k)$ with its neighbors and updates $\mathcal{P}_i(k+1)$ according to

$$\mathcal{P}_i(k+1) = \mathcal{P}_i(k) \bigcap_{j \in \mathcal{M}_i} \mathcal{P}_j(k),$$

where $\mathcal{M}_i$ represents the set of communication neighbors of robot i. As long as the communication graph has certain connectivity property, the iteration converges in finite steps and leads to a unique solution p, that is the sensing graph at time t. Thus, the switching control law (2) can be determined.

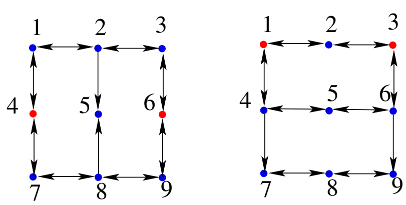
Finally, we discuss a possible extension to formation maneuvering (i.e., reach and maintain a formation shape while moving). Suppose a reference velocity, say $v_0(t)$ is known to all robots, or they synchronize their reference velocities by a distributed consensus control law. Then a group of n robots can achieve formation maneuvering with a common velocity $v_0(t)$ by the following distributed control laws:

$$u_i = d_i(\sigma(t)) \sum_{j \in \mathcal{N}_i(t)} w_{ij}(\sigma(t))(z_j - z_i) + v_0(t). \qquad (8)$$

IV. Simulation and Experiment

A. A simulation result

We consider an example consisting of 9 robots. Suppose the sensing graph switches over $\{\mathcal{G}_p : p = 1, 2\}$ periodically. The graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ are shown in Fig. 4, both of which are 2-rooted with roots $\{4, 6\}$ for $\mathcal{G}_1$ and roots $\{1, 3\}$ for $\mathcal{G}_2$. The switching signal is shown in Fig. 5.



(a) $\mathcal{G}_1$ (b) $\mathcal{G}_2$

Fig. 4. A family of sensing graphs for our simulation.

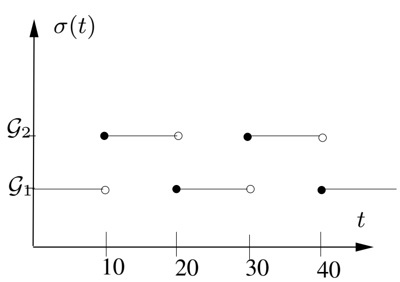


Fig. 5. A switching signal $\sigma(t)$.

Consider a formation shape described by $\xi = [-1 + \iota,\ \iota,\ 1 + \iota,\ -1,\ 0,\ 1,\ -1 - \iota,\ -\iota,\ 1 - \iota]^T$, which is a 3-by-3 grid. We design $D_1 L_1$ and $D_2 L_2$ by the homotopy Newton iteration method ([21]) such that the eigenvalues of both them lie in $\{0.9 + 0.01\iota,\ 0.9 - 0.01\iota,\ 1 + 0.02\iota,\ 1 - 0.02\iota,\ 1.1 + 0.05\iota,\ 1.1 - 0.05\iota,\ 0.8 + 0.020\iota,\ 0,\ 0\}$. It also makes the dwell time condition hold. In the simulation, we suppose all the agents take the control law (8) with a common reference velocity $v_0 = 4 + 3\iota$.

Fig. 6 records the trajectories of the 9 robots using the proposed distributed control law. The blue circles are the initial positions of the robots and the red diamonds represent the positions of the robots at the end of the simulation. Moreover, the trajectories in blue indicate that during that interval, the sensing graph is $\mathcal{G}_1$ while the trajectories in red indicate that during that interval, the sensing graph is $\mathcal{G}_2$.

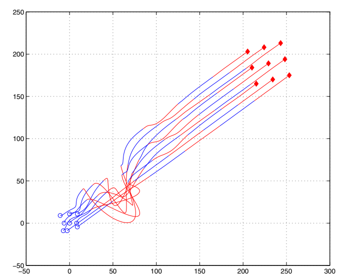
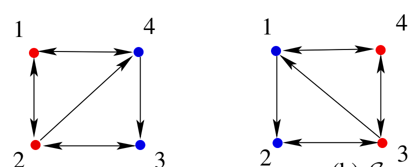


Fig. 6. Simulated trajectories under the proposed formation control law.

B. An experiment result

Our proposed formation control law is also implemented on a group of Rovio mobile robots under switching sensing topologies. The Rovio robots contain a true-track beacon, with which they can localize themselves [9], but in the experiment we convert the absolute location information to relative positions as specified by the directed graphs in Fig. 7. The Rovio robot is equipped with three omni-directional wheels so that they can move freely in the plane like a point mass.



(a) $\mathcal{G}_1$ (b) $\mathcal{G}_2$

Fig. 7. A family of sensing graphs for our experiment.

We consider four Rovio robots for our formation control experiment. The switching signal is also the periodic one shown in Fig. 5, which switches the sensing graph between $\mathcal{G}_1$ and $\mathcal{G}_2$ in Fig. 7. We consider a square formation shape, described by $\xi = [1 + 2\iota,\ -1 + 2\iota,\ -1,\ 1]^T$. The formation control law is implemented in a sampled-data setup. The complex Laplacian matrices for $\mathcal{G}_1$ and $\mathcal{G}_2$ are given below:

$$L_1 = \begin{bmatrix} 0.4 - 0.4\iota & 0.4\iota & 0 & -0.4 \\ 0.6\iota & -0.6 - 0.6\iota & 0.6 & 0 \\ 0 & -0.8 & 0.8 - 0.8\iota & 0.8\iota \\ -0.4 + 0.4\iota & -0.4\iota & 0 & 0.4 \end{bmatrix} \text{ and}$$

$$L_2 = \begin{bmatrix} -0.4 & 0 & -0.4\iota & 0.4+0.4\iota \\ -0.6\iota & 0.6+0.6\iota & -0.6 & 0 \\ 0 & -0.8 & 0.8-0.8\iota & 0.8\iota \\ 0.4 & 0 & 0.4\iota & -0.4-0.4\iota \end{bmatrix}.$$

To meet the dwell time condition, $D_1 = \mathrm{diag}\{0.9244 + 1.1354\iota,\ -0.4930 - 0.6996\iota,\ 1,\ 1\}$ and $D_2 = \mathrm{diag}\{-1.1660 - 1.1709\iota,\ 1.4711 - 0.2516\iota,\ 1,\ 1\}$ are designed to assign the eigenvalues of D_1L_1 and D_2L_2 at $\{0.95 + 0.5\iota,\ 0.95 - 0.5\iota,\ 0,\ 0\}$.

With a common reference velocity $v_0 = 5 + 5\iota$, the trajectories of the four Rovio robots are plotted in Fig. 8, which shows the four Rovio robots asymptotically converge to form and maintain a square formation shape. A snapshot when the square formation is reached is given in Fig. 9.

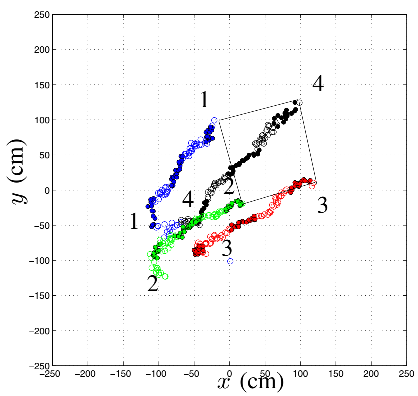


Fig. 8. The experimental trajectories of four Rovio robots under switching sensing topologies with $v_0 = 5 + 5\iota$.

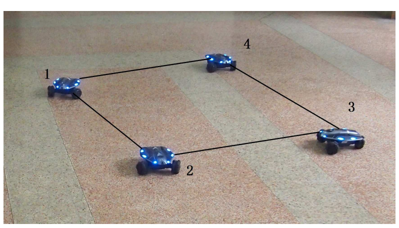


Fig. 9. A snapshot of square formation achieved in our experiment.

V. CONCLUSION

The paper developed a distributed control strategy for formation shape control of autonomous robots under switching sensing topologies. It is practical that a dwell time or average dwell time condition is often satisfied. Thus, with a known dwell time (average dwell time) constant, we showed that our proposed distributed control schemes are effective with properly designed control gains. However, in this work we assumed that the sensing graph switches over a family of 2-rooted graphs. Certainly, the sensing graph may become not 2-rooted sometimes in practical setting and it is more interesting to see whether our distributed control strategies still work for the situation that the sensing graph switches among all possible topologies.

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