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Distributed algorithm for dynamic economic power dispatch with energy storage in smart grids

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Abstract: The dynamic economic dispatch problem with energy storage in a smart grid scenario is studied, which aims at minimising the aggregate generation costs over multiple periods on condition that the time-varying demand is met, while physical constraints on generation and storage as well as system spinning reserve requirement are satisfied. In our model, energy storage devices are incorporated for not only inter-temporal energy arbitrage to reduce total generation cost, but also providing spinning reserve to share generators' burden. To solve this problem, we assume that the communication networks are strongly connected directed graphs and propose a fully distributed algorithm based on the 'consensus-like' iterative algorithm and the alternating direction method of multipliers. Our algorithm is distributed in the sense that no leader or master nodes are needed, while all the nodes conduct local computation and communicate merely with their neighbours. Numerical simulation is included to show the effectiveness of the proposed algorithm.

1 Introduction

The economic dispatch problem (EDP) is of vital importance to the power industry and has been intensively investigated for decades [1]. The static economic dispatch problem (SEDP) focuses on minimising the total operational costs of fossil-fired generation units at one single period. Nevertheless, since in practice electricity demand is time-varying, generators need to dispatch their generation outputs to achieve the minimum total operational costs over multiple periods, which is referred to as the dynamic economic dispatch problem (DEDP). The major distinction between the SEDP and the DEDP is that the DEDP must take into consideration generators' ramp rate constraints for security reasons [2]. Ramp rate constraints prevent generators from changing their generation outputs rapidly, hence prolonging generators' lifespan. However, ramp rate constraints lead to temporal correlations of generation outputs at adjacent periods, rendering the DEDP more complicated to solve than the SEDP [3].

Various algorithms have been proposed to solve the DEDP. In [3], a robust heuristic method and an adaptive look-ahead-based algorithm are proposed to find a feasible solution and the optimal solution, respectively. The authors of [4] propose an algorithm based on evolutionary programming and sequential quadratic programming to solve the DEDP. In [5], the multiple tabu search (MTS) is applied to the DEDP, and through simulation the authors show that the proposed method based on MTS outperforms the conventional approaches based on simulated annealing, genetic algorithm, tabu search, and particle swarm optimisation (PSO) with higher efficiency and less CPU time.

High penetration of renewable energy sources (RESs) poses a great challenge to the reliable and efficient operation of future smart grids due to their volatile nature. One possible way to deal with the uncertainties of RESs is the system-wide integration of energy storage, e.g. batteries [6]. Besides, energy storage can also be used for not only inter-temporal energy arbitrage to reduce total generation costs, i.e. charging during off-peak periods at a lower marginal cost and discharging during on-peak periods at a higher marginal cost but also providing other ancillary services, e.g. spinning reserve. Recently, some researchers have investigated the DEDP with energy storage (DEDP-S). In [7], the DEDP-S for microgrids is formulated as an optimal control problem and a dynamic programming solution is proposed. Nikmehr and Ravadanegh [8] study the DEDP-S with demand uncertainties in a multi-microgrids scenario, where a probabilistic model of RESs is included and the PSO technique is applied to solve the DEDP-S.

Distributed algorithms for control [9, 10], optimisation [11] and estimation [12, 13] have received tremendous attention from researchers due to the fact that centralised algorithms may be unscalable for large-scale systems, e.g. future smart grid [14]. Featuring the incorporation of communication network, advanced metering infrastructures, and advanced control technologies (cyber layer) onto the power network (physical layer), the smart grid is also a cyber physical system [15]. Compared with centralised algorithms, distributed algorithms have the advantages including enhanced robustness, less (or no) dependence on global information, and more uniform communication and computational burdens for each agent etc. Following the trend of distributed algorithms, many researchers have proposed distributed algorithms to solve the EDP in a smart grid scenario. In [16, 17], the authors propose an incremental cost consensus algorithm to solve the SEDP, where an average consensus algorithm on undirected graphs is used to guarantee the balance between demand and supply. In [18], the authors propose a consensus- based decentralised algorithm, which enables the generators to collectively learn the mismatch between demand and total supply for feedback. Two fully distributed algorithms for the SEDP are also proposed in our previous works [19, 20], respectively. The algorithm proposed in [19] deals with the SEDP with quadratic cost functions on connected undirected graphs, and it is extended to deal with the SEDP with general convex functions on strongly connected directed graphs in [20]. But to the best of the authors' knowledge, no distributed algorithms have yet been proposed for the DEDP with or without energy storage.

In this paper, we study the DEDP-S in a smart grid scenario. It is assumed that besides inter-temporal energy arbitrage, energy storage also provides spinning reserve service to share generators' burden and reduce their opportunity costs. We formulate the DEDP-S as a resource-constrained discrete-time optimal control problem and propose a fully distributed algorithm, which is based on the consensus-like algorithm and the alternating direction method of multipliers (ADMM) for optimisation. We note that the DEDP-S is also a dynamic programme, which may be subject to the 'curse of dimensionality'. To overcome this issue, we decompose the DEDP-S into simpler subproblems by adopting the idea of the ADMM. To facilitate the implementation of our distributed algorithm, we establish a cyber layer over the physical power grid. The proposed algorithm is fully distributed in the sense that it does not rely on any leader or master node, and all the nodes (generators and energy storage devices) conduct local computation and merely communicate with their neighbours to iteratively find the global optimal solution.

Compared with existing works, the novel features of this paper are summarised as follows. Our major contribution is that this paper is the first to propose a fully distributed algorithm for DEDP-S. Specifically, we take advantage of the mathematical structure of the DEDP-S and decompose it using the idea of ADMM. The subproblems are solved by the distributed bisection method and local computation. Although existing works [16–20] have also designed various distributed algorithms for the EDP, their algorithms are designed for SEDP and cannot be applied to the DEDP-S. We also propose a resource-constrained discrete-time optimal control formulation of the DEDP-S where energy storage is assumed to provide services of energy arbitrage as well as spinning reserve. To the authors' knowledge, none of existing works has proposed such mathematical formulation. For instance, spinning reserve from energy storage is neglected in [7, 8]. We generally assume that the communication networks in this paper are strongly connected directed graphs, while [16, 17, 19] assume undirected graphs which can be treated as a special case of directed graphs and are more restrictive in practice, since communication may be subject to packet losses or device failures. Besides, it is more theoretically challenging to design distributed algorithms on directed graphs. We assume that the cost functions of generators are generic convex functions, while simple quadratic functions are assumed in [16–19], etc.

This paper is organised as follows: In Section 2, we introduce the dynamic model of a single energy storage device and formulate the DEDP-S. Some basics on graph theory, the consensus-like algorithm, and the ADMM are presented in Section 3. In Section 4, we rewrite DEDP-S in the ADMM form and then present our distributed algorithm. A numerical example based on the IEEE 14-bus system is given in Section 5 to show the performance of the proposed algorithm. We conclude our paper in Section 6.

2 Preliminaries

2.1 Graph theory

The directed graph $\mathscr{G} = (\mathscr{N}, \mathscr{E})$ associated with a non-negative matrix $W \in \mathbb{R}^{n \times n}$ comprises a non-empty finite set of nodes $\mathscr{N} = \{1, 2, \dots, n\}$ and a finite set of ordered edges $\mathscr{E} = \{(j, i) | w_{ij} > 0\}$, where w_{ij} is the entry in the ith row and jth column of W. For node $i \in V$, its in-neighbour set and out-neighbour set are denoted by $\mathscr{V}_i^- = \{j \in \mathscr{N} - \{i\} : (j, i) \in \mathscr{E}\}$ and $\mathscr{V}_i^+ = \{j \in \mathscr{N} - \{i\} : (i, j) \in \mathscr{E}\}$, i.e. node i receives information from its in-neighbours and sends out information to its out-neighbours. Let us define $\deg_i^- = |\mathscr{V}_i^-|$ and $\deg_i^+ = |\mathscr{V}_i^+|$ as the in-degree and out-degree of node i, respectively, where $| \cdot |$ represents the cardinality of a set. A graph is strongly connected if there is a path from any node to any other node in the graph.

2.2 Consensus-like algorithm

We first introduce a lemma on non-negative matrices and then present the consensus-like algorithm.

Lemma 1 [21]: A non-negative matrix $A \in \mathbb{R}^{n \times n}$ is primitive, if and only if its associated graph is strongly connected and aperiodic.

Let us define a non-negative matrix W associated with a strongly connected graph with self-loops $\mathscr{G} = (\mathscr{N}, \mathscr{E})$ as

$$w_{ij} = \begin{cases} \dfrac{1}{\deg_j^+ + 1} & \text{for } (j, i) \in \mathscr{E}, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Since the graph $\mathscr{G}$ is strongly connected and with self-loops (which implies the aperiodicity), from Lemma 1, we have that W is primitive. Furthermore, it can be readily verified that W is column stochastic, i.e. each column sums to one, and consequently W^{T} is row stochastic. Therefore we have that $W^{\mathrm{T}}\underline{1} = \underline{1}$, where $\underline{1} = [1, 1, \dots, 1]^{\mathrm{T}}$ is the eigenvector of W^{T} associated with the eigenvalue one. According to Perron–Frobenius Theorem [22], it follows that

$$\lim_{\kappa \to \infty} W^{\kappa} = \lim_{\kappa \to \infty} ((W^{\mathrm{T}})^{\kappa})^{\mathrm{T}} = (\underline{1}\zeta^{\mathrm{T}})^{\mathrm{T}} = \zeta\underline{1}^{\mathrm{T}}, \tag{2}$$

where $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_n]^{\mathrm{T}}$ is the right eigenvector of W associated with the eigenvalue one, and $\zeta_i > 0$ for all i and $\underline{1}^{\mathrm{T}}\zeta = 1$.

Let us consider the following iteration with the iteration index denoted by κ and initial value $\theta(0)$:

$$\theta(\kappa + 1) = W\theta(\kappa), \tag{3}$$

where $\theta(\kappa) = [\theta_1(\kappa), \dots, \theta_n(\kappa)]^{\mathrm{T}}$ and $\theta_i(\kappa)$ is the local variable associated with node i. From a local perspective, the iteration (3) is equivalent to

$$\theta_i(\kappa + 1) = w_{ii}\theta_i(\kappa) + \sum_{j \in \mathscr{V}_i^-} w_{ij}\theta_j(\kappa). \tag{4}$$

Let us denote by θ^* the fixed point of the iteration (3) when it converges. From (2), we have that

$$\theta^* = \lim_{\kappa \to \infty} \theta(\kappa) = \zeta\underline{1}^{\mathrm{T}}\theta(0) = \left(\sum_{i=1}^{n} \theta_i(0)\right)\zeta, \tag{5}$$

2.3 Alternating direction method of multipliers

Consider the following optimisation problem:

$$\begin{aligned} \min \quad & f(x) + g(y), \\ \text{s.t.} \quad & Ax + By = c, \end{aligned} \tag{6}$$

with optimisation variables $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, where $A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{p \times m}$ and $c \in \mathbb{R}^p$.

We make the following assumptions for the problem (6):

Assumption 1: The functions $f(x): \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ and $g(y): \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ are proper, closed and convex.

Assumption 2: The matrices A and B have full column ranks.

The augmented Lagrangian of problem (6) is

$$\begin{aligned} L_\rho(x, y, \lambda) = {} & f(x) + g(y) + \lambda^{\mathrm{T}}(Ax + By - c) \\ & + \rho/2 \parallel Ax + By - c \parallel_2^2, \end{aligned} \tag{7}$$

where $\lambda \in \mathbb{R}^p$ is the Lagrange multiplier, and $\rho > 0$ is a penalty parameter. The alternating direction method of multipliers for problem (6) is given as follows [23]:

$$x^{k+1} = \underset{x}{\arg\min}\, L_\rho(x, y^k, \lambda^k); \tag{8}$$

$$y^{k+1} = \underset{y}{\arg\min}\, L_\rho(x^{k+1}, y, \lambda^k); \tag{9}$$

$$\lambda^{k+1} = \lambda^k + \rho(Ax^{k+1} + By^{k+1} - c)\,. \qquad (10)$$

Define

$$\epsilon^k = Ax^k + By^k - c \text{ and } \varepsilon^k = \rho A^{\mathrm{T}}B(y^{k+1} - y^k)$$

as the primal residual and the dual residual at step k, respectively. The convergence properties of ADMM are given as follows.

Lemma 2 [23]: If Assumptions 1 and 2 and $\rho > 0$ hold, then the ADMM iterations (8)–(10) converge to the optimal solution x^*, y^* and the optimal Lagrange multiplier λ^* of problem (6), with

$$\lim_{k \to \infty} \parallel \epsilon^k \parallel_2 = 0 \text{ and } \lim_{k \to \infty} \parallel \varepsilon^k \parallel_2 = 0.$$

3 System modelling

3.1 Dynamic model of energy storage device

Assume that in total there are n^{s} energy storage devices in the network and let $t = 0, 1, \ldots, \tau$ denote the scheduling intervals with sampling resolution $\Delta\tau$. For the ith energy storage device, denote by $E_i(t)$ and $S_i(t)$, the current amount of energy at time t and the constant rate of energy conversion during time $t-1$ and t, respectively. Note that $S_i(t)$ is positive when the device is charging and negative when it is discharging. Let us define η_i^{c} and η_i^{d} as the charging efficiency and the discharging efficiency, respectively. With $E_i(0)$ denoting the initial amount of energy at $t = 0$, the dynamics of the ith energy storage device is given by

$$E_i(t) = E_i(t-1) + \Delta\tau S_i(t)\eta_i(t) \quad \forall t = 1, \ldots, \tau, \qquad (11)$$

where the ratio of energy conversion $\eta_i(t)$ is defined as

$$\eta_i(t) = \begin{cases} \eta_i^{\mathrm{c}} < 1 & \text{for } S_i(t) \geqslant 0, \\ \dfrac{1}{\eta_i^{\mathrm{d}}} > 1 & \text{for } S_i(t) < 0. \end{cases} \qquad (12)$$

Since $\eta_i(t)$ is conditioned on the sign of $S_i(t)$, model (11) is non-linear. For the ith device let us further define $S_i^{\mathrm{c}}(t)$ and $S_i^{\mathrm{d}}(t)$ as the rates of charging and discharging, and then substitute

$$S_i(t) = S_i^{\mathrm{c}}(t) - S_i^{\mathrm{d}}(t) \quad \forall t = 1, \ldots, \tau\,. \qquad (13)$$

into (11), which yields

$$E_i(t) = E_i(t-1) + \Delta\tau\left(S_i^{\mathrm{c}}(t)\eta_i^{\mathrm{c}} - S_i(t)^{\mathrm{d}}/\eta_i^{\mathrm{d}}\right). \qquad (14)$$

One can easily verify that (14) is equivalent to (11) and (12) if and only if

$$S_i^{\mathrm{c}}(t)S_i^{\mathrm{d}}(t) = 0 \quad \forall t = 1, \ldots, \tau\,.$$

However, we can drop the above non-linear equality constraints without any harm because they can be automatically satisfied in the context of economic dispatch. Since there is energy loss in energy conversion both ways, from the economic perspective, energy storage devices are prevented from charging and discharging simultaneously by the objective of minimising the total generation costs.

The charging/discharging rates $S_i^{\mathrm{c}}(t)$ and $S_i^{\mathrm{d}}(t)$ are bounded by

$$0 \leqslant S_i^{\mathrm{c}}(t) \leqslant \bar{S}_i^{\mathrm{c}} \quad \forall t = 1, \ldots, \tau, \qquad (15)$$

$$0 \leqslant S_i^{\mathrm{d}}(t) \leqslant \bar{S}_i^{\mathrm{d}} \quad \forall t = 1, \ldots, \tau, \qquad (16)$$

where $\bar{S}_i^{\mathrm{c}}$ and $\bar{S}_i^{\mathrm{d}}$ are the maximum rates of charging and discharging, respectively. Furthermore, the energy stored in the device cannot exceed its capacity $\bar{E}_i$ or drop below zero, i.e.

$$0 \leqslant E_i(t) \leqslant \bar{E}_i \quad \forall t = 1, \ldots, \tau\,. \qquad (17)$$

It is further required that at the end of the last scheduling period, the amount of energy stored in the ith device must be greater than a given amount $\underline{E}_i$ in case it is used later for other purposes. Therefore, for the ith energy storage device we have

$$\underline{E}_i \leqslant E_i(\tau)\,. \qquad (18)$$

Remark 1: We note that (12) is not 'modelling friendly' in its current form. For the convenience of modelling, we introduce two auxiliary variables $S_i^{\mathrm{c}}(t)$ and $S_i^{\mathrm{d}}(t)$, one of which is bound to be zero at optimality (the most economic dispatch) due to the positive electricity price and energy losses. Therefore, the above model does not contradict the fact that usually a battery cannot charge and discharge simultaneously.

3.2 Problem formulation of DEDP-S

Suppose that in total there are n^{g} generators in the network and denote by $\mathcal{N}^{\mathrm{g}}$ and $\mathcal{N}^{\mathrm{s}}$ the sets consisting of generators and energy storage devices, respectively. Define $P_i(t)$ as the ith generator's output at time t, then the aggregated variable is $P = [P_1^{\mathrm{T}}, \ldots, P_{n^{\mathrm{g}}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n^{\mathrm{g}}\tau}$ (In this paper, the superscript 'T' represents the transpose of a matrix or vector.), where $P_i = [P_i(1), \ldots, P_i(\tau)]^{\mathrm{T}} \in \mathbb{R}^{\tau}$. The objective of the DEDP-S is given by

$$C^{\mathrm{total}}(P) = \sum_{t=1}^{\tau}\sum_{i=1}^{n^{\mathrm{g}}} C_i(P_i(t)), \qquad (19)$$

where $C_i(P_i(t))$ is the cost function associated with the ith generator.

In most cases, it is assumed that the cost functions are in the following quadratic term:

$$C_i(P_i(t)) = \frac{1}{2}\alpha_i P_i(t)^2 + \beta_i P_i(t) + \gamma_i, \qquad (20)$$

where $\alpha_i > 0$, β_i and γ_i are cost coefficients of the ith generator. Since in practice cost functions are derived via curve fitting based on the data obtained from heat rate tests or from the plant design engineers, non-quadratic functions may be used for better fitting performance [1]. Therefore, in this paper we consider generic cost functions satisfying the assumption as follows:

Assumption 3: For every $1 \leqslant i \leqslant n$, $C_i(P_i(t)) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable and

$$C_i''(P_i(t)) \geqslant 0, \ \ \forall P_i(t) \in \mathbb{R}_+,$$

where $C_i''(P_i(t))$ is the second derivative of $C_i(P_i(t))$, $\mathbb{R}_+$ denotes the set of non-negative real numbers, and the equality holds at isolated points only.

The balance between demand and supply yields the equality constraints:

$$\sum_{i=1}^{n^{\mathrm{g}}} P_i(t) - \sum_{j=1}^{n^{\mathrm{s}}} S_j(t) = P^{\mathrm{D}}(t) \quad \forall t = 1, \ldots, \tau, \qquad (21)$$

where $P^{\mathrm{D}}(t)$ is the time-varying electricity demand.

We assume that at each time t the system operator requires a certain amount of reserve from generators and energy storage devices. For $i \in \mathcal{N}^{\mathrm{g}} \cup \mathcal{N}^{\mathrm{s}}$, define $R_i(t)$ and $\bar{R}_i(t)$ as the reserve contribution and the capacity of reserve at time t, respectively. We then have the following constraints:

$$\underline{P}_i \leqslant P_i(t) \quad \forall i \in \mathcal{N}^{\mathrm{g}}, \quad t = 1, \ldots, \tau, \tag{22}$$

$$P_i(t) + R_i(t) \leqslant \bar{P}_i \quad \forall i \in \mathcal{N}^{\mathrm{g}}, \quad t = 1, \ldots, \tau, \tag{23}$$

$$0 \leqslant R_i(t) \leqslant \bar{R}_i(t) \quad \forall i \in \mathcal{N}^{\mathrm{g}} \cup \mathcal{N}^{\mathrm{s}}, \quad t = 1, \ldots, \tau, \tag{24}$$

$$S_i^{\mathrm{d}}(t) + R_i(t) \leqslant \bar{S}_i^{\mathrm{d}} \quad \forall i \in \mathcal{N}^{\mathrm{s}}, \quad t = 1, \ldots, \tau, \tag{25}$$

$$\bar{R}_i(t) \leqslant E(t-1)\eta_i^{\mathrm{d}}/\Delta\tau \quad \forall i \in \mathcal{N}^{\mathrm{s}}, \quad t = 1, \ldots, \tau, \tag{26}$$

$$\sum_{i \in \mathcal{N}^{\mathrm{g}} \cup \mathcal{N}^{\mathrm{s}}} R_i(t) = SR(t) \quad \forall t = 1, \ldots, \tau, \tag{27}$$

where $SR(t)$ is the system spinning reserve requirement at time t; $\underline{P}_i$ and $\bar{P}_i$ are the lower and upper bounds of the ith generator's output, respectively. Note that for each generator, the reserve capacity $\bar{R}_i(t)$ is time-invariant and in U.S. markets the 10-min ramp rate is usually used to define reserve capacity. However, the reserve capacity of an energy storage device at time t depends on the amount of residual energy at $t-1$, as described by constraints (26).

For security, each generator is also subject to ramp rate constraints, given by

$$\Delta\underline{P}_i \leqslant P_i(t+1) - P_i(t) \leqslant \Delta\bar{P}_i \quad \forall t = 1, \ldots, \tau - 1, \tag{28}$$

where $\Delta\underline{P}_i$ and $\Delta\bar{P}_i$ are the lower and upper bounds of the ramp rate of the ith generator.

With other factors (e.g. line capacity, transmission losses) ignored, the DEDP-S can be formulated as follows:

$$\begin{aligned} \min \quad & C^{\mathrm{total}}(P), \\ \text{over} \quad & P, R, S, \bar{R}, S^{\mathrm{c}}, S^{\mathrm{d}} \text{ and } E, \\ \text{s.t.} \quad & (13)-(18) \text{ and } (21)-(28), \end{aligned} \tag{29}$$

where R, S, $\bar{R}$, S^{c}, S^{d} and E are aggregated variables with similar data structure to that of vector P, respectively.

Remark 2: From a control perspective, the DEDP-S (29) can be viewed as a resource-constrained discrete-time optimal control problem, of which the objective is to minimise the total operational cost (generation cost) over finite periods subject to constrained resources (limited generation capacities, balance between demand and supply etc.). The DEDP-S is analysed and solved in a distributed fashion based on optimisation theory which is an important tool for deriving optimal control policies.

Remark 3: The ramp constraints (28) which only involve the generation outputs has been widely used in the literature, but we comment that such constraints cannot 100% guarantee the provision of spinning reserves. For instance, suppose for generator i, its optimal energy outputs at time t and $t+1$ are such that $P_i^*(t+1) = P_i^*(t) + \Delta\bar{P}_i$, i.e. at time $t+1$ generator i is operating fully at its maximum ramp rate. In this case, the spinning reserve that generator i can provide at $t+1$ is zero, but with the constraints (28) it is still assumed that generator i can provide spinning reserve up to $R_i(t+1) = \bar{P}_i - P_i^*(t+1)$.

An improved formulation of ramp constraints which can ensure the provision of spinning reserves is

$$\Delta\underline{P}_i + R_i(t) \leqslant P_i(t+1) - P_i(t) \leqslant \Delta\bar{P}_i - R_i(t+1), \tag{30}$$

However, constraints (30) may cause over-conservatism and unnecessarily increase the opportunity costs of generation. This market design issue is beyond the scope of this paper and the reader is referred to [24] for further information. Furthermore, we note that the distributed algorithm proposed in this paper applies to both cases, since mathematically the separability of the DEDP-S model is not subject to the type of ramp constraints.

Remark 4: Although spinning reserve is not explicitly included in the objective function, the opportunity costs of providing spinning reserve get implicitly added to the objective (generation costs) through the correlative constraints on power output and spinning reserve. Based on economic theory, one can verify that the DEDP-S model which is a cost-minimisation optimisation problem, is equivalent to a competitive market model where each market participant gets rewarded for providing spinning reserve at the same price and aims at maximising their own net revenue subject to market clearing constraints, as their Karush–Kuhn–Tucker conditions are exactly identical. We further note that in US markets there is a controversy over whether providing spinning reserve results in additional costs beyond the opportunity costs. However, this market design problem is beyond the scope of this paper.

Remark 5: Recently energy storage allocation problem has been intensively investigated [25]. However in this paper it is assumed that energy storage devices have been properly allocated to the grid to optimally achieve some objective, e.g. profit maximising [26]. Besides the economic benefits, energy storage can also relieve the stress of transmission/distribution networks, e.g. managing transmission congestions. Therefore we reasonably neglect the line capacity constraints in the DEDP-S model studied in this paper. It is our future work to investigate more complicated models where more practical factors are considered.

4 Distributed algorithm for DEDP-S

4.1 Rewriting DEDP-S in ADMM form

Before rewriting the DEDP-S (29) in the ADMM form, we first present some analysis on the mathematical structure of the DEDP-S:

- Firstly, generators and energy storage devices are spatially coupled only by the system balance constraint (21) and the system spinning reserve constraint (27). The remaining constraints are local and only lead to the temporal correlations of local variables.
- The variables directly coupled by the system-wide constraints (21) and (27) include P, S and R. Besides, the objective function does not involve any other variables except P. Therefore, the remaining variables can be viewed as auxiliary variables for defining the feasible region of P, S and R.

Let us define four convex sets:

$$\begin{aligned} \Psi^{PS} &= \left\{ (P, S) \mid \sum_{i=1}^{n^{\mathrm{g}}} P_i(t) - \sum_{j=1}^{n^{\mathrm{s}}} S_j(t) = P^{\mathrm{D}}(t)\, \forall t \right\}, \\ \Psi^{R} &= \left\{ R \mid SR(t) = \sum_{i \in \mathcal{N}^{\mathrm{g}} \cup \mathcal{N}^{\mathrm{s}}} R_i(t)\, \forall t \right\}, \\ \Psi &= \Psi^{PS} \times \Psi^{R}, \\ \Omega &= \{ (P, S, R) \mid (13)-(18), (22)-(26) \text{ and } (28) \}. \end{aligned}$$

Based on the above analysis, we can rewrite the DEDP-S (29) as

$$\begin{aligned} \min_{P,S,R} \quad & C^{\mathrm{total}}(P), \\ \text{s.t.} \quad & (P, S, R) \in \Psi \cap \Omega. \end{aligned} \tag{31}$$

For simplicity, let us further define $X = [P^{\mathrm{T}}, S^{\mathrm{T}}, R^{\mathrm{T}}]^{\mathrm{T}}$. Using the trick that one can eliminate a constraint by introducing a corresponding penalty in the objective function, we rewrite the DEDP-S (29) equivalently in the ADMM form as follows:

$$\begin{aligned} \min_{X,Y} \quad & C^{\text{total}}(X) + G_\Psi(X) + G_\Omega(Y), \\ \text{s.t.} \quad & X - Y = 0, \end{aligned} \tag{32}$$

where $Y = [P_Y^{\mathrm{T}}, S_Y^{\mathrm{T}}, R_Y^{\mathrm{T}}]$ is the duplicate of X; G_Ψ and G_Ω are the indicator functions for the sets Ψ and Ω, respectively, i.e.

$$G_\Psi(X) = \begin{cases} 0, & \text{if } X \in \Psi, \\ +\infty, & \text{otherwise}, \end{cases}$$

$$G_\Omega(Y) = \begin{cases} 0, & \text{if } Y \in \Omega, \\ +\infty, & \text{otherwise}. \end{cases}$$

Since the indicator function of a convex set is proper, closed and convex, the functions $C^{\text{total}}(X) + G_\Psi(X)$ and $G_\Omega(Y)$ satisfy Assumption 1. Also, in this case we have $A = I$ and $B = -I$, where I is the identify matrix with proper dimensions, so Assumption 2 is satisfied as well.

Applying the ADMM to the DEDP-S yields

$$X^{k+1} = \underset{X}{\arg\min}\, L_\rho(X, Y^k, \lambda^k), \tag{33}$$

$$Y^{k+1} = \underset{Y}{\arg\min}\, L_\rho(X^{k+1}, Y, \lambda^k), \tag{34}$$

$$\lambda^{k+1} = \lambda^k + \rho(X^{k+1} - Y^{k+1}), \tag{35}$$

where $L_\rho(X, Y, \lambda)$ is the augmented Lagrangian for the DEDP-S, given by

$$\begin{aligned} L_\rho(X, Y, \lambda) = {} & C^{\text{total}}(X) + G_\Psi(X) + G_\Omega(Y) \\ & + \lambda^{\mathrm{T}}(X - Y) + \rho/2 \parallel X - Y \parallel_2^2 . \end{aligned} \tag{36}$$

4.2 Collecting demand and reserve information in distributed manner

A cyber layer is necessary for solving the DEDP-S in a distributed fashion, as in our distributed algorithm each node needs the update from their neighbours to perform the next iteration. Let us define two sets $\mathcal{N}^a$ and $\mathcal{N}^b$, where $\mathcal{N}^a = \mathcal{N}^{\mathrm{g}} \cup \mathcal{N}^{\mathrm{s}}$ consists of the buses associated with generation and energy storage, while $\mathcal{N}^b$ consists of all the buses in the power system. Note that cyber agents (nodes) are assigned to each generator and each energy storage device. For instance, if a bus contains a generator and a battery, then two nodes are assigned to this bus. Each node has its own variables, constraints and cost functions depending on their types (generation or energy storage). To facilitate our distributed algorithm, let us establish two communication networks $\mathcal{G}_a = (\mathcal{N}_a, \mathcal{E}_a)$ and $\mathcal{G}_b = (\mathcal{N}_b, \mathcal{E}_b)$. We assume that both $\mathcal{G}_a$ and $\mathcal{G}_b$ are strongly connected directly graphs with self-loops. With recent advances in communication technologies, it is feasible and inexpensive to fulfill our network assumption.

The problem formulation in Section 3.2 is based on an implicit assumption that the aggregate demand $P^{\mathrm{d}}(t)$ and the system spinning reserve requirement $SR(t)$ are known to each generator and energy storage device. Nevertheless, in practical power systems demand and reserve requirement are spatially distributed at almost all the buses, i.e.

$$P^{\mathrm{D}}(t) = \sum_{j \in \mathcal{N}^b} P_j^{\mathrm{D}}(t) \quad \text{and} \quad SR(t) = \sum_{j \in \mathcal{N}^b} SR_j(t),$$

where $P_j^{\mathrm{D}}(t)$ and $SR_j(t)$ are the demand and reserve requirement of bus j at time t, respectively.

To collect the scaled information, for every node $i \in \mathcal{N}_b$, we establish three variables $p_i(\kappa)$, $r_i(\kappa)$ and $s_i(\kappa)$, respectively, initialised by

$$p_i(0) = P_i^{\mathrm{D}}(t), \quad r_i(0) = SR_i(t) \quad \forall i \in \mathcal{N}^b,$$

$$\text{and} \quad s_i(0) = \begin{cases} 1, & i \in \mathcal{N}^a, \\ 0, & i \in \mathcal{N}^b \setminus \mathcal{N}^a. \end{cases}$$

Run the following consensus-like algorithms simultaneously and iterate until convergence:

$$p(\kappa + 1) = W_b p(\kappa)$$

$$r(\kappa + 1) = W_b r(\kappa),$$

$$s(\kappa + 1) = W_b s(\kappa),$$

where W_b is defined w.r.t. (The acronym 'w.r.t.' stands for 'with respect to' in this paper.) graph $\mathcal{G}_b$ using (1). When converged, we have that

$$p^* = \lim_{\kappa \to \infty} p(\kappa) = P^{\mathrm{D}}(t)\zeta_b,$$

$$r^* = \lim_{\kappa \to \infty} r(\kappa) = SR(t)\zeta_b,$$

$$s^* = \lim_{\kappa \to \infty} s(\kappa) = (n^{\mathrm{g}} + n^{\mathrm{s}})\zeta_b,$$

where ζ_b is the right eigenvector of W_b associated with the eigenvalue one. Then every node $i \in \mathcal{N}_a$ establishes two variables $v_i(\kappa)$ and $z_i(\kappa)$ initialised by

$$v_i(0) = p_i^*/s_i^* = P^{\mathrm{D}}(t)/(n^{\mathrm{g}} + n^{\mathrm{s}}),$$

$$z_i(0) = r_i^*/s_i^* = SR(t)/(n^{\mathrm{g}} + n^{\mathrm{s}}),$$

respectively. Run the following consensus-like algorithms and iterate until convergence:

$$v(\kappa + 1) = W_a v(\kappa),$$

$$z(\kappa + 1) = W_a z(\kappa),$$

where W_a is defined w.r.t. graph $\mathcal{G}_a$ using (1). When converged, we have that

$$v^* = \lim_{\kappa \to \infty} v(\kappa) = P^{\mathrm{D}}(t)\zeta_a,$$

$$z^* = \lim_{\kappa \to \infty} z(\kappa) = SR(t)\zeta_a,$$

where ζ_a is the right eigenvector of W_a associated with the eigenvalue one. We will use v^* and z^* as the scaled information of demand and system spinning reserve requirement in the remaining of the proposed algorithm.

4.3 X-update

In this section, we present the distributed implementation of X-update (33). Let us define $u^k = \lambda^k/\rho$ and $X^{PS} = [P^{\mathrm{T}}, S^{\mathrm{T}}]^{\mathrm{T}}$. Combining (33) with (36), we have

$$
\begin{aligned}
X^{k+1} &= \underset{X}{\arg\min}\, L_\rho(X, Y^k, \lambda^k) \\
&= \underset{X}{\arg\min}\Big(C^{\text{total}}(X) + G_\Psi(X) + (\lambda^k)^{\mathrm{T}}(X - Y^k) \\
&\quad +(\rho/2)\parallel X - Y^k \parallel_2^2\Big) \\
&= \underset{X \in \Psi}{\arg\min}\Big(C^{\text{total}}(X) + \frac{\rho}{2}\parallel X - Y^k + u^k \parallel_2^2\Big) \\
&= \underset{\substack{X^{PS} \in \Psi^{PS} \\ R \in \Psi^R}}{\arg\min} \sum_{t=1}^{\tau} \sum_{i \in \mathscr{N}^a} \Big(F_i(X_i^{PS}(t)) + H_i(R_i(t))\Big),
\end{aligned} \tag{37}
$$

where

$$
F_i(X_i^{PS}(t)) = \begin{cases} C_i(P_i(t)) + \dfrac{\rho}{2}\Big(P_i^2(t) + u_i^k(t) - P_{Yi}^k(t)\Big)^2 \; \forall i \in \mathscr{N}^{\mathrm{g}}, \\ \dfrac{\rho}{2}\Big(S_i^2(t) + u_i^k(t) - S_{Yi}^k(t)\Big)^2 \; \forall i \in \mathscr{N}^{\mathrm{s}}, \end{cases}
$$

$$
H_i(R_i(t)) = \frac{\rho}{2}\Big(R_i^2(t) + u_i^k(t) - R_{Yi}^k(t)\Big)^2 \; \forall i \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}.
$$

One can easily verify that X-update (37) consists of two independent optimisation subproblems:

$$
\min_{X^{PS} \in \Psi^{PS}} \sum_{t=1}^{\tau} \sum_{i \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}} F_i(X_i^{PS}(t)),
$$

$$
\min_{R \in \Psi^R} \sum_{t=1}^{\tau} \sum_{j \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}} H_j(R_j(t)).
$$

Each of the above subproblems can be further split into τ subproblems, i.e. $\forall t = 1, \ldots, \tau$,

$$
\begin{aligned}
\min \quad & \sum_{i \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}} F_i(X_i^{PS}(t)), \\
\text{s.t.} \quad & \sum_{i \in \mathscr{N}^{\mathrm{g}}} P_i(t) - \sum_{j \in \mathscr{N}^{\mathrm{s}}} S_j(t) = P^{\mathrm{D}}(t),
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
\min \quad & \sum_{i \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}} H_i(R_i(t)), \\
\text{s.t.} \quad & \sum_{i \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}} R_i(t) = SR(t).
\end{aligned} \tag{39}
$$

Therefore, X-update includes 2τ convex subproblems. Note that although in terms of the physical meanings, subproblem (38) involves two variables P an S, from a mathematical perspective they can be viewed as one variable. Therefore subproblems (38) and (39) are identical in mathematical structure. The following lemma gives the optimally conditions for subproblems (38) and (39).

Lemma 3 [27]: The optimal conditions for subproblem (38) and (39) are

$$
F_i'(P_i^*(t)) = -F_j'(S_j^*(t)) = \nu^* \quad \forall i \in \mathscr{N}^{\mathrm{g}}, \; j \in \mathscr{N}^{\mathrm{s}},
$$

$$
\sum_{i \in \mathscr{N}^{\mathrm{g}}} P_i^*(t) - \sum_{j \in \mathscr{N}^{\mathrm{s}}} S_j^*(t) = P^{\mathrm{D}}(t);
$$

and $H_j'(R_j^*(t)) = H_i'(R_i^*(t)) = \omega^* \quad \forall i, j \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}$,

$$
\sum_{i \in \mathscr{N}^{\mathrm{g}} \cup \mathscr{N}^{\mathrm{s}}} R_i^*(t) = SR(t);
$$

where $P_i^*(t)$'s, $S_i^*(t)$'s and $R_i^*(t)$'s are the optimal solutions; ν^* and ω^* are the optimal Lagrange multipliers, respectively.

Due to limited space and considering the fact that subproblems (38) and (39) are identical in mathematical structure, we only present the distributed algorithm for the subproblem (38), whereas it can also be applied to solve (39).

According to Assumption 1, for any generator i, the first derivative of its cost function, denoted by $C_i'(P_i(t))$, is monotonically increasing w.r.t. $P_i(t)$. Combining $\rho > 0$, the first derivative of $F_i(X_i^{PS}(t))$, denoted by $F_i'(X_i^{PS}(t))$, is monotonically increasing w.r.t. $X_i^{PS}(t)$. From Lemma 3, we infer that $P_i^*(t)$ and $-S_i^*(t)$ are monotonically increasing with the optimal Lagrange multiplier ν^*, which motivates us to adopt the idea of *bisection*. To solve problem (38) in a fully distributed fashion on the directed graph $\mathscr{G}_a$, we present the *distributed bisection* method.

Step 1: Let us define two variables $\bar{\nu}(\iota)$ and $\underline{\nu}(\iota)$, where $\iota = 0, 1, \ldots$ denotes the bisection steps. Note that $\bar{\nu}(\iota)$ and $\underline{\nu}(\iota)$ are commonly shared by each node $i \in \mathscr{N}_a$ and their initial values $\underline{\nu}(0)$ and $\bar{\nu}(0)$ should guarantee that $\underline{\nu}(0) \leqslant \nu^* \leqslant \bar{\nu}(0)$ holds.

Step 2: Each node computes

$$
\nu(\iota) = \frac{\underline{\nu}(\iota) + \bar{\nu}(\iota)}{2},
$$

$$
\mu_i(0) = \begin{cases} J_i(\nu(\iota)) & \text{for } i \in \mathscr{N}^{\mathrm{g}}, \\ -J_i(-\nu(\iota)) & \text{for } i \in \mathscr{N}^{\mathrm{s}}, \end{cases}
$$

where $J_i(\cdot)$ is the inverse function of $F_i'(\cdot)$.

Step 3: Each node establishes a variable $\mu_i(\kappa)$ which is initialised by $\mu_i(0)$, and run the following consensus-like algorithm till convergence:

$$
\mu(\kappa + 1) = W_a \mu(\kappa). \tag{40}
$$

When converged, we have for any $i \in \mathscr{N}_a$,

$$
\mu^* = \lim_{\kappa \to \infty} \mu(\kappa) = \left(\sum_{i=1}^{n} J_i(\nu(\iota))\right)\zeta_a.
$$

Step 4: Each node i updates $\bar{\nu}(\iota + 1)$ and $\underline{\nu}(\iota + 1)$ according to

$$
\begin{cases} \bar{\nu}(\iota + 1) = \nu(\iota), \; \underline{\nu}(\iota + 1) = \underline{\nu}(\iota) & \text{for } \mu_i^* > v_i^*, \\ \bar{\nu}(\iota + 1) = \bar{\nu}(\iota), \; \underline{\nu}(\iota + 1) = \nu(\iota) & \text{for } \mu_i^* \leqslant v_i^*. \end{cases} \tag{41}
$$

Step 5: Go back to Step 2 and loop until convergence.

Subproblems (39) can also be solved by the above distributed bisection method. Furthermore, subproblems (38) and (39) can be solved in parallel, as they are independent of each other. With the 2τ subproblems solved by the distributed bisection method, the X-update is accomplished in a distributed manner. Due to length restrictions, the reader is referred to our previous work [20] for detailed analysis on the distributed bisection method.

4.4 Y-update

Combining (34) and (36), we have

$$
\begin{aligned}
Y^{k+1} &= \underset{Y}{\arg\min}\, L_\rho(X^{k+1}, Y, \lambda^k) \\
&= \underset{Y}{\arg\min}\,(G_\Omega(Y) + (\rho/2)\parallel X^{k+1} - Y + u^k \parallel_2^2) \\
&= \underset{Y \in \Omega}{\arg\min}\,(\rho/2)\parallel X^{k+1} - Y + u^k \parallel_2^2 \\
&= \mathscr{P}_\Omega(X^{k+1} + u^k),
\end{aligned} \tag{42}
$$

where $\mathscr{P}_\Omega$ is the projection operator onto the convex set Ω.

Note that the constraints which define the set Ω are local and they only cause the temporal coupling of local variables. For

Require: $P_i^D(t)$ and $SR_i(t), \forall i \in \mathcal{N}_b,\ t$;
Ensure: P^*, S^*, and R^*;
1: Each node collects the information of demand and reserve requirement in a distributed fashion;
2: **for** $k = 0, 1, 2, \ldots$ **do**
3: Each node performs X-update using the distributed bisection method;
4: Each node performs Y-update via local computation;
5: Each node performs λ-update using (35) ;
6: **end for**

Fig. 1 *Algorithm 1: Distributed algorithm for DEDP-S*

Table 1 Cost coefficients of five generators

Generator	Bus	α_i	β_i	$\bar{P}_i$	ΔP_i	$\bar{R}_i$
1	1	0.08	2	80	10	10
2	2	0.06	3	90	20	20
3	3	0.07	4	120	25	25
4	6	0.06	4	130	20	30
5	8	0.08	2.5	80	10	20

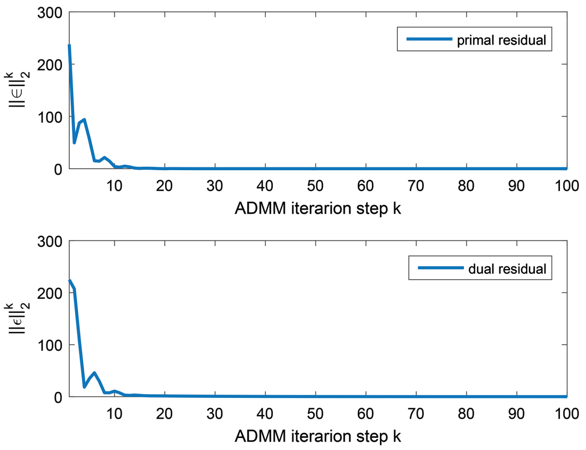

Fig. 2 *Convergence result of the proposed algorithm*

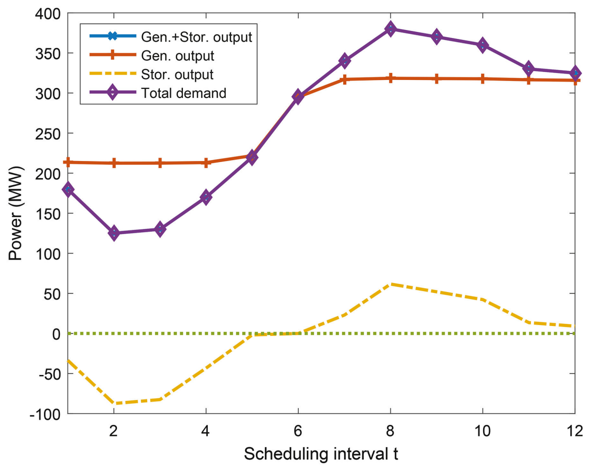

Fig. 3 *Total demand and the outputs from generation and storage*

$i \in \mathcal{N}^g \cup \mathcal{N}^s$, denoting by Ω_i the set defined by its local constraints, we have

$$\Omega = \prod_{i \in \mathcal{N}^g \cup \mathcal{N}^s} \Omega_i,$$

where $\prod$ represents the Cartesian product of sets. Since projection onto linearly constrained sets is a quadratic programme (QP), the Y-update consists of $n^g + n^s$ local QP subproblems. Therefore, the Y-update only requires local computation and it can be accomplished in a distributed fashion simply by applying existing QP methods, e.g. interior point method [28], to each local QP subproblem.

4.5 λ-update

After X^{k+1} and Y^{k+1} are obtained, λ^{k+1} can be solved using (35) by local computation.

4.6 Overall sketch of proposed algorithm

For clarity, we summarise the above procedures in the algorithm (Fig. 1).

5 Simulation

In this section, we present a numerical case performed on the IEEE 14-bus system [29] to show the effectiveness of the proposed algorithm. Consider a DEDP-S with $\tau = 12$ scheduling intervals. The system configuration is given as follows. There are in total five generators in this system, for which we homogenously set $\underline{P}_i = 10\,\text{MW}$ and $\Delta\underline{P}_i = -\Delta\bar{P}_i = -\Delta P_i$. Every generator's cost function contains a quadratic term of which the parameters are given in Table 1. Additionally, we introduce a non-quadratic term to generators 1 and 3, given by

$$50\exp\left(\frac{P_1(t) + 40}{100}\right) \text{ and } 7 \times 10^6 P_3^3(t),$$

respectively. One could easily verify that all the cost functions satisfy Assumption 3. We also place in total five energy storage devices in buses $\{1, 3, 5, 8, 10\}$. For simplicity, we assume that they have homogenous parameters except for their initial amounts of energy, where $\bar{E} = 20\,\text{MW}\Delta\tau$, $\bar{S}^c = \bar{S}^d = 30\,\text{MW}$, $\eta^c = \eta^d = 90\%$ (round-trip efficiency $\eta^c\eta^d = 81\%$) and $E(0) = [0, 40, 80, 120, 160]^T\,\text{MW}\Delta\tau$. Furthermore, at the end of the last scheduling interval, each energy storage is required to maintain an identical minimum amount of energy $E(\tau) = 80\,\text{MW}\Delta\tau$. The system reserve requirement is $SR(t) = 0.25P^D(t)\,\forall t = 1, \ldots, 12$.

To initialise the proposed algorithm, we set $\rho = 1$, Y^0=0 and u^0 =0. Note that the power transmission network does not necessarily align with communication graphs. We do not assign node to bus 7, as it is not associated with generation, load or energy storage. Define two corresponding node sets $\mathcal{N}_a = \mathcal{N}^g \cup \mathcal{N}^s = \{1, 2, 3, 5, 6, 8, 10\}$ and $\mathcal{N}_b = \{1, 2, \ldots, 6, 8, 9, \ldots, 14\}$. For $\mathcal{G}_a$ and $\mathcal{G}_b$, the edge sets $\mathcal{E}_a$ and $\mathcal{E}_b$ are properly chosen to set up two strongly connected directed graphs with self-loops.

The convergence of the proposed algorithm is demonstrated in Fig. 2. The proposed algorithm converges fast, as the primal and dual residuals ϵ^k and ε^k converge to zero after around 15 ADMM iterations.

According to Fig. 3, the generators and the energy storage devices cooperatively supply electricity to meet the time-varying demand. The generators' outputs and the charging/discharging rates of energy storage devices are shown in Figs. 4 and 5, respectively. We can see that the generators' outputs are successfully kept low during on-peak intervals when energy storage devices discharge to share generators' workload.

The evolutions of energy stored in the five devices are shown in Fig. 6, where at the end of the 12th interval, every device maintains the common energy level 80 MW $\Delta\tau$. Finally, Fig. 7 shows that generators and energy storage devices cooperatively provide the required spinning reserve. Specifically, energy storage devices provide more than half of the spinning reserve during off-peak intervals, which greatly mitigates generators' pressure of providing spinning reserve.

6 Concluding remarks

In this paper, we investigate the DEDP-S and propose a distributed algorithm based on the ADMM and the consensus-like algorithm.

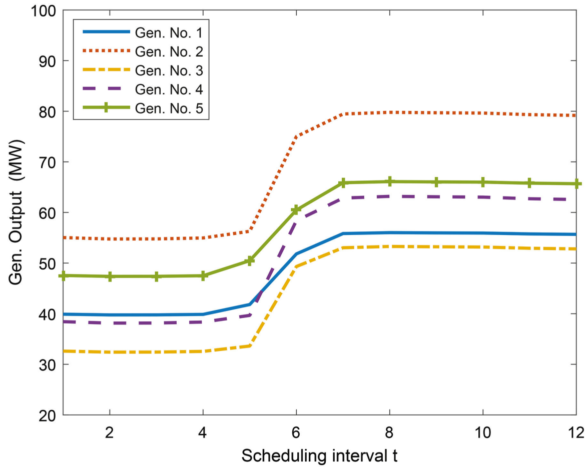

Fig. 4 *Outputs of five generators*

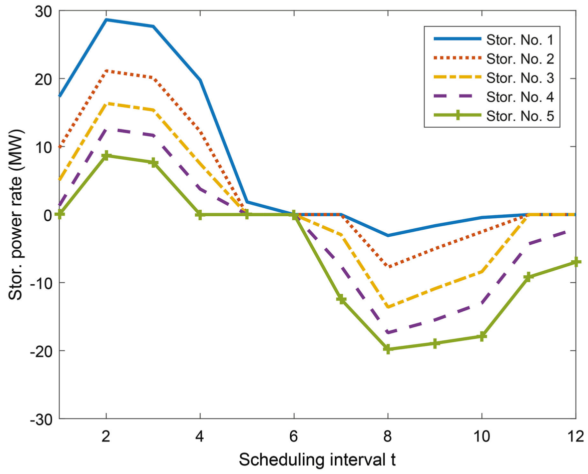

Fig. 5 *Charging/discharging rates of five energy storage devices*

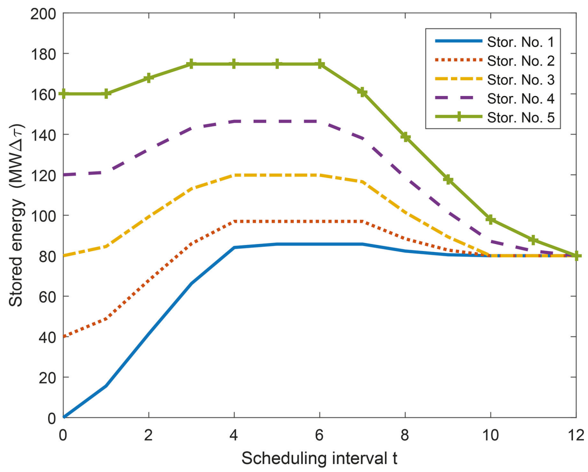

Fig. 6 *Evolutions of energy stored in five energy storage devices*

To the authors' knowledge, this paper is the first not only to propose the mathematical formulation of the DEDP-S where energy storage also provides spinning reserve but also to propose a distributed algorithm to solve the DEDP-S. Our algorithm is fully distributed in the sense that no leader or master nodes are needed, while all the nodes (generators) conduct local computation and merely communicate with their neighbours. Through simulation we show the performance of the proposed method. Future work may include more practical constraints, e.g. transmission losses and line capacity.

7 Acknowledgments

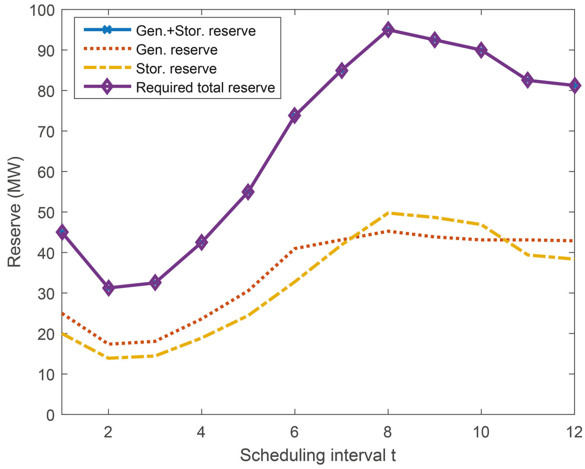

Fig. 7 *System spinning reserve requirement and the reserves from generation and storage.*

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