

# Linear LMS Compensation for Timing Mismatch in Time-Interleaved ADCs

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**Abstract**—The time-interleaved architecture permits implementing high frequency analog-to-digital converters (ADCs) by multiplexing the output of several time-shifted low frequency ADCs. An issue in the design of a time-interleaved ADC is the compensation of timing mismatch, which is the difference between the ideal and the real sampling times. In this paper we propose a compensation method which, as opposite to other approaches, do not make a bandlimited assumption on the signal to be sampled. The proposed compensation is designed in a statistically optimal sense, to minimize the power of the reconstruction error in the samples, for a given input signal power spectrum. Due to the non-bandlimited assumption, perfect reconstruction is not possible in general. However, if the input signal is bandlimited, the proposed method achieves perfect reconstruction, if no constraints are made in the order of the compensation. Simulation results show that the proposed compensation outperforms the other methods, in terms of the reconstruction error power, for a given fixed compensation order, except for signal having large zero regions in their power spectrum. A generalization of the proposed method solves this drawback, and will be addressed in a journal version.

## I. INTRODUCTION

A high speed analog-to-digital converter (ADC) can be realized by using the so-called time-interleaved architecture [1]. In this technique, a  $D$  channel time-interleaved ADC consists of  $D$  ADCs (called channel ADCs) having the same sampling rate but different sampling phases, as if they were a single converter operating at a  $D$  times higher sampling rate. Recent high-speed ADCs using this technology achieve sampling rates of up to 20 GS/s [2].

In spite of its conceptual simplicity, a drawback of the time-interleaved technique is that mismatches between different channel ADCs deteriorate the overall signal-to-noise-and-distortion ratio (SINAD). Compensation methods for different types of mismatches (gain, offset, jitter and timing) are available, for which a survey can be found in [3].

In this paper we consider the problem of timing mismatch compensation, which is the difference between the ideal and the real sampling time of each channel ADC. A first step in a timing mismatch compensation strategy consists in estimating the mismatches. To this end, a number of methods are available [4], [5], [6]. The timing mismatch information is then used to design a compensation, which can be done using different available approaches. An early method carries out

the compensation in the frequency domain [7]. The drawback of this approach is that, being a frequency domain method, it theoretically requires the batch processing of the “whole history” of the sampled signal, which prevents its usage for real time applications. To go around this issue, a number of methods have been proposed which carry out the compensation using multirate filterbanks [8], [9], [10]. We give an overview of these methods in Section III.

A common assumption of the methods in [8], [9], [10] is that the signal to be sampled is bandlimited. Under this assumption, all methods are able to achieve perfect reconstruction (i.e., the contribution of the timing mismatch to the overall SINAD is completely removed), if the order of the compensation is not truncated. The arguable point of doing so is that this might not be a realistic assumption in many applications. To address this issue, we propose in this paper a filterbank-based method which uses the knowledge of the power spectrum of the signal to be sampled, to carry out a compensation in a statistically optimal (least-mean-squares (LMS)) sense. The proposed compensation is derived as a (matrix) Wiener filter [11]. However, we point out in Section V that, like the methods in [8], [9], [10], it is equivalent to a filterbank compensation. Since the proposed method is designed for non-bandlimited signals, it obviously cannot achieve perfect reconstruction in general. However, as we show in Section VI, it does so in the bandlimited case, if no constraints are made in the order of the compensation.

Throughout the paper we use the following:

**Notation:** Continuous-time signals are denoted using non-bold letters (e.g.,  $X(t)$ ) and discrete-time signals using bold letters (e.g.,  $\mathbf{X}(k)$ ). The  $z$ -transform of a discrete-time signal is denoted by  $\mathbf{X}(z)$ , and  $\mathbf{X}^*(z)$  denotes the transpose conjugate of  $\mathbf{X}(z)$ . Finally,  $\mathbf{X} * \mathbf{Y}$  denotes the convolution of discrete-time signals, i.e.,

$$(\mathbf{X} * \mathbf{Y})(k) = \sum_{l \in \mathbb{Z}} \mathbf{X}(l) \mathbf{Y}(k - l)$$

**Matrix pseudoinverse:** Let  $M^\dagger$  denote the (Moore-Penrose) pseudoinverse [12] of the matrix  $M$ . For any  $M$ , the following conditions hold:

$$MM^\dagger M = M \tag{1}$$

$$M^\dagger MM^\dagger = M^\dagger \tag{2}$$

$$(M^\dagger)^* = (M^*)^\dagger =: M^{\dagger*} \tag{3}$$

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Also, if  $M$  has a left inverse (i.e., there exists  $M^L$  such that  $M^L M = I$ ), then

$$(MN)^\dagger = N^\dagger M^\dagger \quad (4)$$

## II. TIME-INTERLEAVED ADCS

The time-interleaved ADC scheme is depicted in Fig. ???. The continuous-time input signal  $x(t)$  is a stationary random process which is sampled using  $D$  slow rate channel ADCs, operating at sampling frequency  $1/DT$ , but having different sampling phases. The  $d$ -th ADC's sampling phase is denoted by  $t_d$ , i.e., its output  $y_d(k)$  is given by

$$y_d(k) = x(t_d + kDT) \quad (5)$$

The outputs  $y_d(k)$ ,  $d = 1, \dots, D$  are then multiplexed to generate the time-interleaved ADC output  $y(k)$ , which has an average rate of  $1/T$ .

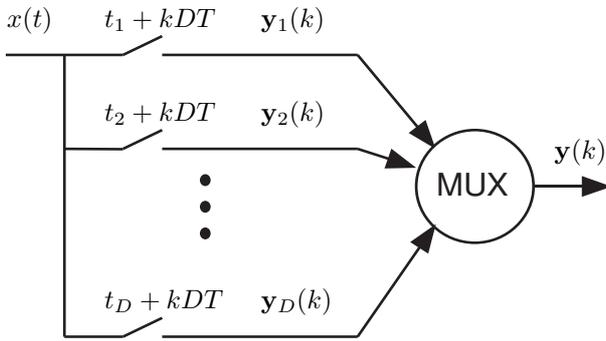


Fig. 1. Time-interleaved analog-to-digital converter scheme.

Obviously, if the sampling phases  $t_d$  satisfy

$$t_d = (d-1)T, \quad d = 1, \dots, D \quad (6)$$

then  $y(k)$  equals the samples  $z(k)$  that would be obtained by using a fast ADC of sampling frequency  $1/T$ , i.e.,

$$y(k) = z(k) := x(kT) \quad (7)$$

However, (6) cannot always be guaranteed in practice, and therefore, an estimate  $\hat{z}(k)$  of the regular samples  $z(k)$  need to be constructed from the available samples  $y_d(k)$ ,  $d = 1, \dots, D$ .

## III. METHODS FOR TIMING MISMATCH COMPENSATION

As mentioned in Section I, a number of filterbank-based methods have been proposed to address the timing mismatch compensation problem described in Section II. A general scheme describing all of them is shown in Fig. ???. In this scheme, the arrangement of  $M$  channel ADCs, with its associated sampling phases, is considered as an analysis filterbank with continuous-time input and discrete-time output, formed by the filters  $e^{t_m s}$ ,  $m = 1, \dots, M$  (with  $s$  denoting the Laplace variable), whose outputs are synchronously sampled at frequency  $1/DT$ . Notice that this scheme permits the use of oversampling (i.e.,  $M > D$ ) in the design of the time-interleaved ADC. The compensation is then done by using a

synthesis filterbank, which is implemented by an upsampling operation (i.e.,  $D-1$  zero valued samples are added between every two samples), then filtering each component using the array of filters  $f_m(z)$   $m = 1, \dots, M$ , and finally adding together all the resulting signals.

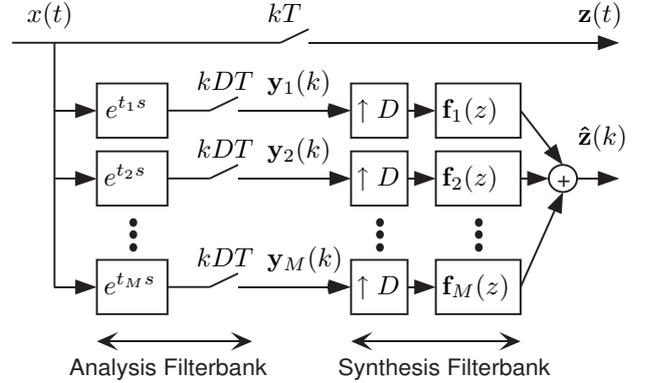


Fig. 2. Filterbank-based timing mismatch compensation scheme.

Using the scheme in Fig. ??, we give a brief overview of the available methods below.

**EO:** Yao and Thomas proposed in [13] a formula for perfect reconstruction of a bandlimited continuous-time signal which is irregularly sampled with an average rate higher than or equal to the Nyquist rate. It was shown in [8] that if the sampling grid is periodic (as is the case in a time-interleaved ADC), and the continuous-time signal needs only be reconstructed in a regular grid (as in (7)), then the reconstruction formula is equivalent to a filterbank-based reconstruction, as shown in Fig. ???. In the resulting filterbank,  $M = D$  and the filters  $f_m(z)$   $m = 1, \dots, M$  are given by

$$f_m(e^{j\omega}) = \frac{1}{T} \hat{f}_m\left(\frac{\omega}{T}\right) e^{-j\frac{\omega t_m}{T}}, \quad |\omega| < \pi$$

where  $\hat{f}_m(\omega)$  denotes the Fourier transform of

$$f_m(t) = \text{sinc}\left(\frac{\pi t}{MT}\right) \prod_{n=1, n \neq m}^M \frac{\sin\left(\frac{\pi(t+t_m-t_n)}{MT}\right)}{\sin\left(\frac{\pi(t_m-t_n)}{MT}\right)}$$

and  $\text{sinc}(x) = \sin(x)/x$ .

**JL:** In [9] the synthesis filterbank is designed using digital fractional delay filters. More precisely,

$$f_m(e^{j\omega}) = a_m e^{-j\frac{\omega t_m}{T}}, \quad m = 1, \dots, M, \quad |\omega| < \pi$$

and the coefficients  $a_m$ ,  $m = 1, \dots, M$  are designed to minimize the reconstruction error. It is shown that if the input signal  $x(t)$  is bandlimited to  $|f| \leq f_0$  and

$$f_0 \leq \frac{D+1}{4MT}, \quad (8)$$

the proposed scheme achieves perfect reconstruction.

**PLH:** A different approach is used in [10], where in order to design the compensation, the continuous-input/discrete-output analysis filterbank in Fig. ?? is replaced by a discrete-input/discrete-output filterbank, formed by digital fractional delay filters, i.e.,

$$\mathbf{h}_m(e^{j\omega}) = e^{j\frac{\omega t_m}{T}}, \quad m = 1, \dots, M, \quad |\omega| < \pi$$

Then, the synthesis filterbank is designed to minimize the reconstruction error of a hypothetical discrete-time signal applied to the input of the filters  $\mathbf{h}_m$ ,  $m = 1, \dots, M$ . This analysis filterbank substitution introduces no error if  $x(t)$  is bandlimited to  $|f| \leq 1/T$ , in which case, this scheme is able to achieve perfect reconstruction if  $M \geq D$ .

#### IV. PROPOSED METHOD

The measured sampling grid  $\{t_m + kDT : k \in \mathbb{Z}, m = 1, \dots, M\}$  is irregular and periodic with period  $DT$ , while the desired grid  $\{kT : k \in \mathbb{Z}\}$  is regular with rate  $1/T$ . Now, both grids can be turned into regular sampling grids of rate  $1/DT$  taken on the vector signals

$$\begin{aligned} \mathbf{Y}(t) &= [x(t_1 + t), x(t_2 + t), \dots, x(t_M + t)]^T \\ \mathbf{Z}(t) &= [x(t), x(T + t), \dots, x((D-1)T + t)]^T \end{aligned}$$

Hence, we can restate the problem as that of estimating  $\mathbf{Z}(k) = \mathbf{Z}(kDT)$  from  $\mathbf{Y}(k) = \mathbf{Y}(kDT)$ . This is a classical problem in estimation theory, and since the (vector) signals  $\mathbf{Y}(k)$  and  $\mathbf{Z}(k)$  are stationary random processes, the solution is given by the so-called Wiener filter [11]. More precisely, the estimate  $\hat{\mathbf{Z}}(k)$  of  $\mathbf{Z}(k)$  is given by

$$\hat{\mathbf{Z}}(k) = (\mathbf{W} * \mathbf{Y})(k)$$

where  $\mathbf{W}(k)$  denotes the Wiener filter (matrix) impulse response, which is calculated to minimize the power of the reconstruction error signal  $\mathbf{E}(k) = \mathbf{Z}(k) - \hat{\mathbf{Z}}(k)$ , i.e.,

$$\mathbf{W} = \arg \min_{\mathbf{W}'} \mathcal{E} \left\{ |\mathbf{Z}(0) - (\mathbf{W}' * \mathbf{Y})(0)|^2 \right\} \quad (9)$$

where  $\mathcal{E}\{\cdot\}$  denotes expected value. The resulting scheme is shown in Fig. ??.

To derive an expression for  $\mathbf{W}(k)$ , let  $\mathbf{R}_{\mathbf{Y}}(k)$  and  $\mathbf{R}_{\mathbf{ZY}}(k)$  denote the correlation matrix of  $\mathbf{Y}(k)$  and the cross-correlation matrix between  $\mathbf{Z}(k)$  and  $\mathbf{Y}(k)$ , respectively, i.e.,

$$\mathbf{R}_{\mathbf{Y}}(k) = \mathcal{E}\{\mathbf{Y}(k)\mathbf{Y}^*(0)\} \quad (10)$$

$$\mathbf{R}_{\mathbf{ZY}}(k) = \mathcal{E}\{\mathbf{Z}(k)\mathbf{Y}^*(0)\} \quad (11)$$

and let  $r_x(t)$  denote the autocorrelation of  $x(t)$ , i.e.,

$$r_x(t) = \mathcal{E}\{x(t)\overline{x(0)}\}$$

We have that, for each  $m, n = 1, \dots, M$ ,

$$\begin{aligned} [\mathbf{R}_{\mathbf{Y}}]_{m,n}(k) &= \mathcal{E}\{[\mathbf{Y}]_m(k)\overline{[\mathbf{Y}]_n(0)}\} \\ &= \mathcal{E}\{x(t_m + kMT)\overline{x(t_n)}\} \\ &= r_x(kMT + t_m - t_n) \end{aligned}$$

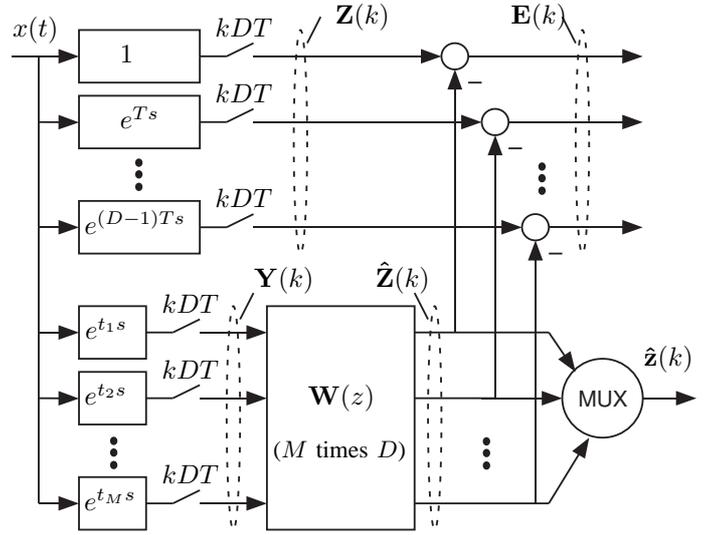


Fig. 3. Proposed compensation scheme.

and for each  $d = 1, \dots, D$ ,

$$\begin{aligned} [\mathbf{R}_{\mathbf{ZY}}]_{d,m}(k) &= \mathcal{E}\{[\mathbf{Z}]_d(k)\overline{[\mathbf{Y}]_m(0)}\} \\ &= \mathcal{E}\{x((d-1)T + kDT)\overline{x(t_m)}\} \\ &= r_x(kDT + (d-1)T - t_m) \end{aligned}$$

Then, the solution of (9) is given by

$$\mathbf{W}(z) = \mathbf{S}_{\mathbf{ZY}}(z)\mathbf{S}_{\mathbf{Y}}^\dagger(z) \quad (12)$$

where

$$\begin{aligned} \mathbf{S}_{\mathbf{Y}} &= \mathcal{Z}\{\mathbf{R}_{\mathbf{Y}}\} \\ \mathbf{S}_{\mathbf{ZY}} &= \mathcal{Z}\{\mathbf{R}_{\mathbf{ZY}}\} \end{aligned}$$

#### V. INTERPRETATION AS A FILTERBANK-BASED COMPENSATION METHOD

As mentioned in Section I, the proposed compensation method is equivalent to a filterbank-based method as depicted in Fig. ?. Using the so-called polyphase representation of a filterbank [14], we can interpret the  $M \times D$  transfer matrix  $\mathbf{W}(z)$  as the polyphase representation of a synthesis filterbank with upsampling factor  $D$  and filters

$$\mathbf{f}_m(z) = \sum_{d=0}^{D-1} z^d \mathbf{W}_{m,d}(z^D), \quad m = 1, \dots, M \quad (13)$$

where  $\mathbf{W}_{m,d}(z^D)$  denotes the  $(m, d)$ -entry of the transfer matrix  $\mathbf{W}(z)$ . Then, the proposed method provides, for any value of  $M$  and  $D$ , an optimal reconstruction of  $\mathbf{z}(k)$  from the samples  $\mathbf{y}(k)$ , and for a prescribed input power spectrum  $\phi_x = \mathcal{Z}\{r_x\}$ .

A common feature of the methods in Section III is that they achieve perfect reconstruction if  $x(t)$  is bandlimited. In the next section we show that the proposed method also enjoys this property if  $M \geq D$ .

## VI. BANDLIMITED CASE

Let  $x(t)$  be bandlimited with support in  $|f| < 1/T$ . Then, we can write

$$x(t) = \sum_k x(kT) \text{sinc} \left( \frac{t}{T} - K \right)$$

Consider the discrete-time  $M \times D$  transfer matrix  $\mathbf{A}(z)$ , whose  $(m, d)$ -entry  $\mathbf{A}_{m,d}(z)$  has impulse response

$$\mathbf{A}_{m,d}(k) = \text{sinc} \left( kD + \frac{t_m}{T} - (d-1) \right)$$

Then, it is straightforward to see that

$$\mathbf{S}_Y(z) = \mathbf{A}(z)\mathbf{S}_Z(z)\mathbf{A}^*(z) \quad (14)$$

$$\mathbf{S}_{ZY}(z) = \mathbf{S}_Z(z)\mathbf{A}^*(z) \quad (15)$$

We need the following lemma:

*Lemma 1:* If  $x(t)$  is bandlimited with support in  $|f| < 1/T$  and  $M \geq D$ , then

$$\mathbf{S}_{ZY}(z)\mathbf{S}_Y^\dagger(z)\mathbf{S}_{YZ}(z) = \mathbf{S}_Z(z)$$

*Proof:* It was shown in [13] that a bandlimited signal can be perfectly reconstructed from its irregular samples if the average sampling rate is greater than or equal to the Nyquist rate. In our context, this implies that  $\mathbf{A}(z)$  has a left inverse on the unit circle. Hence, for all  $|z| = 1$ ,

$$\mathbf{A}^\dagger(z)\mathbf{A}(z) = \mathbf{I}$$

Now, from (14) and (4), it follows that

$$\mathbf{S}_Y^\dagger(z) = \mathbf{A}^{\dagger*}(z)\mathbf{S}_Z^\dagger(z)\mathbf{A}^\dagger(z) \quad (16)$$

Combining (15) and (16) we have that

$$\begin{aligned} \mathbf{S}_{ZY}(z)\mathbf{S}_Y^\dagger(z)\mathbf{S}_{YZ}(z) &= \mathbf{S}_Z(z)\mathbf{S}_Z^\dagger(z)\mathbf{S}_Z(z) \\ &= \mathbf{S}_Z(z) \end{aligned}$$

where the second equality follows from (1).  $\blacksquare$

The  $z$ -transform  $\mathbf{S}_E(z)$  of the autocorrelation matrix  $\mathbf{R}_E$  of the error signal  $\mathbf{E}(k) = \mathbf{Z}(k) - \hat{\mathbf{Z}}(k)$  is given by

$$\begin{aligned} \mathbf{S}_E(z) &= \mathbf{S}_Z(z) - \mathbf{S}_{ZY}(z)\mathbf{W}^*(z) \\ &\quad - \mathbf{W}(z)\mathbf{S}_{YZ}(z) + \mathbf{W}(z)\mathbf{S}_Y(z)\mathbf{W}^*(z) \end{aligned} \quad (17)$$

Now, using (12) and Lemma 1, we have that

$$\begin{aligned} \mathbf{W}(z)\mathbf{S}_{YZ}(z) &= \mathbf{S}_{ZY}(z)\mathbf{S}_Y^\dagger(z)\mathbf{S}_{YZ}(z) \\ &= \mathbf{S}_Z(z) \end{aligned} \quad (18)$$

and

$$\begin{aligned} \mathbf{S}_{ZY}(z)\mathbf{W}^*(z) &= (\mathbf{W}(z)\mathbf{S}_{YZ}(z))^* \\ &= \mathbf{S}_Z^*(z) \\ &= \mathbf{S}_Z(z) \end{aligned} \quad (19)$$

Also, from (2) and Lemma 1 it follows that

$$\begin{aligned} \mathbf{W}(z)\mathbf{S}_Y(z)\mathbf{W}^*(z) &= \mathbf{S}_{ZY}(z)\mathbf{S}_Y^\dagger(z)\mathbf{S}_Y(z)\mathbf{S}_Y^\dagger(z)\mathbf{S}_{YZ}(z) \\ &= \mathbf{S}_{ZY}(z)\mathbf{S}_Y^\dagger(z)\mathbf{S}_{YZ}(z) \\ &= \mathbf{S}_Z(z) \end{aligned} \quad (20)$$

Finally, putting (18), (19) and (20) in (17), we have that

$$\begin{aligned} \mathbf{S}_E(z) &= \mathbf{S}_Z(z) - \mathbf{S}_Z(z) - \mathbf{S}_Z(z) + \mathbf{S}_Z(z) \\ &= 0 \end{aligned}$$

implying that the proposed compensation achieves perfect reconstruction.

## VII. SIMULATION

In order to evaluate the proposed compensation method we compare its performance with those of the methods EO, JL and PLH described in Section III. To this end we consider the example used in [9], [10], which uses 5 channel ADCs ( $M = 5$ ) with no oversampling ( $D = 5$ ), and with sampling phases  $[0, 0.96T, 2.02T, 2.99T, 4.03T]$ . This corresponds to the following timing mismatches  $[0, -0.04T, 0.02T, -0.01T, 0.03T]$ . For simplicity, the sampling period is  $T = 1$ . However, as opposite to the example in [9], [10], we consider the input signal to be a random process instead of a sum of sinusoids. This is a more realistic assumption on the input signal, which corresponds to the stochastic setting used to derive the proposed compensation method.

The filters  $\mathbf{f}_m(z)$   $m = 1, \dots, M$  in the EO, JL and PLH methods, as well as the Wiener filter  $\mathbf{W}(z)$  in the proposed method, have in theory infinite order, and therefore need to be truncated. Following the design in [10] we have truncated the synthesis filters to 150 taps, and we have truncated the order of  $\mathbf{W}(z)$  so that its equivalent synthesis filterbank (13) has the same number of taps.

In order to quantify the performance we use the inverse of the SINAD, i.e.,

$$\text{SINAD}^{-1} = 10 \log_{10} \left( \frac{\sum_{t=1}^N |\mathbf{z}(t) - \hat{\mathbf{z}}(t)|^2}{\sum_{t=1}^N |\mathbf{z}(t)|^2} \right)$$

In the first simulation we compare the performances of the different methods, for several values of  $\omega_c$ , when the input signal is generated as filtered white noise using a Butterworth lowpass filter (input filter) of 5-th order. The frequency response of one of such filters, with cutoff frequency  $\omega_c = 0.3Hz$ , is shown in Fig. ???. The result is shown in Fig. ???. We see that the proposed method outperforms the EO, JL and PLH methods, in the whole cutoff frequency range, but its extra performance is not so clear for high cutoff frequency values. The reason for this is that, for these cutoff frequency values, the performance of all methods is limited by the amount signal power above the Nyquist frequency.

In the second simulation we repeat the experiment considering a Butterworth lowpass filter of 20-th order for the input filter. In this case, we observe that the performance of the proposed method deteriorates at low cutoff frequency

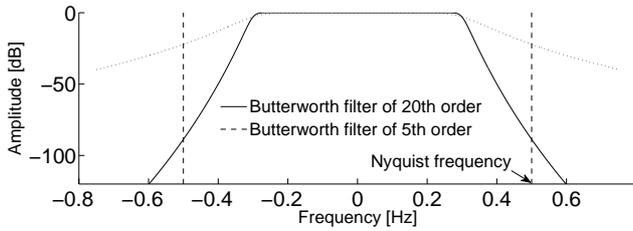


Fig. 4. Frequency response of a 5-th and 20-th order Butterworth input filter with cutoff frequency 0.3Hz.

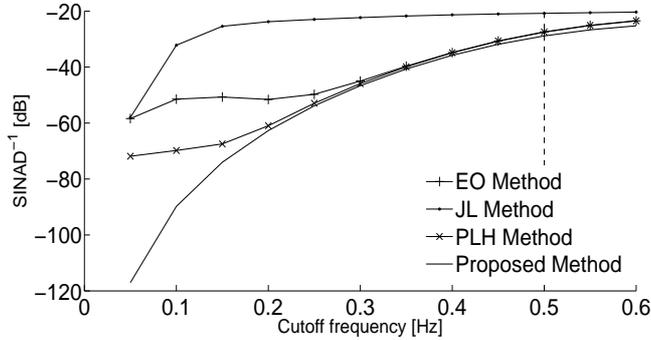


Fig. 5. Performance comparison of different compensation methods, using a 5-th order Butterworth input filter.

values. This due to the truncation of the very high order Wiener filter  $\mathbf{W}(z)$ , which is induced, via  $\mathbf{S}_Y^T(z)$ , by the large zero regions in the power spectrum of the input signal. This drawback is not serious from a practical point of view, since it corresponds to a rather unusual application of a high frequency sampler. However, it illustrates how the performance of the proposed method can be undermined if the input power spectrum has large zero regions. If necessary, this drawback can be eliminated, by designing  $\mathbf{W}(z)$  as a constrained Wiener filter, taking into account the desired truncation order. Due to space limitations, this issue will be addressed in a journal version.

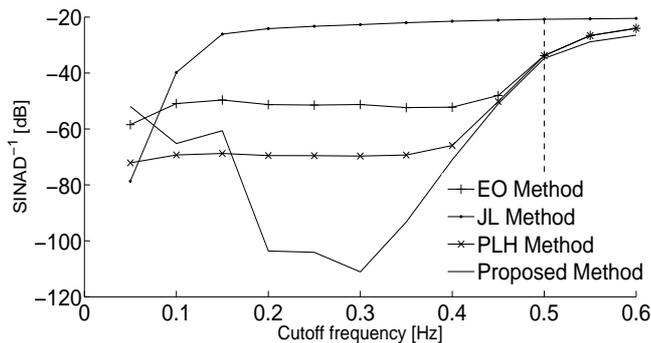


Fig. 6. Performance comparison of different compensation methods, using a 20-th order Butterworth input filter.

**Remark:** The performance of the JL method is rather poor in the high frequency range. The reason for this is that,

according to (8), the method is suitable for signals with cutoff frequency smaller than  $0.3Hz$  (instead of  $0.5Hz$  as for the EO and PLH methods). On the other hand, for very low frequencies, it outperforms the other methods.

## VIII. CONCLUSION

We have proposed a compensation method for timing mismatches in time-interleaved ADCs. As opposite to other approaches, the proposed method does not require the input signal to be bandlimited, hence, perfect reconstruction is not possible in general. To deal with the non-bandlimited assumption, the compensation was designed in a statistically optimal (LMS) sense, i.e., to minimize the power of the reconstruction error in the samples, for a prescribed input signal power spectrum. While the compensation was designed as a Wiener filter, we showed that it is equivalent to a filterbank-based compensation. Also, we showed that, under the bandlimited assumption, the proposed method achieves perfect reconstruction, if no constraints are made in the order of the compensation. Simulation results show that the proposed compensation outperforms the other methods, in terms of the SINAD, for a given fixed compensation order, except for signal having large spectral zero regions. Due to space limitations, this unusual situation will be dealt with in a journal version.

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