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Reduced-order H_∞ filtering for discrete-time singular systems with lossy measurements

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Abstract: In this study, the authors consider an H_∞ filtering problem for discrete-time singular systems with lossy measurements. The authors introduce the stochastic variable satisfying Bernoulli random binary distribution to model the measured outputs. This measurement mode can be used to characterise the effect of data-loss in information transmissions across limited bandwidth communication channels over a wide area. The authors design a filter to cope with the losses, which ensures not only the mean-square stochastic stability but also a prescribed H_∞ filtering performance for filtering the error singular system. They also derive sufficient conditions for the existence of such a filter. Finally, the authors give a numerical example to illustrate the effectiveness of the proposed approach.

1 Introduction

Packet-based transmission of data over a wireless network increases bit error rates relative to wired links. The use of a wireless network will lead to measurement losses or delays of the communicated information and may deteriorate the performance or cause instability as pointed out in [1–4]. Recently, there has been some attention to the research of systems with lossy measurements, many results have appeared in the literature to model measurement losses or network delays. A randomly varying delayed sensor mode was first introduced by Ray [5]. Since then, empirical observations have also been used to develop probabilistic characterisations of measurement losses [6, 7]. Nilsson *et al.* [8] assumed that measurement losses have statistically mutually independent transfer-to-transfer probability distribution. And more recently, measurement losses have been considered as white in nature with Bernoulli random binary distribution [9, 10]. In the meantime, Markov chains are used to describe probabilistic losses in [2, 3, 11], which is assumed that the transmitted packet can be classified as lost or received at the receiving end of channel, the expected Markov Chain state can be described in terms of a probability on its state space. A Markov chain with a large number of states can be used to represent the complicated loss behaviour.

It is worth mentioning that the plants are assumed to be regular in the above mentioned literature. However, they may occur in a singular way since singular systems have extensive applications in large-scale systems, economic systems and electrical networked systems [12]. A singular system is also referred to as descriptor system, generalised state-space system or semistate system. Over the past decades, there has been a growing interest in the research of singular systems, some fundamental results based on the theory of normal systems have been successfully extended to singular systems, such as controllability and observability [13], H_∞ control [14–16], positive realness [17] and so on. Recently, the study of H_∞ filtering problem for singular systems has gained growing interest. In [18], an LMI-based filter design approach was proposed for impulsive stochastic systems, and based on the projection lemma, the reduced-order H_∞ filtering problem was investigated in [19]. On the basis of the admissibility assumption of the uncertain singular systems, an H_∞ singular filter design method is proposed in [20] whereas a delay-dependent result for this problem has been reported in [21–23]. However, to our knowledge, there are no research results on H_∞ filtering for discrete-time singular systems with lossy measurements up to now.

Motivated by the works in [9–11], this paper focuses on the reduced-order H_∞ filtering problem for discrete-time singular systems with lossy measurements. The system plant is discrete-time singular and lossy measurements are subject to stochastic variables satisfying Bernoulli random binary distribution. We are interested in designing reduced-order or zeroth-order filters to cope with the losses such that the filtering error singular system is regular, causal and exponentially mean-square stable and a prescribed H_∞ filtering performance is achieved. We show that the reduced-order filtering problem can effectively be solved by solving the non-convex optimisation problem based on alternating projections which are dependent on the probability of event packet-loss, and the zeroth-order H_∞ filtering problem can be solved by solving a convex minimisation problem. We also obtain that a better H_∞ filtering performance is achieved when less packet-loss occurs by an illustrative numerical example.

The rest of this paper is organised as follows. Section 2 formulates the H_∞ filtering problem. In Section 3, a reduced-order H_∞ filtering approach is proposed. A numerical example is given to demonstrate the effectiveness of the proposed method in Section 4, which is followed by conclusions in Section 5.

Notations: Throughout this paper, $\mathcal{C}$ denotes the complex plane; Z^+ denotes the set of positive integers; $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. A real symmetric matrix $P > 0\ (\geq 0)$ denotes P being a positive definite (or positive semi-definite) matrix, and $A > (\geq) B$ means $A - B > (\geq) 0$. I denotes an identity matrix of appropriate dimension. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations. The superscript 'T' represents the transpose. For a matrix N, $N^{-\mathrm{T}}$ stands for the transpose of matrix N^{-1}. * is used as an ellipsis for terms that are induced by symmetry. The notation $l_2[0, \infty)$ represents the space of square summable infinite vector sequences with the usual norm $\|\cdot\|_2$. A sequence $v = \{v_k\} \in l_2[0, \infty)$ if $\|v\|_2 = \sqrt{\sum_{i=1}^{\infty} v_k^{\mathrm{T}} v_k} < \infty$. $\mathrm{Prob}\{\cdot\}$ stands for the occurrence probability of an event; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure. For a matrix $M \in \mathbb{R}^{n\times m}$ with rank r, the orthogonal complement $M^{\perp}$ is defined as (possibly non-unique) $(n-r)\times n$ matrix such that $M^{\perp}M = 0$ and $M^{\perp}M^{\perp\mathrm{T}} > 0$. $D_{\mathrm{int}}(0, 1)$ is the interior of the unit disk with centre at the origin; $\sigma(E, A) = \{z|\det(zE - A) = 0\}$.

2 Problem formulation and preliminaries

Consider the networked filtering system with measurements communicated from a remote sensor shown in Fig. 1.

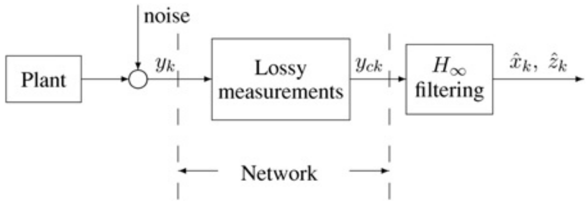

Figure 1 *Filtering using measurements communicated from a remote sensor*

The plant is discrete-time singular system described by

$$Ex_{k+1} = Ax_k + A_w w_k \tag{1}$$
$$z_k = Lx_k + L_w w_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state; $w(k) \in \mathbb{R}^w$ is the disturbance signal in $l_2[0, \infty)$; $z_k \in \mathbb{R}^q$ is the signal to be estimated; A, A_w, L and L_w are known real matrices with appropriate dimensions, and E may be singular matrix, we shall assume that rank $E = r \leq n$. The measurement with data-loss is described by

$$\begin{cases} y_k = Cx_k + Dv_k \\ y_{ck} = (1-\theta_k)y_k + \theta_k y_{k-1} \end{cases} \tag{3}$$

where $y_k \in \mathbb{R}^p$ is the output, $y_{ck} \in \mathbb{R}^p$ is the measured output, $v_k \in \mathbb{R}^v$ is measured noise in $l_2[0, \infty)$, C is a known matrix; the stochastic variable θ_k is a Bernoulli distributed white sequence taking value on 0 and 1, which stands for the effect of data-loss, when data are lost, $\theta_k = 1$, when no data-loss occurs, $\theta_k = 0$. It is described as

$$\mathrm{Prob}\{\theta_k = 1\} \equiv \mathcal{E}\{\theta_k\} = \rho \tag{4}$$
$$\mathrm{Prob}\{\theta_k = 0\} \equiv \mathcal{E}\{1-\theta_k\} = 1-\rho \tag{5}$$

where $\rho \in [0, 1]$ and is a known constant.

Remark 1: In [2, 11], the random packet losses are assumed to be governed by multi-state Markov chains, where transmitted packet can be classified as lost (L) or received (R) at the receiving end of channel, and $\theta_k \in \{L, R\}$. The expected Markov Chain state at time index k can be described in terms of a probability on its state space. Therefore a Markov chain with a large number of states can be used to represent a more complicated loss behaviour. In (4) and (5), a Bernoulli distributed white sequence (see [9]) taking on values 0 and 1 is used to describe the random packet losses in this paper, that is, when data are lost, $\theta_k = 1$, when no data-loss occurs, $\theta_k = 0$. Furthermore, only the probability of $\theta_k = 1$ and $\theta_k = 0$ is considered, which thus lead to the fact that this is not a more precise model than that in [2, 11] for network transmission. However, the binary random model has gained considerable research interests because of its simplicity and practicality in describing networked-induced lossy measurements (see [5, 9, 10]). The system measurements with data-loss modelled in (4) and (5) was first introduced in [5] and has been used

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to characterise the effect of communication delays and/or data-loss in information transmissions across limited bandwidth communication channels over a wide area such as navigating a vehicle based on the estimations from a sensor web of its current position and velocity [1].

Remark 2: The output y_k produced at a time k is sent to the observer through a communication channel. If no packet-loss occurs, the measurement output y_{ck} takes value y_k; otherwise, the measurement output y_{ck} takes value y_{k-1}. When the probability of event packet-loss occurring is assumed as ρ, the measurement output y_{ck} in (4) thus takes the value y_k with probability $1-\rho$, and the value y_{k-1} with probability ρ.

Throughout this paper, we use the following definitions.

Definition 1 [9]: The singular system (1) is said to be exponentially mean-square stable if with $w_k = 0$, there exist constants $\alpha > 0$ and $\tau \in (0, 1)$ such that

$$\mathcal{E}\{\|x_k\|^2\} \leq \alpha\tau^k \mathcal{E}\{\|x_0\|^2\}, \quad \text{for all } x_0 \in \mathbb{R}^n, \quad k \in Z^+$$

Definition 2 [15]:

(1) The singular system (1) with $w_k = 0$ is said to be regular, that is, the pair (E, A) is regular if $\det(zE - A)$ is not identically zero.

(2) The singular system (1) with $w_k = 0$ is said to be causal, that is, the pair (E, A) is casual if $\deg(\det(zE - A)) = \operatorname{rank} E$.

(3) The singular system (1) with $w_k = 0$ is said to be stable, that is, the pair (E, A) is stable if $\sigma(E, A) \subset D_{\text{int}}(0, 1)$.

(4) The singular system (1) with $w_k = 0$ is admissible if the pair (E, A) is regular, casual and stable.

Definition 3: The singular system (1) is exponentially mean-square admissible if it is regular, casual and exponentially mean-square stable.

For the delayed sensor mode (3), we assume that $x_{-1} = 0$, which implies that $y_{-1} = 0$.

We are concerned with the following reduced-order filter with order $\hat{n}$ for the estimation of z_k

$$\begin{cases} \hat{x}_{k+1} = A_f\hat{x}_k + B_f y_{ck} \\ \hat{z}_k = C_f\hat{x}_k + D_f y_{ck} \end{cases} \tag{6}$$

where $\hat{x}_k \in \mathbb{R}^{\hat{n}}$, $\hat{z}_k \in \mathbb{R}^q$ and $\hat{n} \leq n$. A_f, B_f, C_f and D_f are matrices to be determined. Combining (1)–(3) and (6) the filtering error dynamics is given by

$$\begin{cases} \bar{E}\bar{x}_{k+1} = \mathcal{A}(\theta_k)\bar{x}_k + \mathcal{A}_1(\theta_k)H\bar{x}_{k-1} + \mathcal{A}_\varpi(\theta_k)\varpi_k \\ \bar{z}_k = \mathcal{L}(\theta_k)\bar{x}_k + \mathcal{L}_1(\theta_k)H\bar{x}_{k-1} + \mathcal{L}_\varpi(\theta_k)\varpi_k \end{cases} \tag{7}$$

where

$$\bar{x}_k = \begin{bmatrix} x_k^T & \hat{x}_k^T \end{bmatrix}^T, \bar{z}_k = z_k - \hat{z}_k, \quad \varpi_k = \begin{bmatrix} w_k^T & v_k^T & v_{k-1}^T \end{bmatrix}^T,$$
$$H = \begin{bmatrix} I, & 0 \end{bmatrix} \tag{8}$$

(see (9))

The H_∞ filtering problem addressed in this paper is to design a filter in the form of (6) such that for a given scalar γ and all non-zero ϖ_k, the filtering error singular system (7) is regular, casual and exponentially mean-square stable and under the zero initial condition, the filtering error $\bar{z}_k$ satisfies

$$\sum_{k=0}^{\infty} \mathcal{E}\{\|\bar{z}_k\|^2\} \leq \gamma^2 \sum_{k=0}^{\infty} \|\varpi_k\|^2 \tag{10}$$

In such a case, the filtering error singular system is said to be exponentially mean-square admissible with H_∞ filtering performance γ.

Remark 3: We assume that the discrete-time singular system (1) is regular, casual and stable and the filter (6) is stable with the order $\hat{n}$ satisfying $\hat{n} \leq n$ throughout this paper.

Remark 4: If $\hat{n} = 0$, then the reduced-order filter in (6) becomes

$$\hat{z}_k = D_f y_{ck} \tag{11}$$

and the reduced-order H_∞ filtering problem reduces to the static or zeroth-order H_∞ filtering problem.

Now, we establish a condition of mean-square stability and H_∞ performance for the filtering error dynamics (7), which will play a key role in the derivation of our H_∞ filter design method.

Lemma 1: Given a scalar $\gamma > 0$, the filtering error singular system (7) is exponentially mean-square admissible with a guaranteed H_∞ filtering performance γ, if there exist matrices P and Q such that

$$\bar{E}^T P \bar{E} \geq 0 \tag{12}$$

$$\begin{cases} \bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \mathcal{A}(\theta_k) = \begin{bmatrix} A & 0 \\ (1-\theta_k)B_f C & A_f \end{bmatrix}, \quad \mathcal{A}_1(\theta_k) = \begin{bmatrix} 0 \\ \theta_k B_f C \end{bmatrix} \\ \mathcal{A}_\varpi(\theta_k) = \begin{bmatrix} A_w & 0 & 0 \\ 0 & (1-\theta_k)B_f D & -\theta_k B_f D \end{bmatrix}, \quad \mathcal{L}_\varpi(\theta_k) = \begin{bmatrix} L_w - (1-\theta_k)D_f D - \theta_k D_f D \end{bmatrix} \\ \mathcal{L}(\theta_k) = \begin{bmatrix} L - (1-\theta_k)D_f C, & -C_f \end{bmatrix}, \quad \mathcal{L}_1(\theta_k) = -\theta_k D_f C \end{cases} \tag{9}$$

$$\begin{bmatrix} P & 0 & P\mathcal{A}(\rho) & P\mathcal{A}_1(\rho) & P\mathcal{A}_{\varpi}(\rho) \\ * & I & \mathcal{L}(\rho) & \mathcal{L}_1(\rho) & \mathcal{L}_{\varpi}(\rho) \\ * & * & \bar{E}^{\mathrm{T}}P\bar{E} - H^{\mathrm{T}}QH & 0 & 0 \\ * & * & * & Q & 0 \\ * & * & * & * & \gamma^2 I \end{bmatrix} > 0 \quad (13)$$

where $*$ denotes the corresponding transposed block matrix due to symmetry and ρ-dependent matrices are defined as in (9) with θ_k replaced by ρ.

Proof: Firstly, we shall establish the regularity, causality of the filtering error singular system (7) with $\varpi_k = 0$. In this case, the dynamic equation in (7) becomes

$$\bar{E}\bar{x}_{k+1} = \mathcal{A}(\theta_k)\bar{x}_k + \mathcal{A}_1(\theta_k)H\bar{x}_{k-1} \quad (14)$$

Set

$$X_k = [\bar{x}_k^{\mathrm{T}} \quad \bar{x}_{k-1}^{\mathrm{T}}]^{\mathrm{T}} \quad (15)$$

then system (14) can be rewritten as

$$\hat{E}X_{k+1} = \hat{A}X_k \quad (16)$$

where (see (17))

It is easy to show that the regularity, causality of system (14) is equivalent to that of system (16). Now, we establish the regularity, causality of system (16). According to (9) and (17), we have

$$\begin{aligned} \det(z\hat{E} - \hat{A}) &= z^{\hat{n}+n} \cdot \det(z\bar{E} - \mathcal{A}(\theta_k) - z^{-1}\mathcal{A}_1(\theta_k)H) \\ &= z^{\hat{n}+n} \cdot \begin{vmatrix} zE - A & 0 \\ -(1-\theta_k + z^{-1}\theta_k)B_f C & zI - A_f \end{vmatrix} \\ &= z^{\hat{n}+n} \cdot |zE - A| \cdot |zI - A_f| \end{aligned} \quad (18)$$

From the assumption in Remark 3, we can obtain that the pair (E, A) is regular, causal and stable and A_f is Hurwitz, which implies that for sufficiently large z, $|zE - A| \neq 0$ and $|zI - A_f| \neq 0$. Therefore there exists a scalar $z \in \mathcal{C}$ such that $\det(z\hat{E} - \hat{A}) \neq 0$, this implies that the pair $(\hat{E}, \hat{A})$ is regular, that is, system (14) is regular. Furthermore, from (18), it can be seen that

$$\begin{aligned} \deg(\det(z\hat{E} - \hat{A})) &= 2\hat{n} + n + \mathrm{rank}E \\ &= \mathrm{rank}\bar{E} + \hat{n} + n = \mathrm{rank}\hat{E} \end{aligned}$$

Hence, it follows from Definition 2, that if the pair (E, A) is causal, this implies that system (14) is causal.

Next, we shall prove the exponentially mean-square stability of system (14). To this end, define a Lyapunov functional candidate as

$$V_k = \bar{x}_k^{\mathrm{T}}\bar{E}^{\mathrm{T}}P\bar{E}\bar{x}_k + \bar{x}_{k-1}^{\mathrm{T}}H^{\mathrm{T}}QH\bar{x}_{k-1} \quad (19)$$

Let $\mathcal{F}_k$ be the minimal σ-algebra generated by $\{\hat{x}_i, \ 0 \le i \le k\}$. By (14), (19) and some algebraic manipulations, we have

$$\mathcal{E}\{V_{k+1}|\mathcal{F}_k\} - V_k = \eta_k^{\mathrm{T}}\varpi\eta_k \quad (20)$$

where (see equation at the bottom of the page)

We can obtain from (13) and Schur complement formula that $\varpi < 0$, this implies from (20) that

$$\mathcal{E}\{V_{k+1}|\mathcal{F}_k\} - V_k = \eta_k^{\mathrm{T}}\varpi\eta_k \le -\lambda_{\min}(\varpi)\eta_k^{\mathrm{T}}\eta_k < -\alpha\eta_k^{\mathrm{T}}\eta_k \quad (21)$$

where

$$0 < \alpha < \min\{\lambda_{\min}(-\varpi), \sigma\}$$

$$\sigma = \max\{\lambda_{\max}(\bar{E}^{\mathrm{T}}P\bar{E}), \lambda_{\max}(Q)\}$$

Therefore it can be deduced from Lemma 1 in [9] that system (14) is exponentially mean-square stable.

$$\hat{E} = \left[\begin{array}{c|c} \bar{E} & 0_{(\hat{n}+n)\times(\hat{n}+n)} \\ \hline 0_{(\hat{n}+n)\times(\hat{n}+n)} & I_{(\hat{n}+n)\times(\hat{n}+n)} \end{array}\right], \quad \hat{A} = \left[\begin{array}{c|c} \mathcal{A}(\theta_k) & \mathcal{A}_1(\theta_k)H \\ \hline I_{(\hat{n}+n)\times(\hat{n}+n)} & 0_{(\hat{n}+n)\times(\hat{n}+n)} \end{array}\right] \quad (17)$$

$$\eta_k = [\bar{x}_k^{\mathrm{T}} \quad x_{k-1}^{\mathrm{T}}]^{\mathrm{T}}$$

$$\varpi = \begin{bmatrix} \mathcal{A}^{\mathrm{T}}(\rho)P\mathcal{A}(\rho) + H^{\mathrm{T}}QH - \bar{E}^{\mathrm{T}}P\bar{E} & \mathcal{A}^{\mathrm{T}}(\rho)P\mathcal{A}_1(\rho) \\ * & \mathcal{A}_1^{\mathrm{T}}(\rho)P\mathcal{A}_1(\rho) - Q \end{bmatrix}$$

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Finally, we shall show that filtering error $\bar{z}_k$ satisfies (10). Let

$$J_N = \sum_{k=0}^{N-1} [\mathcal{E}\{\|\bar{z}_k\|^2\} - \gamma^2 \|\varpi_k\|^2] \tag{22}$$

where N is a positive integer.

For any non-zero $\varpi_k \in l_2[0, +\infty)$ and zero initial condition, we have

$$J_N = \sum_{k=0}^{N-1} [\mathcal{E}\{\|\bar{z}_k\|^2\} - \gamma^2 \|\varpi_k\|^2 + \mathcal{E}\{V_{k+1} - V_k\}] \\ + \mathcal{E}V_0 - \mathcal{E}V_N \le \sum_{k=0}^{N-1} \mathcal{E}\{\xi_k^{\mathrm{T}} \mathcal{N}(\rho)\xi_k\} \tag{23}$$

where

$$\xi_k = \begin{bmatrix} \bar{x}_k^{\mathrm{T}}, & x_{k-1}^{\mathrm{T}}, & \varpi_k^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{24}$$

$$\mathcal{N}(\rho) = \begin{bmatrix} \mathcal{A}^{\mathrm{T}}(\rho) \\ \mathcal{A}_1^{\mathrm{T}}(\rho) \\ \mathcal{A}_\varpi^{\mathrm{T}}(\rho) \end{bmatrix} P \begin{bmatrix} \mathcal{A}(\rho) & \mathcal{A}_1(\rho) & \mathcal{A}_\varpi(\rho) \end{bmatrix} \\ + \begin{bmatrix} \mathcal{L}^{\mathrm{T}}(\rho) \\ \mathcal{L}_1^{\mathrm{T}}(\rho) \\ \mathcal{L}_\varpi^{\mathrm{T}}(\rho) \end{bmatrix} \begin{bmatrix} \mathcal{L}(\rho) & \mathcal{L}_1(\rho) & \mathcal{L}_\varpi(\rho) \end{bmatrix} \\ + \begin{bmatrix} H^{\mathrm{T}}QH - \bar{E}^{\mathrm{T}}P\bar{E} & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} \tag{25}$$

It follows from (13) and by Schur complement formula that $\mathcal{N}(\rho) < 0$. This implies that for any N, $J_N < 0$, which leads to that the filtering error $\bar{z}_k$ satisfies condition (10). This completes the proof. □

Remark 5: Lemma 1 provides a sufficient condition of exponentially mean-square admissibility and H_∞ performance for the filtering error dynamics (7). It should be pointed out that the inequality (13) is a non-linear matrix inequality such that it cannot be used to solve the parameters of the filter by the Matlab/LMI toolbox. Furthermore, in the case when $E = I$, that is, singular system (1) reduces to state-space one, the corresponding condition of exponentially mean-square stability and H_∞ performance is given by inequality (13) with $P > 0$, $Q > 0$ and $\bar{E}$ replaced by I.

Next, we shall analyse the problem of zeroth-order filtering proposed in Remark 3. Combining the zeroth-order filter (11) and system (1), the corresponding zeroth-order filtering error dynamics is described as

$$\begin{cases} Ex_{k+1} = Ax_k + A_{z\varpi}\varpi_k \\ \bar{z}_k = L_z(\theta_k)x_k + L_{z1}(\theta_k)x_{k-1} + L_{z\varpi}(\theta_k)\varpi_k \end{cases} \tag{26}$$

where

$$A_{z\varpi} = \begin{bmatrix} A_w & 0 & 0 \end{bmatrix}, \quad L_z(\theta_k) = L - (1-\theta_k)D_{\mathrm{f}}C \\ L_{z1}(\theta_k) = -\theta_k D_{\mathrm{f}} C, \\ L_{z\varpi}(\theta_k) = \begin{bmatrix} L_w & -(1-\theta_k)D_{\mathrm{f}}D & -\theta_k D_{\mathrm{f}} D \end{bmatrix}$$

Similar to Lemma 1, we present the results on the exponentially mean-square stability and H_∞ performance for zeroth-order filtering error dynamics (26) shown as the following lemma.

Lemma 2: Given a scalar $\gamma > 0$, the filtering error singular system (26) is exponentially mean-square admissible with a guaranteed H_∞ filtering performance γ, if there exist matrices P and Q such that

$$E^{\mathrm{T}}PE \ge 0 \tag{27}$$

$$\begin{bmatrix} P & 0 & PA & 0 & PA_{z\varpi} \\ * & I & L_z(\rho) & L_{z1}(\rho) & L_{z\varpi}(\rho) \\ * & * & E^{\mathrm{T}}PE - Q & 0 & 0 \\ * & * & * & Q & 0 \\ * & * & * & * & \gamma^2 I \end{bmatrix} > 0 \tag{28}$$

where ρ-dependent matrices are defined as in (26) with θ_k replaced by ρ.

Proof: The proof is similar to Lemma 1 and thus is omitted. □

3 Reduced-order H_∞ filtering

In this section, we shall present the results on the solvability of the reduced-order and zeroth-order H_∞ filtering problem based on Lemmas 1 and 2, respectively.

The following lemma is useful in the derivation of our main results in this section.

Lemma 3 [24, 25]: Given a symmetric matrix $\Lambda \in \mathbb{R}^{n\times n}$ and two matrices $\mathcal{X} \in \mathbb{R}^{n\times m}$ and $\mathcal{Y} \in \mathbb{R}^{k\times n}$ with rank $\mathcal{X} < n$ and rank $\mathcal{Y} < n$. Consider the problem of finding some matrix Δ such that

$$\Lambda + \mathcal{X}\Delta\mathcal{Y} + (\mathcal{X}\Delta\mathcal{Y})^{\mathrm{T}} > 0 \tag{29}$$

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Then (29) is solvable for Δ if and only if

$$\mathcal{X}^{\perp}\Lambda\mathcal{X}^{\perp\mathrm{T}} > 0, \quad (\mathcal{Y}^{\mathrm{T}})^{\perp}\Lambda(\mathcal{Y}^{\mathrm{T}})^{\perp\mathrm{T}} > 0 \tag{30}$$

Now, we shall give a sufficient condition on the solvability for the reduced-order H_∞ filtering problem shown as follows.

Theorem 1: There exists an $\hat{n}$-order filter in the form of (6) such that the reduced-order H_∞ filtering problem for the filtering error singular system (7) with lossy measurements is solvable if there exist matrices $X > 0$, $Y > 0$ and $Q > 0$ satisfying

$$E^{\mathrm{T}}XE \geq 0 \tag{31}$$

$$E^{\mathrm{T}}YE \geq 0 \tag{32}$$

$$\begin{bmatrix} Y & YA & 0 & YA_w & 0 & 0 \\ A^{\mathrm{T}}Y^{\mathrm{T}} & E^{\mathrm{T}}YE - Q & 0 & 0 & 0 & 0 \\ 0 & 0 & Q & 0 & 0 & 0 \\ A_w^{\mathrm{T}}Y^{\mathrm{T}} & 0 & 0 & \gamma^2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma^2 I & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma^2 I \end{bmatrix} > 0 \tag{33}$$

(see (34))

$$X - Y \geq 0 \tag{35}$$

and

$$\mathrm{rank}(X - Y) \leq \hat{n} \tag{36}$$

where (see equation at the bottom of the page)

In this case, the parameters of all desired filters with order $\hat{n}$ corresponding to a feasible solution (X, Y, Q) to (31)–(36)

$$\mathcal{M}(\rho)\left[\begin{array}{cc|c|cccc} X & 0 & XA & XA_w & 0 & 0 \\ 0 & I & L & L_w & 0 & 0 \\ \hline A^{\mathrm{T}}X^{\mathrm{T}} & L^{\mathrm{T}} & E^{\mathrm{T}}XE & 0 & 0 & 0 \\ \hline A_w^{\mathrm{T}}X^{\mathrm{T}} & L_w^{\mathrm{T}} & 0 & \gamma^2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma^2 I & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma^2 I \end{array}\right]\mathcal{M}^{\mathrm{T}}(\rho) > 0 \tag{34}$$

$$X - Y \geq 0$$

$$\mathcal{M}(\rho) = \left[\begin{array}{cc|c|ccc} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (1-\rho)C^{\mathrm{T}\perp} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\rho)D^{\mathrm{T}\perp} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho D^{\mathrm{T}\perp} \end{array}\right]$$

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are given by

$$\begin{bmatrix} D_f & C_f \\ B_f & A_f \end{bmatrix} = [-W^{-1}\Psi^T\Pi\Phi_r^T(\Phi_r\Pi\Phi_r^T)^{-1} + W^{-1}\Xi^{1/2} \times \Upsilon(\Phi_r\Pi\Phi_r^T)^{-1/2}]\Phi_l^+ + \Theta - \Theta\Phi_l\Phi_l^+ \quad (37)$$

where (see equation at the bottom of the page)

and Θ is any matrix with appropriate dimensions; Υ is any matrix satisfying $\bar{\sigma}(\Upsilon) < 1$, where $\bar{\sigma}(\cdot)$ is maximum singular value of a matrix; Φ_l and Φ_r are any full rank factors of Φ, that is $\Phi = \Phi_l\Phi_r$; and Φ_l^+ is the Moore–Penrose of Φ_l. Moreover, $X_1 \in \mathbb{R}^{n\times\hat{n}}$, $X_2 \in \mathbb{R}^{\hat{n}\times\hat{n}}$, $W > 0$ and $X_2 > 0$ satisfy

$$\Pi > 0, \quad X - Y = X_1X_2^{-1}X_1^T \geq 0$$

$$\Xi = W - \Psi^T\left[\Pi - \Pi\Phi_r^T(\Phi_r\Pi\Phi_r^T)^{-1}\Phi_r\Pi\right]\Psi$$

$$\Pi = (\Psi W^{-1}\Psi^T - \varpi)^{-1}$$

$$\varpi = \left[\begin{array}{cc|c|cc|c|ccc} X & X_1 & 0 & XA & 0 & 0 & XA_w & 0 & 0 \\ X_1^T & X_2 & 0 & X_1^TA & 0 & 0 & X_1^TA_w & 0 & 0 \\ \hline 0 & 0 & I & L & 0 & 0 & L_w & 0 & 0 \\ \hline A^TX^T & A^TX_1 & L^T & E^TXE - Q & E^TX_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_1^TE & X_2 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & Q & 0 & 0 & 0 \\ \hline A_w^TX & A_w^TX_1 & L_w^T & 0 & 0 & 0 & \gamma^2I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^2I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^2I \end{array}\right]$$

$$\Psi = \begin{bmatrix} 0 & X_1 \\ 0 & X_2 \\ -I & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0 & 0 & 0 & (1-\rho)C & 0 & \rho C & 0 & (1-\rho)D & \rho D \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}$$

Proof: It can be deduced from (12) and (13) that P is a positive definite symmetric matrix. Set

$$P = \begin{bmatrix} X & X_1 \\ X_1^{\mathrm{T}} & X_2 \end{bmatrix}, P^{-1} = \begin{bmatrix} Z & Z_1 \\ Z_1^{\mathrm{T}} & Z_2 \end{bmatrix} \tag{38}$$

then, we obtain from (12)

$$\begin{bmatrix} E^{\mathrm{T}}XE & E^{\mathrm{T}}X_1 \\ X_1^{\mathrm{T}}E & X_2 \end{bmatrix} \geq 0 \tag{39}$$

which implies that

$$E^{\mathrm{T}}XE \geq 0, \quad X_2 > 0 \tag{40}$$

$$E^{\mathrm{T}}(X - X_1X_2^{-1}X_1^{\mathrm{T}})E \geq 0 \tag{41}$$

Furthermore, by simple calculation, one have

$$Z^{-1} = X - X_1X_2^{-1}X_1^{\mathrm{T}} \tag{42}$$

On the other hand, it is easy to show from (9) and (13) that the ρ-dependent matrices $\mathcal{A}(\rho)$, $\mathcal{A}_1(\rho)$, $\mathcal{L}(\rho)$ and $\mathcal{L}_1(\rho)$ can be rewritten as

$$\begin{aligned} \mathcal{A}(\rho) &= \bar{A} + \bar{F}G\bar{H}(\rho), \quad \mathcal{A}_1(\rho) = \bar{F}G\bar{N}(\rho) \\ \mathcal{L}(\rho) &= \bar{L} + \bar{S}G\bar{H}(\rho), \quad \mathcal{L}_1(\rho) = \bar{S}G\bar{N}(\rho) \\ \mathcal{A}_{\varpi}(\rho) &= \bar{A}_{\varpi} + \bar{F}G\bar{H}_{\varpi}(\rho), \quad \mathcal{L}_{\varpi}(\rho) = \bar{L}_{\varpi} + \bar{S}G\bar{H}_{\varpi}(\rho) \end{aligned} \tag{43}$$

with

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \bar{L} = \begin{bmatrix} L & 0 \end{bmatrix}, \quad G = \begin{bmatrix} D_{\mathrm{f}} & C_{\mathrm{f}} \\ B_{\mathrm{f}} & A_{\mathrm{f}} \end{bmatrix},$$

$$\bar{A}_{\varpi} = \begin{bmatrix} A_w & 0 \\ 0 & 0 \end{bmatrix}, \bar{L}_{\varpi} = \begin{bmatrix} L_w & 0 & 0 \end{bmatrix}$$

$$\bar{F} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \quad \bar{S} = \begin{bmatrix} -I & 0 \end{bmatrix},$$

$$\bar{H}(\rho) = \begin{bmatrix} (1-\rho)C & 0 \\ 0 & I \end{bmatrix}$$

$$\bar{N}(\rho) = \begin{bmatrix} \rho C \\ 0 \end{bmatrix}, \bar{H}_{\varpi}(\rho) = \begin{bmatrix} 0 & (1-\rho)D & \rho D \\ 0 & 0 & 0 \end{bmatrix}$$

Substituting (43) into inequality (13), we can deduce from Lemma 1 that

$$\varpi_c + \Psi_c G \Phi_c + (\Psi_c G \Phi_c)^{\mathrm{T}} > 0 \tag{44}$$

where

$$\varpi_c = \begin{bmatrix} P & 0 & P\bar{A} & 0 & P\mathcal{A}_w \\ * & I & \bar{L} & 0 & L_w \\ * & * & \bar{E}^{\mathrm{T}}P\bar{E} - H^{\mathrm{T}}QH & 0 & 0 \\ * & * & * & Q & 0 \\ * & * & * & * & \gamma^2 I \end{bmatrix},$$

$$\Psi_c = \begin{bmatrix} P\bar{F} \\ \bar{S} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{45}$$

$$\Phi_c = \begin{bmatrix} 0 & 0 & \bar{H}(\rho) & \bar{N}(\rho) & \bar{H}_{\varpi}(\rho) \end{bmatrix}$$

By Lemma 3, it is easy to see that a necessary and sufficient condition for inequality (44) to have a solution G is that the following two inequalities hold simultaneously

$$\Psi_c^{\perp} \varpi_c \Psi_c^{\perp\mathrm{T}} > 0 \tag{46}$$

$$\Phi_c^{\mathrm{T}\perp} \varpi_c \Phi_c^{\perp} > 0 \tag{47}$$

Then, by some calculations, we choose (see (48))

$$\Psi_c^{\perp} = \left[\begin{array}{cc|c|cc|c|ccc} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{array}\right] \begin{bmatrix} P^{-1} & & & & \\ & I & & & \\ & & I & & \\ & & & I & \\ & & & & I \end{bmatrix} \tag{48}$$

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and (see (49))

$$\Psi_c^{\perp}\varpi_c\Psi_c^{\perp\mathrm{T}} = \begin{bmatrix} Z & A & 0 & 0 & A_w & 0 & 0 \\ A^{\mathrm{T}} & E^{\mathrm{T}}XE - Q & E^{\mathrm{T}}X_1 & 0 & 0 & 0 & 0 \\ 0 & X_1^{\mathrm{T}}E & X_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q & 0 & 0 & 0 \\ A_w^{\mathrm{T}} & 0 & 0 & 0 & \gamma^2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma^2 I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma^2 I \end{bmatrix} > 0 \tag{50}$$

$$\Phi_c^{\mathrm{T}\perp}\varpi_c\Phi_c^{\perp} = \begin{bmatrix} I & \\ & \mathcal{M}(\rho) \end{bmatrix} \times \begin{bmatrix} X & X_1 & 0 & XA & XA_w & 0 & 0 \\ X_1^{\mathrm{T}} & X_2 & 0 & X_1^{\mathrm{T}}A & X_1^{\mathrm{T}}A_w & 0 & 0 \\ 0 & 0 & I & L & L_w & 0 & 0 \\ A^{\mathrm{T}}X & A^{\mathrm{T}}X_1 & L^{\mathrm{T}} & E^{\mathrm{T}}XE & 0 & 0 & 0 \\ A_w^{\mathrm{T}}X & A_w^{\mathrm{T}}X_1 & L_w^{\mathrm{T}} & 0 & \gamma^2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma^2 I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma^2 I \end{bmatrix} \times \begin{bmatrix} I & \\ & \mathcal{M}^{\mathrm{T}}(\rho) \end{bmatrix} > 0 \tag{51}$$

by using the Schur complement Lemma, we can easily deduce that (50) is equivalent to

$$\begin{bmatrix} E^{\mathrm{T}}XE - Q - E^{\mathrm{T}} X_1X_2^{-1}X_1^{\mathrm{T}}E & & & & \\ & Q & & & \\ & & \gamma^2 I & & \\ & & & \gamma^2 I & \\ & & & & \gamma^2 I \end{bmatrix} - \begin{bmatrix} A^{\mathrm{T}} \\ 0 \\ A_w^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} Z^{-1}\begin{bmatrix} A & 0 & A_w & 0 & 0 \end{bmatrix} > 0 \tag{52}$$

Considering (42), let

$$Y = Z^{-1} = X - X_1X_2^{-1}X_1^{\mathrm{T}} \tag{53}$$

pre- and post-multiplying the LMI in (52) by diag(Y, I, I) and using Schur complement Lemma again, we obtain the LMI in (33).

Furthermore, by using the Schur complement Lemma and considering (51) and

$$\begin{bmatrix} X & X_1 \end{bmatrix} P^{-1} \begin{bmatrix} X^{\mathrm{T}} \\ X_1^{\mathrm{T}} \end{bmatrix} = X \tag{54}$$

we also obtain the LMI in (34).

It follows from (40)–(42) and (53) that the inequalities (31), (32), (35) and (36) hold. In addition, when (31)–(36) are satisfied, we can obtain the parameters of all desired filters with order $\hat{n}$ corresponding to a feasible solution by using the results in [24, 25]. This completes the proof. □

Remark 6: Theorem 1 provides a sufficient condition for the solvability of the reduced-order H_∞ filtering problem for discrete-time singular systems with lossy measurements. It should be noted that the inequalities in (31)–(36) are non-convex due to the fact that the rank constraint in (36) with respect to variables X and Y is not linear, although the constraints in (31)–(35) are convex. Fortunately, to solve these non-convex inequalities, an efficient numerical algorithm based on alternating projections with bisection iterations given in [26] can be resorted to. Similar to [26], the solution of the optimal H_∞ model reduction problem is

$$\Phi_c^{\mathrm{T}\perp} = \left[\begin{array}{cc|c|cc|c|ccc} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & (1-\rho)C^{\mathrm{T}\perp} & 0 & (1-\rho)C^{\mathrm{T}\perp} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\rho)D^{\mathrm{T}\perp} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho D^{\mathrm{T}\perp} \end{array}\right] \tag{49}$$

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obtained by solving the following non-convex minimisation problem

$$\min_{(X,Y,Q)} \delta \quad \text{subject to (31)–(36) with } \delta = \gamma^2$$

Then, the corresponding optimal H_∞ filtering performance γ^* is given by $\gamma^* = (\min \delta)^{1/2}$.

A bisection iterative algorithm based on alternating projections onto the constraint sets (31)–(36) is similar to [26] and thus is omitted.

Remark 7: It can be seen from the definition of the orthogonal complement that the different orthogonal complement matrices of a given matrix are equivalent through non-singular linear transformation. Thus, although $C^{T\perp}$ and $D^{T\perp}$ may be non-unique, the different $C^{T\perp}$ and $D^{T\perp}$ are equivalent in the sense of non-singular linear transformation. This implies that it is possible that the different choices of $C^{T\perp}$ and $D^{T\perp}$ produce the same minimal H_∞ filtering performance γ^* according to the equivalence of LMI.

Now, we discuss the problem of the solvability of the zeroth-order H_∞ filtering for dynamics (26).

Noting that the inequalities in (27) and (28) are linear with respect to matrix variables P, Q and D_f, therefore, we can easily obtain the condition on the solvability of the zeroth-order H_∞ filtering based on Lemma 2 shown as the following theorem.

Theorem 2: There exists a zeroth-order filter in the form of (11) such that the zeroth-order H_∞ filtering problem for the filtering error singular system (26) with lossy measurements is solvable if there exist matrices $X > 0$, $Q > 0$ and D_f satisfying

$$E^T X E \geq 0 \tag{55}$$

(see (56))

$$\begin{bmatrix} X & 0 & XA & 0 & XA_w & 0 & 0 \\ * & I & L-(1-\rho)D_f C & -\rho D_f C & L_w & -(1-\rho)D_f D & -\rho D_f D \\ * & * & E^T X E - Q & 0 & 0 & 0 & 0 \\ * & * & * & Q & 0 & 0 & 0 \\ * & * & * & * & \gamma^2 I & 0 & 0 \\ * & * & * & * & * & \gamma^2 I & 0 \\ * & * & * & * & * & * & \gamma^2 I \end{bmatrix} > 0 \tag{56}$$

In this case, the parameter D_f of zeroth-order filter in the form of (11) can be obtained by finding a feasible solution (X, Q, D_f) to LMIs in (55) and (56).

Remark 8: Note that constraints (55) and (56) are convex constraints; therefore, the zero-order H_∞ filtering problem is a convex feasibility problem. The optimal zeroth-order H_∞ filtering problem is a convex minimisation problem

$$\min_{(X,Q,D_f)} \delta \quad \text{subject to (55)–(56) with } \delta = \gamma^2$$

The corresponding optimal H_∞ filtering performance γ^* is given by $\gamma^* = (\min \delta)^{1/2}$.

4 Illustrative example

In this section, an illustrative example is provided to demonstrate the applicability and effectiveness of the proposed approach.

Consider the plant described in (1) and (2) with parameters as

$$E = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0.5 & 1 \\ -1 & -0.3 & 1 \\ 0.5 & 0 & 1 \end{bmatrix},$$

$$A_w = \begin{bmatrix} -0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$L = \begin{bmatrix} -3.2 & 0 & 3.2 \\ 3.2 & 0 & 1.6 \\ 0 & 0 & 3.2 \end{bmatrix}, \quad L_w = \begin{bmatrix} -0.1 \\ 0.5 \\ 0.1 \end{bmatrix}$$

and lossy measurements described in (3) with parameters as

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

The purpose of this example is to design a filter in the form of (6) for this system such that the filtering error system (7) is exponentially mean-square stable and an optimal H_∞ filtering performance γ^* is achieved.

By some calculations, it can be obtained that

$$\sigma(E, A) = \{0.6203,\ 0.4116\} \subset D_{\text{int}}(0, 1)$$

Thus, it can be easily checked that the pair (E, A) is

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Table 1 Reduced-order optimal H_∞ filtering performance

ρ	1	0.8	0.5	0.3	0.1	0
γ^*	2.9773	1.1429	0.8017	0.3072	0.0027	0.0005

admissible. Now, we choose

$$C^{T\perp} = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}, \quad D^\perp = \begin{bmatrix} 0 & 1 & 0 \\ 0.01 & 0.9870 & -0.01 \end{bmatrix}$$

By using bisection iterative algorithm based on alternating projections given in [26] to solve the non-convex minimisation problem as stated in Remark 5, we obtain the optimal H_∞ filtering performance γ^* for different values of ρ, summarised in Table 1.

It is shown that γ^* decreases as ρ decreases. In other words, a better H_∞ filtering performance is achieved when less data-loss occurs. In the case when $\rho = 0.5$, $\gamma^* = 0.8017$, the corresponding solution is obtained

$$X = \begin{bmatrix} 7.1987 & -3.6754 & 0.3893 \\ -3.6754 & 7.3567 & 1.9439 \\ 0.3893 & 1.9439 & 6.4780 \end{bmatrix}$$

$$Y = \begin{bmatrix} 3.8914 & -2.3257 & 0.3893 \\ -2.3257 & 5.3415 & 1.9439 \\ 0.3893 & 1.9439 & 6.4780 \end{bmatrix},$$

$$Q = 10^{-3} \times \begin{bmatrix} 0.1016 & -0.1079 & 1.0275 \\ -0.1079 & 0.5316 & -0.1003 \\ 1.0275 & -0.1003 & 0.0020 \end{bmatrix}$$

When $\rho = 0.5$, if we choose

$$C^{T\perp} = \begin{bmatrix} 5 & -5 & 0 \end{bmatrix}, \quad D^\perp = 10^{-2} \times \begin{bmatrix} 1 & 2 & -1 \\ -2 & 4 & 2 \end{bmatrix}$$

it can be found that the γ^* is still 0.8017, the corresponding solution (X, Y, Q) is the same too.

Furthermore, when $\rho = 0.5$, $\gamma^* = 0.8017$, it can be shown that

$$X - Y = \begin{bmatrix} 3.3073 & -1.3497 & 0 \\ -1.3497 & 2.0152 & 0 \\ 0 & 0 & 0 \end{bmatrix} \geq 0$$

which implies that

$$\text{rank}(X - Y) = 2$$

Therefore we can choose

$$X_1 = \begin{bmatrix} 1.8186 & 0 \\ -0.7422 & 1.2101 \\ 0 & 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$$

In this case, it is easy to see that matrix Φ is of full rank, and thus, we can set Φ_l to be an identity matrix, and $\Phi_r = \Phi$. Furthermore, if we choose $Y = \text{diag}\{0.23, 0.23, 0.23, 0.23, 0.23\}$, $W = \text{diag}\{0.073, 0.073, 0.073, 0.073, 0.073\}$, then, according to Theorem 1, a desired reduced-order filter is given by

$$\begin{cases} \hat{x}_{k+1} = \begin{bmatrix} -0.0406 & -0.0478 \\ -0.0493 & -0.1289 \end{bmatrix} \hat{x}_k \\ \qquad + \begin{bmatrix} 0.0228 & -0.0248 & 0.0004 \\ -0.0039 & -0.0002 & -0.0001 \end{bmatrix} y_{ck} \\ \hat{z}_k = 10^{-3} \times \begin{bmatrix} 0.3564 & 0.5468 \\ 0.1279 & -0.8789 \\ -0.5753 & -0.9655 \end{bmatrix} \hat{x}_k + 10^{-3} \\ \qquad \times \begin{bmatrix} 2.2563 & 0.6444 & -0.0036 \\ 0.3588 & -0.5711 & -1.2001 \\ 0.0641 & 0.2101 & 0.1402 \end{bmatrix} y_{ck} \end{cases}$$

It can be seen that the filter is stable and the error dynamic system meets H_∞ filtering performance with $\gamma = 0.8017$.

The zeroth-order optimal H_∞ filter is obtained by solving the convex optimisation problem as stated in Remark 9. The optimal H_∞ filtering performance γ^* for different values of ρ is shown in Table 2.

When $\rho = 0.5$, $\gamma^* = 0.9663$, the matrix parameters solution X, Q and D_f are

$$X = 10^7 \times \begin{bmatrix} 7.4768 & -2.2127 & -3.5123 \\ -2.2127 & 3.6469 & -0.9793 \\ -3.5123 & -0.9793 & 4.6412 \end{bmatrix},$$

$$Q = 10^4 \times \begin{bmatrix} 0.0450 & 0.0064 & -0.0028 \\ 0.0064 & 0.2729 & 0.0239 \\ -0.0028 & 0.0239 & 0.1075 \end{bmatrix}$$

$$D_f = \begin{bmatrix} -1.1084 & 0.0330 & 0.2069 \\ -0.0860 & -0.0124 & 0.1089 \\ -0.1146 & -0.0815 & -3.7703 \end{bmatrix}$$

From Tables 1 and 2, it can be found that the optimal H_∞ performance γ^* increases as the order of the filter decreases. This result is similar to [27].

Table 2 Zeroth-order optimal H_∞ filtering performance

ρ	1	0.8	0.5	0.3	0.1	0
γ^*	3.3379	1.6581	0.9663	0.6118	0.1109	0.0015

5 Conclusions

In this paper, we have investigated the reduced-order H_∞ filtering problem for a class of discrete-time singular systems with lossy measurements. The purpose is to design a filter such that the filtering error system is mean-square stable and a prescribed level of H_∞ filtering performance is guaranteed for lossy measurements. A filter design approach to cope with lossy measurements has been developed for this class of systems. It has been shown that the reduced-order H_∞ filtering problem can be solved by solving a non-convex minimisation problem based on alternating projections, and the zeroth-order H_∞ filtering problem can be solved by solving a convex minimisation problem. A numerical example has been provided to illustrate the effectiveness of the proposed approach.

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