Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Fast Distributed Power Regulation Method via Networked Thermostatically Controlled Loads

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Abstract: This paper studies the problem of countering the fluctuations of renewable power supply in smart grid. A fully distributed algorithm to control a network of thermostatically controlled loads (TCLs) is proposed to match, in real time, the aggregated power consumption of the TCLs and the forecast power supply. The algorithm is developed by converting the control problem into a consensus problem of individual utility functions. We then show that the problem is equivalent to a convex optimization problem and an algorithm based on distributed bisection method is presented to solve the problem. The proposed algorithm converges fast and is fully distributed, requiring only local information for each TCL and limited communication with neighboring TCLs, yet the available power supply is fairly dispatched among different TCLs. A numerical example is given to illustrate the effectiveness of the algorithm when applied to a network of variable-frequency air-conditioners (VFACs).

1. INTRODUCTION

Research on utilization of renewable energy in smart grid has received tremendous attention from scientific communities due to environmental and economic concerns (Ipakchi and Albuyeh, 2009), (Subramanian et al., 2012). Although many techniques for collecting solar and wind energy have been developed, the integration of distributed generation with conventional generation remains a great challenge, for the power generated from wind and solar energy is fluctuating and of high uncertainty (Doherty and O'Malley, 2005),(Ito et al., 2004). Methods for managing the fluctuations of distributed renewable generation by using storage devices such as battery banks are available, but they are not suitable for large scale operations due to high costs (Murakami et al., 2006), (Ohtaka et al., 2004).

An alternative approach is to allow the energy management system (EMS) to deploy real-time control of the load so that the fluctuations in the power supply can be absorbed by the variations of the load. This is the approach we will explore in this paper. The basic idea is to manipulate a population of thermostatically controlled loads (TCLs), such as air conditioning and refrigeration systems, to meet a supply and demand balance between the aggregated power consumption of TCLs and fluctuating distributed power generation. This concept was initially proposed in Callaway (2009). More specifically, by changing the power consumption of each TCL via adjusting the temperature set point within the user's comfort zone, the population of TCLs can make an aggregated power demand response on hourly or minutely time scale to follow the power forecast of the distributed generators, especially wind driven generators, whose outputs are relatively easier to predict (McDonald and Bruning, 1979). We want to develop a distributed algorithm for the above power regulation problem.

Distributed algorithms for control, estimation and optimization have been intensively investigated in consideration of the issue of large-scale systems and the development of network technology (Bakule, 2008). Spatially distributed large-scale systems interconnected by network are ubiquitous in the real world, where the traditional centralized control algorithms do not work well. A smart grid with distributed renewable power generation is a typical such large-scale system. In recent years, distributed algorithms are developed and applied in smart grids problems, especially for distributed energy management problems from different perspectives. For example, Dominguez-Garcia et al. (2012) address the problem of optimally dispatching a set of distributed energy resources, Yang et al. (2013) consider the economic dispatch problem in terms of the distributed generation side by a consensus based approach, and Guo et al. (2013) concentrate on decentralized control of aggregated residential responsive loads for load response based on game theory.

Continuing along this booming direction, this paper aims to develop a distributed algorithm for the power regulation problem to counter the fluctuations of renewable power supply using networked thermostatically controlled loads. Compared with previous work mainly by Callaway (2009),

⋆ This work was partly supported by Zhejiang Provincial Natural Science Foundation of China LR13F030002.

Koch et al. (2011), Mathieu et al. (2012), the novel features of our work are as follows. We consider variable frequency air conditioners (VFAC) whose power consumption to be controlled can vary from 0 to some rated power. Our control algorithm is fully distributed and is designed based on a novel distributed bisection method so that the total available power is fairly dispatched among different air conditioners. More specially, the power dispatch is done such that the relative temperature deviation, which is the difference between the achievable temperature and target temperature divided by the comfort zone, is identical for all air conditioners. In addition, our algorithm is based on deterministic and heterogeneous models for air conditioners, and thus can be applied to both large-scale systems and small-scale systems. Finally, our distributed algorithm allows air conditioning systems to join and leave the power regulation network from time to time and converges at a very high speed due to the nature of bisection. In comparison, the work in Callaway (2009) where a hybrid model based on probability distribution of on/off state serves as the aggregated model of TCLs, requires a large number of TCLs to ensure the accuracy of the aggregated model. Moreover, the algorithms in Dominguez-Garcia et al. (2012) and Yang et al. (2013) apply to only quadratic cost functions.

2. PRELIMINARIES AND PROBLEM FORMULATION

In this section, we first introduce the VFAC model and then present the power regulation problem.

2.1 VFAC Model

Let n be the number of VFACs under consideration. We assume that the power of VFAC can vary from zero to its rated power continuously. We consider the cooling process and use the following discrete time model, originally developed in Mortensen and Haggerty (1988):

$$T_i[k+1] = a_i T_i[k] + (1-a_i)(T_{a,i} - \eta_i R_i x_i[k]) + w_i[k] \tag{1}$$

where $T_i[k]$ (°C), $T_{a,i}$ (°C) and $x_i[k]$ (kW), $i = 1, 2, \ldots, n$, are the room temperature, ambient temperature, and the power of $VFAC_i$ at time k, respectively. The sampling time interval is denoted by $\Delta\tau$. The parameters in (1) are as follows: $a_i = \exp(-\Delta\tau / C_i R_i)$ represents the thermal characteristics, where C_i (kWh/°C) is the thermal capacitance, and R_i (°C/kW) is the thermal resistance; η_i is the load efficiency, which equals to the rate of energy transfer between the thermal mass and its environment divided by the power consumption of $VFAC_i$; $w_i[k]$ is assumed to be Gaussian white noise with variance $\Delta\tau\sigma^2$.

In this paper, we take $\Delta\tau = 5$ minutes. Though in reality, the parameters, especially the ambient temperature $T_{a,i}$ and the load efficiency η_i, may vary in a small range, in this paper we assume that all the parameters including C_i, R_i, η_i, $T_{a,i}$ are constant without loss of generality. These parameters can be obtained by direct measurements or system identification.

2.2 Problem Formulation

Consider a network consisting of n VFACs that are connected to a power network where the total power supply may fluctuate over time. Suppose the total power supply forecast $P[k]$ is known to all the users. With a reasonable forecasting period, the power supply is considered to be constant in each interval. An illustrative example is given in Fig. 1.

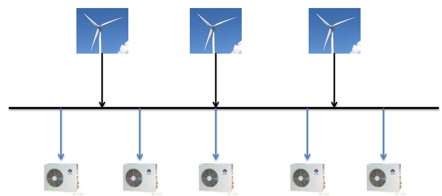

Fig. 1. A network consisting of 10 VFACs and 3 wind power generators.

In order to compensate the power supply fluctuation, the power consumption of n VFACs should satisfy the supply constraint:

$$\sum_{i=1}^{n} x_i[k] = P[k], \ k = 1, 2, \ldots \tag{2}$$

However, for practical use, it may be sufficient to have $|\sum_{i=1}^{n} x_i[k] - P[k]|$ less than a tolerable supply-demand gap. For this purpose, it is assumed that

A1: the forecast of the total power supply $P[k]$ and the total power consumption $D[k-1] = \sum_{i=1}^{n} x_i[k-1]$ at the previous step $k-1$ are known, and the ratio $\gamma[k] = \frac{P[k]}{D[k-1]}$ is broadcast to all users.

This assumption can be met technically in power systems. For the single generator case, the generator can measure the total power consumption at every step and predicts the total power supply it provides in the future. For the multiple generator case, an aggregator (e.g. one of the generators) can collect these information and broadcast the prediction ratio $\gamma[k]$ to users.

On the other hand, another constraint comes from the power constraint of each VFAC ($i = 1, \ldots, n$), i.e.,

$$0 \leqslant x_i[k] \leqslant x_i^{rated}, \ k = 1, 2, \ldots \tag{3}$$

where x_i^{rated} is the rated power of the ith VFAC.

Define an individual utility function $h_i(x_i)$ for each VFAC ($i = 1, \ldots, n$). In order to assure the uniqueness of our problem, we generally assume that

A2: $h_i(x_i)$ is continuously differentiable and satisfies $h_i'(x_i) > 0$, $h_i(-\infty) = -\infty$, and $h_i(\infty) = \infty$, i.e., $h_i(\cdot)$ is a monotonically increasing bijective mapping of $\mathbb{R} \to \mathbb{R}$.

To ensure fairness and equity from the user's perspective, it is desired that after a certain amount of transient time κ, for any pair i and j,

$$h_i(x_i[k]) = h_j(x_j[k]), \ k = \kappa, \kappa+1, \ldots \tag{4}$$

and meanwhile the constraints (2) and (3) are satisfied. In addition, during the transient time period, those VFACs not working with their powers at the lower or upper bounds should meet the equity constraint, i.e., for any pair $i, j \in \{m : 0 < x_m < x_m^{rated}\}$

$$h_i(x_i[k]) = h_j(x_j[k]),\ k = 1, \ldots, \kappa - 1. \quad (5)$$

An individual utility function can be any function of energy cost, room temperature, etc. The only requirement is that it satisfies Assumption A2.

The following is a very simple example of an individual utility function. Suppose every user sets a temperature set-point $T_{s,i}$ and expects the room temperature to remain within a comfort zone $[-\Delta T_i + T_{s,i}, \Delta T_i + T_{s,i}]$. Then the individual utility function can be defined as

$$h_i(x_i[k]) = \frac{T_{s,i} - T_i[k+1]}{\Delta T_i},$$

which means by controlling the power of each VFAC so that the room temperature remains in a comfort zone, the ratio of the temperature error (the difference between the actual room temperature and its set-point) and its tolerable variation reaches consensus for fairness, and all VFACs together counter the power fluctuation. For the solvability of the problem, it is assumed that a solution exists such that $-1 \leqslant h_i(x_i) \leqslant 1$ except those VFACs that either work at their rated power or 0.

From (1), it can be obtained that for $i = 1, \ldots, n$,

$$h_i(x_i(k)) = \alpha_i x_i[k] + \beta_i[k] \quad (6)$$

where

$$\alpha_i = (1 - a_i)\eta_i R_i / \Delta T_i, \text{ and}$$
$$\beta_i[k] = (T_{s,i} - a_i T_i[k] - (1 - a_i) T_{a,i}) / \Delta T_i.$$

3. MAIN RESULTS

In this section, we show the equivalence of our problem and an optimization problem. Then we present a distributed bisection method to solve the problem.

3.1 An Equivalent Optimization Problem

Define $f_i(x_i) = \int_0^{x_i} h_i(y)dy$. Then it is certain that $f_i(x_i)$ is twice continuous differentiable and strictly convex due to the assumption $h_i'(x_i) > 0$. Now consider the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & f(x[k]) = \sum_{i=1}^{n} f_i(x_i[k]) \\ \text{subject to} \quad & \sum_{i=1}^{n} x_i[k] = P[k], \\ & 0 \leqslant x_i[k] \leqslant x_i^{rated},\ i = 1, ..., n. \end{aligned} \quad (7)$$

The following result reveals the equivalence of our problem and the optimization problem (7).

Theorem 1. The solution to the optimization problem (7) satisfies (2)-(3) and

$$h_i(x_i[k]) = h_j(x_j[k])$$

for any i and j in $\mathcal{V}^* = \{m : 0 < x_m[k] < x_m^{rated}\}$.

The proof requires the following lemma.

Lemma 2. (Xiao and Boyd (2006)) $x^* = [x_1^*, \ldots, x_n^*]^T$ is the optimal solution to the optimization problem

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} f_i(x_i) \\ \text{subject to} \quad & \sum_{i=1}^{n} x_i = C \end{aligned}$$

if and only if

$$\sum_{i=1}^{n} x^* = C \text{ and } f_1'(x_1^*) = \cdots = f_n'(x_n^*).$$

Proof of Theorem 1: Denote by $\mathcal{D}$ the constraint set of x in the optimization problem (7) and let x^* be the solution to the optimization problem (7), i.e.,

$$f(x^*[k]) = \min\{f(x[k]) : x[k] \in \mathcal{D}\}.$$

Consider $\mathcal{V}^*$ defined in Theorem 1 and denote

$$\mathcal{V}^\dagger = \{m : x_m^*[k] = 0 \text{ or } x_m^*[k] = x_m^{rated}\}.$$

Then it is clear that

$$f(x^*[k]) = \sum_{i \in \mathcal{V}^*} f_i(x_i^*[k]) + \sum_{j \in \mathcal{V}^\dagger} f_j(x_j^*[k]).$$

Define

$$P^\dagger[k] = \sum_{i \in \mathcal{V}^\dagger} x_i^*[k].$$

Then $x_i^*[k]$, $\forall i \in \mathcal{V}^*$ is the optimal solution to the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{i \in \mathcal{V}^*} f_i(x_i[k]) \\ \text{subject to} \quad & \sum_{i \in \mathcal{V}^*} x_i[k] = P[k] - P^\dagger[k] \end{aligned}$$

Thus, the conclusion follows from Lemma 2. ■

By Theorem 1, we know that our problem is equivalent to the optimization problem (7) and thus the solution is unique. That is, the VFACs will take their unique allocation of power so that the temperature ratios reach consensus in one step unless they are unable to do it due to their power constraints. Take the individual utility function (6) for example. If the current room temperature is beyond the comfort zone too much, then the corresponding air conditioner will work at its rated power, but even working on its rated power, it may still not be able to reduce the temperature ratio to the consensus value.

The optimization problem (7) can also find many applications in economic dispatch problem (EDP) (Yang et al., 2013) and optimal resource allocation problem (ORAP) (Dominguez-Garcia et al., 2012).

3.2 Distributed Bisection Method

In this subsection, we develop a distributed bisection method for our problem as well as for the optimization problem (7). To make our distributed algorithm work, we assume the communication network between VFACs is a connected undirected graph, which is adequate for implementing average consensus algorithm (Olfati-Saber and Murray, 2004). Since our objective is to achieve (4) or (5) while satisfying the constraints (2)-(3), we can start with an initial value λ and let all the utility functions $h_i(x_i[k])$ $(i = 1, \ldots, n)$ equal to λ. By solving $x_i[k] = h_i^{-1}(\lambda)$, each unit computes the average of $x_i[k]$'s by the average consensus algorithm. So each unit has a copy of the average of $x_i[k]$'s (the total power demand), denoted by $\bar{x}_i[k] := \frac{1}{n}\sum_{i=1}^{n} x_i[k]$. Notice that the constraint (2) can also be written as

$$\frac{1}{n}\sum_{i=1}^{n} x_i[k] = \frac{1}{n}P[k].$$

Consequently, each unit compares $\bar{x}_i[k]$ with the average of total power supply $\frac{1}{n}P[k]$ and then sets a new value of λ for new iterations by the bisection method if the feasible interval $[\lambda_{lower}, \lambda_{upper}]$ of λ is known. In practical applications, the feasible interval $[\lambda_{lower}, \lambda_{upper}]$ can be known or estimated *a priori*. For example, for the problem with the utility function defined in (6), all the room temperatures are desired to be kept in the comfort zone $[-\Delta T_i + T_{s,i}, \Delta T_i + T_{s,i}]$ while the air conditioning systems are considered to absorb the power supply fluctuation. The lower and upper bound can be set as $\lambda_{lower} = -1$ and $\lambda_{upper} = 1$ as otherwise the problem does not have a feasible solution.

To deal with the inequality constraint (3), a projection mapping is defined, i.e.,

$$\mathcal{P}_i(x_i) = \begin{cases} x_i & \text{if } 0 < x_i < x_i^{rated} \\ 0 & \text{if } x_i \leqslant 0 \\ x_i^{rated} & \text{if } x_i \geqslant x_i^{rated} \end{cases}$$

The projection mapping resets the value $x_i[k] = h_i^{-1}(\lambda)$ to the constraint set. For simplicity of notation, we use $Q_{ave}[k]$ to denote $\frac{1}{n}P[k]$. The complete description of the distributed bisection method is described in Algorithm 1.

Algorithm 1 Distributed Bisection Method

Input: $Q_{ave}[k]$: the forecast of the average of power supply at time k.
Output: $x_i[k]$: power assignment, $i = 1, ..., n$,
$\bar{x}_i[k]$: the average of total power demand.

- **Initialization:** $\lambda^- = \lambda_{lower}$; $\lambda^+ = \lambda_{upper}$;
- **for** $t = 1, 2, \ldots$ **do**
  - Each unit updates $\lambda = \frac{1}{2}(\lambda^- + \lambda^+)$;
  - Each unit computes $x_i[k] = \mathcal{P}_i\left(h_i^{-1}(\lambda)\right)$.
  - Run a distributed average consensus algorithm to compute $\bar{x}_i[k] = \frac{1}{n}\sum_i x_i[k]$ for each unit;
  - **if** $\bar{x}_i[k] > Q_{ave}[k]$ **then**
    - Each unit updates $\lambda^+ = \lambda$;
  - **else**
    - Each unit updates $\lambda^- = \lambda$;
  - **end if**
- **end for**

Next we show in the following theorem that Algorithm 1 converges to the solution of our problem and also the optimization problem (7).

Theorem 3. Suppose $h_i(x_i)$ satisfies Assumption A2. Then Algorithm 1 converges to the unique optimal solution of Problem (7) as $t \to \infty$.

Proof: Due to Assumption A2, the function $h_i^{-1}(\lambda)$ is monotonically increasing with respect to λ, so is the function $\mathcal{P}_i\left(h_i^{-1}(\lambda)\right)$. Thus, if there is a feasible solution for the problem, then the average $\frac{1}{n}\sum_{i=1}^{n}\mathcal{P}_i\left(h_i^{-1}(\lambda)\right)$ is strictly increasing with respect to λ. Therefore, Algorithm 1 converges. Moreover, since the optimal solution is unique by Theorem 1, Algorithm 1 converges to the unique one. ■

Dealing with unknown total number n of VFACs. In practical applications, the total number n of VFACs in the network may not be known and may vary from time to time. Notice that the average of total power supply, $Q_{ave}[k]$, in Algorithm 1 can be calculated $Q_{ave}[k] = \gamma[k]\bar{x}_i[k-1]$ where $\gamma[k]$ is available (see Assumption A1). So the algorithm does not require to know the total number n in the network. For the initial step at $k = 0$, every VFAC works at their rated power and meanwhile computes the average of total power demand, $\bar{x}_i[0]$, by a distributed average consensus algorithm. Then $Q_{ave}[1] = \gamma[1]\bar{x}_i[0]$ can be calculated and the algorithm can run iteratively, for which only at the initial step the constraint (2) is not satisfied.

Dealing with dynamic networks. In real situations, air conditioners may power on and off from time to time. So it consists of a dynamic network, but our approach still works for such a scenario. That is, when an air conditioner is powered on, it communicates with its neighbors to receive the average of total power demand $\bar{x}_i[k-1]$ of the previous step and then participates in the power regulation control by running Algorithm 1 to get its own power allocation.

Remark 1. In Algorithm 1, each unit only needs to exchange with its neigbhors the value of $x_i[k]$ to compute the average. So it can be implemented in a fully distributed manner. For the same optimization problem (7), two different distributed algorithms are provided in Dominguez-Garcia et al. (2012) and Yang et al. (2013), where the former considers a consensus based approach with an auxiliary variable recording the supply-demand mismatch amount and the latter adopts the idea of using a ratio consensus algorithm to solve its Lagrange dual. However, both approaches can only apply to the quadratic cost functions and are hard to be extended to general cost functions while our algorithm is competent for any function satisfying Assumption A2. Moreover, unlike our algorithm, it seems hard to let the algorithms in Dominguez-Garcia et al. (2012) and Yang et al. (2013) work in a situation where members participating in optimization may vary over time. In addition, in Dominguez-Garcia et al. (2012) the computation and communication package size will explode as the network size grows.

3.3 Distributed Power Regulation

In this subsection, we present the complete process for distributed power regulation via n VFACs.

Algorithm 2 Distributed Power Regulation

- **for** $k = 1, 2, ...$ **do**
  - Each VFAC receives the forecast $\gamma[k]$;
  - Each VFAC measures the current temperature $T_i[k]$ and other necessary physical parameters;
  - Each VFAC computes its own power assignment by distributed bisection method (Algorithm 1);
  - Each VFAC adjusts its working power.
- **end for**

For practical use, each VFAC can stop Algorithm 1 when certain tolerable supply-demand gap is reached, e.g., 1% of the total power supply. Due to the nature of bisection, it can be seen that within ten steps, λ converges to the interval $\frac{1}{2^{10}}|\lambda^{+} - \lambda^{-}|$, which is super fast.

4. NUMERICAL SIMULATIONS

In this section, a numerical example is presented to demonstrate the algorithm developed in the paper.

Just for the demonstration purpose, we consider a small number of VFACs (10 VFACs in a network). Suppose they communicate each other to run an average consensus algorithm according to the connected graph given in Fig. 2 where each node represents a VFAC in the network.

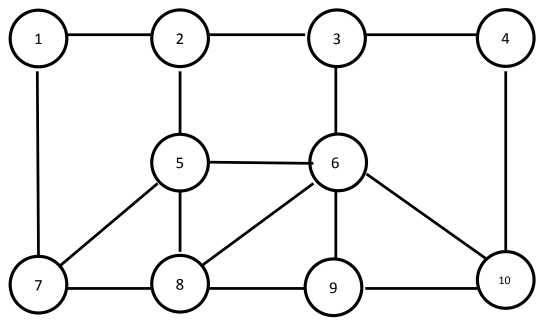

Fig. 2. The Communication Network of VFACs

In the simulation we consider the individual utility function defined in (6) with the following parameters: The thermal capacity C_i (kWh/°C), thermal resistance R_i (°C/kW), ambient temperature $T_{a,i}$ (°C), temperature set point $T_{s,i}$ (°C), thermal efficiency η_i, and ΔT_i (°C), are different for different VFACs and given in a vector form below.

$$
\begin{aligned}
C &= [1.5760\ 1.9222\ 1.8721\ 1.9661\ 1.7104\\
&\quad 1.9218\ 1.5826\ 1.5270\ 1.7037\ 1.6652],\\
R &= [13.091\ 12.177\ 12.834\ 12.586\ 12.252\\
&\quad 12.780\ 12.315\ 12.773\ 12.027\ 12.452],\\
T_a &= [29.8241\ 30.3240\ 30.1523\ 30.0950\ 29.6795\\
&\quad 30.1077\ 30.4365\ 29.6200\ 29.7774\ 30.2843],\\
T_s &= [25\ 27\ 25\ 25\ 25\ 26\ 24\ 25\ 24\ 25],\\
\eta &= [2.3662\ 2.2277\ 2.2583\ 2.6941\ 2.6169\\
&\quad 2.3903\ 2.7701\ 2.2207\ 2.4632\ 2.4289],\\
\Delta T &= [2.0\ 2.0\ 2.0\ 2.5\ 2.0\ 3.0\ 2.5\ 2.0\ 1.5\ 2.5].
\end{aligned}
$$

Fig. 3 shows the forecast of the power supply for $12h$. Fig. 4 shows the evolution of temperature ratios of all the units by applying our proposed strategy. Ideally, all the temperature ratios need to fall within the $\pm 100\%$ region. We see that this is achieved after about 0.7 hour, during which more and more VFAC's temperature ratios become synchronized. Especially, from about 0.7 h to 1.7, all the temperature ratios reach consensus except the 6th unit. During the period between 7.5 h and 10 h, the curves disperse a little, which is mainly because the power supply is very little, thus making many VFACs being assigned zero power (turned off), and their rates of natural warming are different.

We take the 7-th VFAC as an example. Fig. 5 shows the temperature evolution over the 12 hours. The temperature

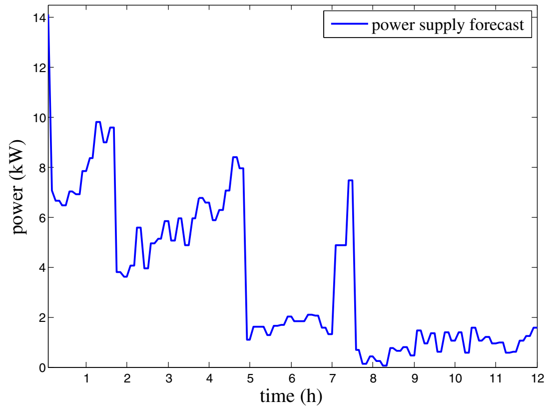

Fig. 3. Power supply forecast for 12 hours.

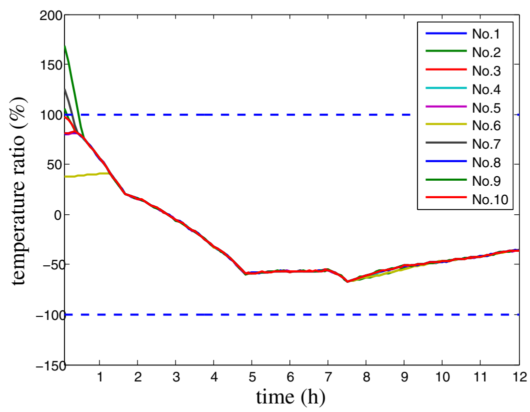

Fig. 4. Temperature ratios of the VFACs.

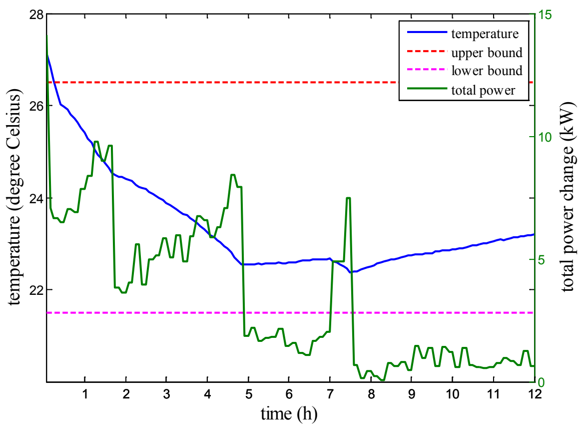

Fig. 5. Temperature for the 7th VFAC and total power supply.

of the 7-th VFAC enters into the comfort zone after about $0.7h$, and stays within the comfort zone from then on.

Also, the demand-supply gap (the difference between the total power supply and total power demand) is plotted in Fig. 6 for the above example, in which the distributed bisection algorithm runs 20 iterations at every step. From the simulation result we can observe that the demand-supply gap arrives below 1% of the total power supply.

Finally, another simulation is conducted with a network of 1000 VFACs. Moreover, the network may change over time as some air conditioners may power off and some may power on from time to time. The algorithm runs only when the air conditions are on, for which Algorithm 1 again runs 20 iterations at each step. The demand-supply gap of this example is plotted in Fig. 7, from which we can see that the demand-supply gap still meets the practical requirement of below 1% the total power supply though the network changes over time.

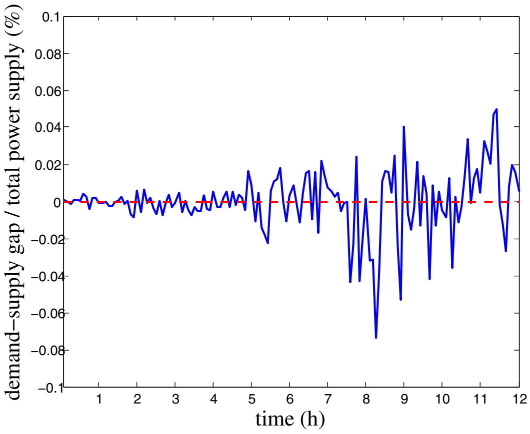

Fig. 6. The demand-supply gap for the example of 10 VFACs in a static network.

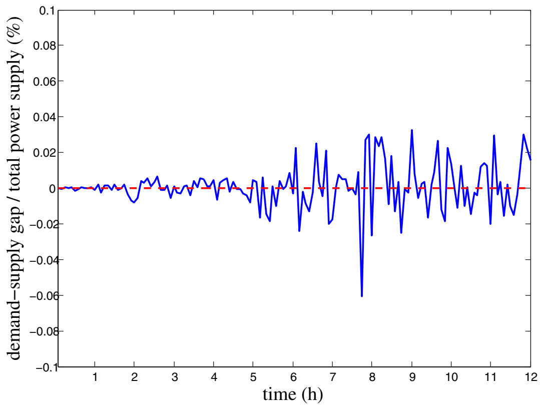

Fig. 7. The demand-supply gap for an example of 1000 VFACs, in which the total number varies from time to time.

5. CONCLUSIONS

In the paper, we develop a distributed scheme for real-time power regulation of fluctuating renewable power generation via aggregated VFACs. We formulate the fair power dispatch problem among VFACs into a convex optimization problem, for which a fully distributed bisection method based on average consensus algorithm is presented. Future work would be applying our algorithm to other types of loads and energy storage devices, such as refrigerators and plug-in electric vehicles.

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