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Optimal state estimation using randomly delayed measurements without time stamping

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SUMMARY

This paper studies an optimal state estimation (Kalman filtering) problem under the assumption that output measurements are subject to random time delays caused by network transmissions without time stamping. We first propose a random time delay model which mimics many practical digital network systems. We then study the so-called *unbiased, uniformly bounded* linear state estimators and show that the estimator structure is given based on the average of all received measurements at each time for different maximum time delays. The estimator gains can be derived by solving a set of recursive discrete-time Riccati equations. The estimator is guaranteed to be optimal in the sense that it is unbiased with uniformly bounded estimation error covariance. A simulation example shows the effectiveness of the proposed algorithm. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The problem of state estimation for systems with random time delays has attracted great attention due to the wide applications in signal processing, control and communication systems [1–5]. The random measurement problem for state estimation was studied as early as in [6]. In the recent years, many results have been reported for networked control systems with random time delays [7–13].

In [8] and [14], the least mean square filtering problem was discussed for systems with a single random sampling delay. Estimation problems for systems with random delays and uncertain measurements were also investigated in [15] and [7]. Zhang and Xie [16] studied the optimal estimation problem for discrete-time systems with time-varying delays in the measurement channel, and the measurements were time stamped. Zhang *et al.* [17] studied linear estimation with random delayed observations. Schenato [18] proposed estimators subject to simultaneous random packet delay and packet dropout, and this allowed packets to arrive in bursts or even out of order at the receiver side, as long as the measurements were time stamped. Moayedi *et al.* [19] considered the state estimation with the models corresponding to the uncertainties of random measurement delays, dropouts, and missing measurements in networked control systems.

Without time stamping, Sun [20] proposed the optimal filtering problem for discrete-time stochastic linear system with multiple random measurement delays. Sun [21] also investigated the estimation problem for systems with bounded random measurement delays and packet dropouts,

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which were described by some binary distributed random variables with known probabilities. But in [21], the network model could receive the same measurement multiple times, and at the same time, an excessively high packet loss rate occurred, which did not fit most communication protocols. In fact, for most network protocols, random time delays mean that more than one measurement may be received at each time instant. That is, measurements are received in bursts of various sizes.

In this paper, we propose the optimal estimation problem where observation packets are subject to bounded random delays. This allows packets to arrive in bursts at the receiver side. We assume that there are no packet dropouts and the packets can not be received repeatedly, but no time stamping is available for the measurements. A new network model is presented to describe the packet transmission. Three cases are considered. The first one assumes that the maximum time delay N is equal to 1. The second case allows $N > 1$ but with the restriction that there is no time reversal among different received data bursts while allowing time reversal for packets within a burst. This is a realistic assumption for most network protocols when time delays are not serious. The third case allows $N > 1$ and time reversal among the received data bursts. For each case in the former discussions, we derive an optimal estimator by minimizing the estimation error covariance subject to the constraints that the estimate is unbiased and that the estimation error covariance is uniformly bounded. The estimator gains are given recursively and are in terms of Riccati equations.

This paper is organized as follows. Section 2 formulates the optimal estimation problem and describes the network model for random delays; Sections 3–5 present solutions to the optimal estimation problem using the new network model. In particular, Section 3 studies the case with maximum delay equal to 1; Section 4 studies the case with larger maximum delay but with the restriction that time reversal for the measurements can occur only within the packets received at the same time; and Section 5 removes this restriction. Section 6 considers the case as in Section 4 but for a stable system. Section 7 gives a simulation example. Concluding remarks are drawn in Section 8.

2. PROBLEM FORMATION

Consider the following discrete-time linear stochastic system:

$$\begin{aligned} x_{k+1} &= Ax_k + v_k \\ y_k &= Cx_k + w_k \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $y_k \in \mathbb{R}^m$ is the measured output, $v_k \in \mathbb{R}^n$ and $w_k \in \mathbb{R}^m$ are the system noise and measurement noise, respectively, $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$. The initial state x_0 and v_k, w_k are Gaussian and independent, and their means and variances are denoted by $(\hat{x}_{0|-1}, 0, 0)$ and $(P_{0|-1}, Q_k, R_k)$, respectively, with $R_k > 0$. Without loss of generality, we assume that $\hat{x}_{0|-1} = 0$ and that the pair (A, C) is observable.

2.1. Network delay model

For the state estimation problem, we study in this paper, the output measurements are transmitted over a digital network and are thus subject to random time delays. We are interested in the scenario where the measurements are transmitted without time stamping (neither by the measurement sensor nor by the network protocol). To establish a suitable time delay model, we consider the following typical features in most modern communication networks (e.g., networks based on the popular IEEE 802.15.4 Standard):

- The network is designed with a low packet loss probability p_l. For wired networks, $p_l \leqslant 10^{-6}$ is typically required. For wireless networks, it is common to require $p_l \leqslant 10^{-3}$.
- The communication protocol ensures that a transmitted packet is received no more than once.

The combined effect of the two properties earlier and random time delay means that the number of packets received at each sampling time (for the discrete-time system) varies, in contrary to many commonly used random time delay models where the number of received packets at each sampling time is constant (typically one measurement).

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With the previous discussions in mind, the following assumptions about the network transmission will be made in the sequel:

A1: The measurements are not time stamped.
A2: There are no packet losses, that is, every measurement arrives at the receiver end.
A3: The maximum time delay $N \geqslant 1$ is finite and is known.
A4: Denoting by τ_k the network transmission delay associated with the output measurement y_k, that is, y_k is received at the time instant $k + \tau_k$, then the delay probabilities $\text{Prob}(\tau_{k=i})$, $i = 0, 1, \cdots, N$, are assumed to be nonzero and independent of the measurement time, that is, $\text{Prob}(\tau_{k=i}) = \rho_i > 0$ for some known ρ_i, with $\rho_0 + \rho_1 + \cdots + \rho_{N=1}$.

With the previous assumptions, we understand that the measurement y_k will be received *once* and *once only* with time delay $0 \leqslant \tau_k \leqslant N - 1$. This implies that at each time *k*, the number of received measurements can vary from 0 to *N*. Denoting the set of received measurements at time *k* by z_k, then its cardinal number ranges from 0 to *N*. We emphasize that z_k is a *set* rather than an ordered sequence because of lack of time stamping in the transmission.

The network delay model introduced earlier applies to situations where transmission delays dominate, whereas problems such as transmission errors, packet losses, and quantization errors are negligible. This model is distinct from two commonly used models in the literature, as detailed in the succeeding discussions.

The first commonly used model assumes that measurements are time stamped. As shown in [1], time-stamped delayed measurements can be re-aligned easily, and the resulting state estimation problem is still more or less a standard Kalman filtering problem. More precisely, at time k, the measurements y_0 to y_{k-N} are fully obtained so that the one-step prediction of x_{k-N+1} (i.e., prediction of x_{k-N+1} based on y_0 to y_{k-N}) can be carried out using the standard Kalman filter, and the estimation error covariance P_{k-N+1} is easily established and deterministic (i.e., independent of the random time delays). Also, between $k - N + 1$ and *k*, there may be some measurements available, and the prediction of x_{k+1} can be carried out using a Kalman filter with missing data. The only technical difficulty is that the measurements between $k - N + 1$ to *k* are available randomly, thus the estimation error covariance at P_{k+1} at time $k + 1$ is random and its stochastic properties may be somewhat difficult to analyze. Nevertheless, P_{k+1} is evolved from P_{k-N+1} linearly, which means that the stability of the state estimator is always guaranteed.

The second commonly used model does not assume time stamping, but assumes instead that, at each time *k*, one and only one randomly delayed measurement is received, that is,

$$\tilde{y}_k = y_{k-\tau_k} \tag{2}$$

where $\tau_k = 0, 1, \cdots, N$ for some N with probabilities $\text{Prob}(\tau_k = i) = \rho_i$ as in the previous discussion. This model has been widely used, as shown in, for example, [20–22]. On the surface, this model appears to be similar to our model, but there is a sharp difference between the two. Indeed, we claim that the model (2) is inappropriate for most communication protocols. To illustrate this, we consider the case where $N = 1$ and $\rho_0 = \rho_1 = 0.5$. With (2), $\tilde{y}_k = y_k$ with probability of 0.5 and $\tilde{y}_{k+1} = y_k$ with probability of 0.5 as well. Because y_k can be received only at k or $k + 1$, it is clear that the probability that y_k obtains lost equals the probability that $\tilde{y}_k = y_{k-1}$ and $\tilde{y}_{k+1} = y_k$, which equals 0.25. It is not possible to perceive any network protocols to be designed to produce such a high *inherent* packet loss probability.

To properly model the transmission time delays, we define m_k to be the number of missing measurements at time k (prior to accepting z_k) and denote the cardinal number of z_k by r_k. It is clear that $0 \leqslant m_k \leqslant N$ and it obeys the following simple dynamic model:

$$m_{k+1} = m_{k+1} - r_k \tag{3}$$

Because of Assumption A2, we have the following constraint for r_k:

$$r_k \leqslant m_k + 1;\ r_k \geqslant \begin{cases} 1 & \text{if } m_k = N; \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

which simply means that the resulting m_{k+1} must be between 0 and *N*.

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When $r_k > 1$, there are $r_k!$ ways to order the measurements in z_k, which will be denoted by $z_k^{(1)}, z_k^{(2)}, \cdots, z_k^{(r_k!)}$, blueand $r_k!$ is the factorial of r_k. Thus, z_k, when viewed as a random vector (instead of a set, which involves some abuse of notation), can be written as

$$z_k = \gamma_k^{(1)} z_k^{(1)} + \gamma_k^{(2)} z_k^{(2)} + \cdots + \gamma_k^{(r_k!)} z_k^{(r_k!)} \tag{5}$$

where $\gamma_k^{(i)}$ are random variables such that only one of them equals 1 and the rest are 0. We denote $\gamma_k = \left\{\gamma_k^{(1)}, \gamma_k^{(2)}, \cdots, \gamma_k^{(r_k!)}\right\}$. To reflect the fact that the particular ordering of the received measurements is unknown due to the lack of time stamping, we further assume that

A5: $p_k^{(i)} = \text{Prob}\left(\gamma_k^{(i)} = 1\right) > 0$ for all $i = 0, 1, \cdots, r_k!$.

We note that $p_k^{(i)}$ are related to ρ_i, but the exact relationship is not important for the study in the sequel.

2.2. State estimation criteria

Denote $Z_k = \bigcup_{i=0}^k z_k$. We seek a linear state estimator for one-step-ahead prediction, that is, we want to compute an estimate $\hat{x}_{k+1|k} = \mathcal{L}_{k+1|k}(Z_k)$ of the state x_k using a linear operator $\mathcal{L}_{k+1|k}$. It is useful also to consider a linear estimate $\hat{x}_{t|k} = \mathcal{L}_{t|k}(Z_k)$ of the state x_t for any time t. We define the estimation error $e_{t|k}$ and estimation error covariance $P_{t|k}$ as follows:

$$e_{t|k} = x_t - \hat{x}_{t|k} \tag{6}$$

$$P_{t|k} = \mathcal{E}\left[\mathrm{e}_{t|k} e_{t|k}^T\right] \tag{7}$$

where the superscript T stands for matrix transpose, $\mathcal{E}$ is the expectation with respect to all v_k, w_k, x_0, and γ_k.

The optimal estimator we seek is required to satisfy the following properties:

C1 : Unbiased estimate, that is, $\mathcal{E}[e_{k+1|k}] = 0$ for all k;
C2 : Uniformly bounded estimation error covariance, as defined in the succeeding discussions; and
C3 : Minimum estimation error covariance $P_{k+1|k}$.

Definition 1
The estimation error covariance $P_{k+1|k}$ is called uniformly bounded if there exists a constant $M > 0$, independent of $P_{0|-1}$, such that

$$P_{k+1|k} \leqslant M, \text{ for all } k = 0, 1, 2, \cdots \tag{8}$$

The associated state estimator will be called *uniformly bounded*.

3. OPTIMAL ESTIMATOR FOR AN UNSTABLE SYSTEM WITH $N = 1$

This section considers the case where the system is unstable and the random delay τ_k is either 0 or 1. It follows that m_k is either 0 or 1, and the state transition diagram of m_k is easily shown in Figure 1, which involves the following cases:

Case 1
$m_k = 0$. This means that measurements $y_0, y_1, \cdots, y_{k-1}$ all have been received. There are two possible cases by the number of received measurements r_k:

Case 1.1 : $r_k = 0$, that is, no measurement is received at time k. This will result in $m_{k+1} = 1$;
Case 1.2 : $r_k = 1$, that is, one measurement is received. Because $m_{k-1} = 0$, this measurement must be y_k, and the resulting $m_{k+1} = 0$.

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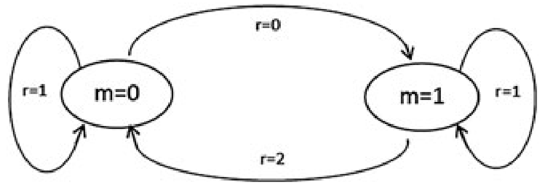

Figure 1. State transition diagram for $N = 1$.

Case 2
$m_k = 1$. This means that y_{k-1} is missing at time $k-1$, and all previous measurements are received (due to Assumptions A2–A3). Again, there are two possible cases by r_k:

Case 2.1 : $r_k = 1$. Because of Assumption A2, the received measurement at time k must be y_{k-1} and $m_{k+1} = 1$ because y_k is still missing;
Case 2.2 : $r_k = 2$. The received measurements must be y_{k-1} and y_k (but without known order). Subsequently, $m_{k+1} = 0$.

Note that for Case 2.2, following (5) and Assumption A5, we have

$$z_k = (1-\gamma_k)\begin{bmatrix} y_{k-1} \\ y_k \end{bmatrix} + \gamma_k \begin{bmatrix} y_k \\ y_{k-1} \end{bmatrix} \tag{9}$$

where γ_k takes value of 0 or 1 with $\text{Prob}(\gamma_k = 1) = p_k$ and $\text{Prob}(\gamma_k = 0) = 1 - p_k$ for some $0 < p_k < 1$.

The main result in this section is given later:

Theorem 1
Consider the one-step-ahead linear state estimation problem for system (1) with unstable matrix A, assumptions A1–A5 and optimality criteria C1–C3. The state estimate update at time $k \geqslant 0$ is given as follows:

- For Case 1.1, the state estimate update is given by

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} \tag{10}$$

with the estimation error covariance update given by

$$P_{k+1|k} = AP_{k|k-1}A^T + Q_k \tag{11}$$

- For Case 1.2, the state estimate update is given by

$$\hat{x}_{k+1|k} = (A - H_k C)\hat{x}_{k|k-1} + H_k y_k \tag{12}$$

with the estimation error covariance update given by

$$P_{k+1|k} = AP_{k|k-1}A^T - H_k\left(CP_{k|k-1}C^T + R_k\right)H_k^T + Q_k \tag{13}$$

where

$$H_k = AP_{k|k-1}C^T\left(CP_{k|k-1}C^T + R_k\right)^{-1} \tag{14}$$

- For Case 2.1, the state estimate update is given by

$$\hat{x}_{k|k-1} = (A - H_k C)\hat{x}_{k-1|k-2} + H_k y_{k-1} \tag{15}$$

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} \tag{16}$$

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with the estimation error covariance update given by

$$P_{k|k-1} = AP_{k-1|k-2}A^T + Q_{k-1} - H_k\left(CP_{k-1|k-2}C^T + R_{k-1}\right)H_k^T \quad (17)$$

where

$$H_k = AP_{k-1|k-2}C^T\left(CP_{k-1|k-2}C^T + R_{k-1}\right)^{-1} \quad (18)$$

and

$$P_{k+1|k} = AP_{k|k-1}A^T + Q_k \quad (19)$$

- For Case 2.2, the state estimate update is given by

$$\hat{x}_{k+1|k} = \left(A^2 - \frac{1}{2}H_k(C + CA)\right)\hat{x}_{k-1|k-2} + H_k\bar{z}_k \quad (20)$$

with the estimation error covariance update given by

$$P_{k+1|k} = A^2P_{k-1|k-2}(A^2)^T - H_kM_kH_k^T + AQ_{k-1}A^T + Q_k \quad (21)$$

where $\bar{z}_k = \frac{1}{2}(y_{k-1} + y_k)$,

$$H_k = \frac{1}{2}\left(A^2P_{k-1|k-2}(C + CA)^T + AQ_{k-1}C^T\right)M_k^{-1} \quad (22)$$

and

$$M_k = \frac{1}{4}\left[(C + CA)P_{k-1|k-2}(C + CA)^T + CQ_{k-1}C^T + R_{k-1} + R_k\right] \quad (23)$$

The proof of Theorem 1 will be deferred as a special case of Theorem 2.

4. OPTIMAL ESTIMATOR FOR AN UNSTABLE SYSTEM WITH $N > 1$ AND RESTRICTED TIME REVERSAL

In this section, we generalize the result of Theorem 1 to allow multiple step delays (i.e., $N > 1$), but with the following assumption:

A6: The measurements are not time stamped, but the received measurement bursts are in order. That is, the received bursts follow the first-in-first-out (FIFO) principle, but the order of the measurements within each burst is not known.

This assumption holds for networks where time delays are not excessive. Also, it is straightforward to see that this assumption is automatically satisfied for $N = 1$ under assumptions A1–A5. The state transition diagram for $N > 1$ is given in Figure 2.

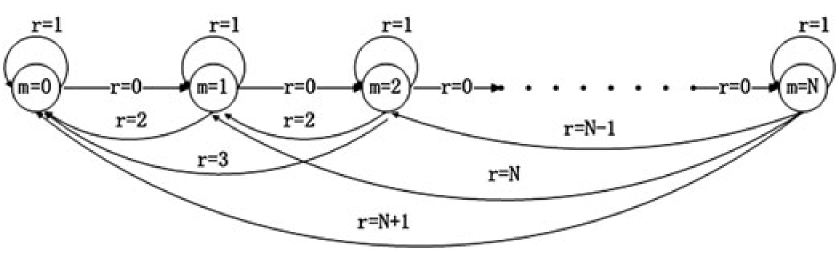

Figure 2. State transition diagram for $N > 1$.

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Because m_k measurements are missing at time k before z_k arrives, all the measurements from time 0 to $k-m_k+r_k-1(=k-m_{k+1})$ will be received after z_k is received (because of assumption A5). Our state estimation problem then involves computing the one-step-ahead linear estimate $\hat{x}_{k+1-m_{k+1}|k}$ and using this to compute the estimate $\hat{x}_{k+1|k}$. We want $\hat{x}_{k+1-m_{k+1}|k}$ (respectively, $\hat{x}_{k+1|k}$) to be a linear function of Z_k, the estimation error

$$e_{k+1-m_{k+1}|k} = x_{k+1-m_{k+1}} - \hat{x}_{k+1-m_{k+1}|k}$$

(respectively, $e_{k+1|k} = x_{k+1} - \hat{x}_{k+1|k}$) to be unbiased and uniformly bounded, and its covariance $P_{k+1-m_{k+1}|k}$ (respectively, $P_{k+1|k}$) to be minimized. The Theorem 1 is given as follows:

Theorem 2

Consider the one-step-ahead linear state estimation problem for system (1) with unstable matrix A, assumptions A1–A6 and optimality criteria C1–C3. The state estimate update at time k is given as follows:

Step 1: If $r_k > 0$, the estimate for $x_{k+1-m_{k+1}}$ is given by

$$\hat{x}_{k+1-m_{k+1}|k} = H_k \bar{z}_k + \left(A^{r_k} - H_k \frac{1}{r_k} \sum_{i=0}^{r_k-1} CA^i \right) \hat{x}_{k-m_k|k^-} \tag{24}$$

where $\bar{z}$ is the average of the elements in z_k, k^- is the time instant just before k with a nonzero r_{k^-},

$$H_k = \frac{1}{r_k} \left(A^{r_k} P_{k-m_k|k^-} \left(\sum_{i=0}^{r_k-1} CA^i \right)^T + \sum_{i=0}^{r_k-2} \sum_{j=0}^{i} A^{i+1} Q_{k-m_k+r_k-2-i} (CA^j)^T \right) M_k^{-1} \tag{25}$$

and

$$M_k = \frac{1}{r_k^2} \left(\left(\sum_{i=0}^{r_k-1} CA^i \right) P_{k-m_k|k^-} \left(\sum_{i=0}^{r_k-1} CA^i \right)^T + \sum_{i=0}^{r_k-1} R_{k-m_k+i} + \sum_{i=0}^{r_k-2} \sum_{j=0}^{i} CA^j Q_{k-m_k+i-j} (CA^j)^T \right) \tag{26}$$

The corresponding estimation error covariance is given by

$$P_{k+1-m_{k+1}|k} = A^{r_k} P_{k-m_k|k^-} (A^{r_k})^T - H_k M_k H_k^T + \sum_{i=0}^{r_k-1} A^i Q_{k-m_k+i} (A^i)^T \tag{27}$$

Step 2: If $m_{k+1} > 0$, the estimate for x_{k+1} is given by

$$\hat{x}_{k+1|k} = A^{m_{k+1}} \hat{x}_{k+1-m_{k+1}|k} \tag{28}$$

with the corresponding estimation error covariance given by

$$P_{k+1|k} = A^{m_{k+1}} P_{k+1-m_{k+1}|k} (A^{m_{k+1}})^T + \sum_{i=0}^{m_{k+1}-1} A^i Q_{k-i} (A^i)^T \tag{29}$$

It is straightforward to check that Theorem 2 includes Theorem 1 as a special case.

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Before proving Theorem 2, we state some properties of the optimal state estimator. Define the innovation ε_k as

$$\varepsilon_k = z_k - \hat{z}_k \tag{30}$$

where $\hat{z}_k$ is the one-step-ahead prediction of z_k, that is,

$$\hat{z}_k = \mathcal{E}[z_k | \hat{x}_{k-m_k|k^-}] \tag{31}$$

where k^- is as defined in Theorem 2. The expression for $\hat{z}_k$ depends on the realization of (5). We take $N = 1$ for example. For Case 1.1, z_k is void, so $\hat{z}_k$ is void; For Case 1.2, we have $z_k = y_k$, so $\hat{z}_k = C\hat{x}_{k|k-1}$; for Case 2.1, we have $z_k = y_{k-1}$, so $\hat{z}_k = C\hat{x}_{k-1|k-2}$; for Case 2.2, z_k is given by (9), so

$$\hat{z}_k = \begin{bmatrix} (1-p_k)C + p_k CA \\ p_k C + (1-p_k)CA \end{bmatrix} \hat{x}_{k-1|k-2} \tag{32}$$

The optimal state estimator given in Theorem 2 enjoys the following properties:

Theorem 3
For any time instants k and t with $r_k > 0$ and $r_t > 0$, we have

- ε_t and ε_k are uncorrelated if $k \neq t$;
- $e_{k+1-m_{k+1}|k}$ and ε_t are uncorrelated if $k \geqslant t$.

We now provide a proof for Theorems 2 and 3 together.

Proof
The proof is carried out recursively. Given any time k with $r_k > 0$, because $\hat{x}_{k+1-m_{k+1}|k}$ is a linear function of Z_k, it is also a linear function of $\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_k$ and $\hat{x}_{k-m_k|k^-}$, that is,

$$\hat{x}_{k+1-m_{k+1}|k} = F_k \hat{x}_{k-m_k|k^-} + \sum_{t=0}^{k} G_k^{(t)} \varepsilon_t \tag{33}$$

for some F_k and $G_k^{(t)}, t = 0, 1, \cdots, k$. We assume that the state estimates prior to k are such that, for any $t < k$, ε_k is uncorrelated with ε_t and $e_{k-m_k|k^-}$ is uncorrelated with ε_t. We will prove that $e_{k+1-m_{k+1}|k}$ is uncorrelated with ε_t for all $t \leqslant k$. Then, letting k^+ be the earliest time after k with $r_{k^+} > 0$, the previous text will imply that ε_{k^+} is uncorrelated with ε_t for all $t \leqslant k$ because ε_{k^+} is a linear function of $e_{k+1-m_{k+1}|k}$ plus some independent noise. Once the above results hold, then by recursion, the results in Theorem 2 holds.

Indeed, from (1) and (33), we obtain

$$e_{k+1-m_{k+1}|k} = A^{r_k} x_{k-m_k} + \sum_{i=0}^{r_k-1} A^i v_{k-i-m_{k+1}} - F_k \hat{x}_{k-m_k|k^-} - \sum_{t=0}^{k} G_k^{(t)} \varepsilon_t$$

To ensure the estimate unbiased, we set $\mathcal{E}\left(e_{k+1-m_{k+1}|k}\right)$ to zero and obtain $F_k = A^{r_k}$. Hence,

$$e_{k+1-m_{k+1}|k} = A^{r_k} e_{k-m_k|k^-} + \sum_{i=0}^{r_k-1} A^i v_{k-i+m_{k+1}} - \sum_{t=0}^{k} G_k^{(t)} \varepsilon_t$$

Now, we consider the condition for the estimate error covariance to be uniformly bounded. Note that, in the previous discussion, all the $v_{k-i-m_{k+1}}$ are independent of ε_t. Recall the expression for z_t in (5) and note that, for $r_t > 1$, z_t depends on γ_t (i.e., the ordering of the packets) but $\hat{z}_t$ does not. Hence, ε_t depends on γ_t, thus is a function of not only the past estimation errors but also the past state of the system (1). It follows that, unless $G_k^{(t)}$ are chosen in a way that $G_k^{(t)}\varepsilon_t$ does not

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depend on γ_t, the estimation error $e_{k+1-m_{k+1}|k}$ must depend on the past state of (1), which in turn depend on x_0, implying that the covariance of $e_{k+1-m_{k+1}|k}$ will depend on $P_{0|-1}$. Thus, due to the instability of the matrix A, to ensure the estimation error covariance uniformly bounded, we must have $G_k^{(t)} = H_k^{(t)}[I\ I\ \ldots\ I]/r_t$ for some $H_k^{(t)}$. Consequently,

$$G_k^{(t)}\varepsilon_t = H_k^{(t)}\bar{\varepsilon}_t$$

where $\bar{\varepsilon}_t$ is the average of the elements in ε_t, thus independent of γ_t. For notational simplicity, we denote $H_k^{(k)}$ by H_k.

By using the orthogonal assumptions on ε_t and $e_{k-m_k|k-}$, we have

$$P_{k+1-m_{k+1}|k} = \operatorname{cov}\left\{A^{r_k}e_{k-m_k|k-} - H_k\bar{\varepsilon}_k + \sum_{i=0}^{r_k-1}A^i v_{k-i-m_{k+1}}\right\} + \sum_{t=0}^{k-1}\operatorname{cov}\left\{H_k^{(t)}\bar{\varepsilon}_t\right\}$$

It is clear from the previous discussion that we must set $H_k^{(t)} = 0$ to minimize $P_{k+1-m_{k+1}|k}$. Now, it is straightforward to compute that

$$\bar{\varepsilon}_k = \left(\frac{1}{r_k}\sum_{i=0}^{r_k-1}CA^i\right)e_{k-m_k|k-} + \frac{1}{r_k}\sum_{i=0}^{r_k-2}\sum_{j=0}^{i}CA^j v_{k-m_k+i-j} + \frac{1}{r_k}\sum_{i=0}^{r_k-1}w_{k-m_k+i}$$

By denoting

$$\Pi_k = A^{r_k} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1}CA^i$$

it follows that

$$\begin{aligned} e_{k+1-m_{k+1}|k} =& \Pi_k e_{k-m_k|k-} + \sum_{i=0}^{r_k-1}A^i v_{k-i+m_{k+1}} \\ & - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-2}\sum_{j=0}^{i}CA^j v_{k-m_k+i-j} - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-1}w_{k-m_k+i} \end{aligned}$$

and

$$\begin{aligned} P_{k+1-m_{k+1}|k} = & \Pi_k P_{k-m_k|k-}\Pi_k^T + \sum_{i=0}^{r_k-1}A^i Q_{k-i+m_{k+1}}A^{iT} + H_k\frac{1}{r_k^2}\sum_{i=0}^{r_k-1}R_{k-m_k+i}H_k^T \\ & + H_k\frac{1}{r_k^2}\sum_{i=0}^{r_k-2}\sum_{j=0}^{i}CA^j Q_{k-m_k+i-j}A^{jT}C^T H_k^T \\ & - \frac{1}{r_k}\sum_{i=0}^{r_k-2}\sum_{j=0}^{i}A^{i+1}Q_{k-m_k+r_k-2-i}(H_k CA^j)^T \\ & - \frac{1}{r_k}H_k\sum_{i=0}^{r_k-2}\sum_{j=0}^{i}CA^j Q_{k-m_k+r_k-2-i}(A^{i+1})^T \end{aligned}$$

Minimizing the previous discussion with respect to H_k results in (27) with M_k and H_k given in (26) and (25). The corresponding state estimate $x_{k+1-m_{k+1}}$ in (24) follows directly. Because H_k is chosen to minimize $P_{k+1-m_{k+1}|k}$, it follows that $e_{k+1-m_{k+1}|k}$ is uncorrelated with $e_{k-m_k|k-}$. Also, $e_{k+1-m_{k+1}|k}$ is uncorrelated with ε_k because the latter is a linear function of $e_{k-m_k|k-}$ plus some independent noise. □

5. OPTIMAL ESTIMATOR FOR AN UNSTABLE SYSTEM WITH $N > 1$ AND ARBITRARY TIME REVERSAL

In this section, we consider the situation where $N > 1$ and the received bursts z_k are allowed to be out of order. That is, we will drop assumption A6. The transition diagram for m_k in Figure 2 is still valid. We denote by L_k the *set location* of z_k, that is, L_k is the set of time indices of z_k. There are four cases at time k:

Case 1: $r_k = 0$.
Case 2: $r_k = 1 + m_k$, which implies that $m_{k+1} = 0$ and L_k is unique.
Case 3: $m_k = N$ and $r_k = 1$, which implies that $m_{k+1} = N$. The received measurement must be y_{k-N} (due to the definition of N);
Case 4: $0 < r_k \leqslant m_k$ and $0 < m_{k+1} < N$. In this case, L_k is not unique.

It is clear that Cases 1–3 have been considered before, so we only need to consider Case 4. In this case, in addition to the non-unique ordering of the elements in z_k as expressed in (5), these r_k elements are taken from $1 + m_k (> r_k)$ locations $y_t, k - m_k \leqslant t \leqslant k$. It is clear that there are $\eta_k = C_{1+m_k}^{r_k}$ (choosing r_k from $1 + m_k$) possible set locations for $z_k^{(i)}$ in (5), which will be denoted by $z_k^{(i,j)}$, $j = 1, 2, \ldots, \eta_k$. Thus, we can write

$$z_k^{(i)} = \delta_k^{i,1} z_k^{(i,1)} + \delta_k^{i,2} z_k^{(i,2)} + \cdots + \delta_k^{i,\eta_k} z_k^{(i,\eta_k)} \tag{34}$$

where $\delta_k^{i,j} = 0$ or 1 with $\delta_k^{i,1} + \delta_k^{i,2} + \cdots + \delta_k^{i,\eta_k} = 1$. We will denote the set of $\delta_k^{i,j}$ for all i and j by δ_k. To reflect the fact that the set location L_k is non-unique, we assume that

A7: For any $1 \leqslant i \leqslant r_k!$, $\text{Prob}\left(\delta_k^{i,j}\right) > 0$ for at least two values of j.

Lemma 1
Consider the one-step-ahead linear state estimation problem for system (1) with unstable matrix A, assumptions A1–A5, A7 and optimality criteria C1–C3. At given time k, suppose Case 4 in the previous discussion occurs. Given any $t > 0$, let the estimate for x_{k-m_k+t} conditioned at time k take the following linear form:

$$\hat{x}_{k-m_k+t|k} = F_{k,t}\hat{x}_{k-m_k|k^-} + G_{k,t} z_k \tag{35}$$

where k^- is the closest time instant before k having nonzero r_{k^-} and $\hat{x}_{k-m_k|k^-}$ is an unbiased estimate of x_{k-m_k}. Then, in order for $\hat{x}_{k-m_k+t|k}$ to be unbiased and the estimation error covariance to be uniformly bounded, we must have

$$F_{k,t} = A^t; \quad G_{k,t} = 0 \tag{36}$$

Proof
We can write z_k as

$$z_k = \Pi(\gamma_k, \delta_k) x_{k-m_k} + \text{noise}[v, w]$$

where $\Pi(\gamma_k, \delta_k)$ is a matrix depending on γ_k and δ_k and noise$[v, w]$ is a zero-mean noise term. The estimation error can be written as

$$\begin{aligned} e_{k-m_k+t|k} &= x_{k-m_k+t} - \hat{x}_{k-m_k+t|k} \\ &= \left(A^t - G_{k,t}\Pi(\gamma_k, \delta_k)\right) x_{k-m_k} \\ &\quad - F_{k,t}\hat{x}_{k-m_k|k^-} + \text{noise}[v, w] \end{aligned}$$

To ensure the unbiased property, we must have

$$F_{k,t} = A^t - G_{k,t}\mathcal{E}[\Pi(\gamma_k, \delta_k)]$$

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Then, due to the instability of the matrix A, to ensure that the estimation error covariance to be uniform bounded, we must have

$$G_{k,t}\Pi(\gamma_k,\delta_k) = G_{k,t}\mathcal{E}[\Pi(\gamma_k,\delta_k)] \triangleq W$$

(Otherwise $e_{k-m_k+t|k}$ would be an explicit function of x_{k-m_k}). By fixing δ_k first and taking the expectation with respect to γ_k, we must have $G_{k,t} = H_{k,t}[I\ I \cdots I]$ (as shown in the proof of Theorem 2). Moreover, because of assumption A7, $H_{k,t}[I\ I \cdots I]\Pi(\gamma_k,\delta_k)$ will still take at least two different values if $H_{k,t} \neq 0$. Thus, $H_{k,t}$ must be zero. □

Combining Theorem 2 and Lemma 1, we have the following result:

Theorem 4

Consider the one-step-ahead linear state estimation problem for system (1) with unstable matrix A, assumptions A1–A5, A7, and optimality criteria C1–C3. The state estimate update at time k is given as follows:

If $r_k = m_k + 1$ (Case 2) or $m_k = N$ and $r_k = 1$ (Case 3), compute $\hat{x}_{k+1-m_{k+1}}$ according to Theorem 2.

If $r_k = 0$ (Case 1), we have $\hat{x}_{k+1|k} = A\hat{x}_{k|k-1}$ and $P_{k+1|k} = AP_{k|k-1}A^T + Q$.

If $0 < r_k \leqslant m_k$ and $0 < m_{k+1} < N$ (Case 4), we have $\hat{x}_{k+1|k} = A\hat{x}_{k|k-1}$ and $P_{k+1|k} = AP_{k|k-1}A^T + Q$. In addition, we need to delay z_k by adding it to z_{k+1} and remove r_k from m_{k+1}, that is, reset $m_{k+1} = 1 + m_k$.

Proof

The proof for Cases 1–3 follows from the proof of Theorems 2–3, thus not repeated here. The proof for Case 4 is based on Lemma 1. More specifically, by Lemma 1, $\hat{x}_{k+1|k} = A^{r_k}\hat{x}_{k-m_k|k-}$. It is clear from Theorem 2 that $\hat{x}_{k|k-1} = A^{r_k-1}\hat{x}_{k-m_k|k-}$, thus $\hat{x}_{k+1|k} = A\hat{x}_{k|k-1}$. The proof for $P_{k+1|k}$ is similar. We point out that, in Lemma 1, $\hat{x}_{k+1|k}$ is limited to a linear function of $\hat{x}_{k-m_k|k-}$ and z_k instead of Z_k. But this is carried out without loss of generality, as shown in the proof of Theorems 2 and 3. Finally, because z_k is not used in Case 4, these measurements need to be saved for time $k+1$. This needs to be continued until the time $\tau > k$ such that $m_{\tau+1} = 1+m_\tau - r_\tau = 0$ (i.e., Case 2). □

6. OPTIMAL ESTIMATOR FOR A STABLE SYSTEM WITH $N > 1$ AND RESTRICTED TIME REVERSAL

The results in Sections 3–5 assume that the system is unstable, which yields a unique common property, that is, the filter gain for each received measurement is identical. This property is needed to ensure the uniform boundedness of the estimation error covariance. For stable systems, uniform boundedness of the estimation error covariance is guaranteed automatically. Thus, better filter gains can be used to further minimize the estimation error covariance.

In this section, we consider the case as in Section 4 but for a stable system. That is, we assume that the packets can arrive in bursts, which are in order but the packets within each burst can be out of order. Our result is given as follows:

Theorem 5

Consider the one-step-ahead linear state estimation problem for system (1) with stable matrix A, assumptions A1–A6 and optimality criteria C1–C3. The state estimate update at time k is given as follows:

Step 1: If $r_k > 0$, the estimate for $x_{k+1-m_{k+1}}$ is given by

$$\hat{x}_{k+1-m_{k+1}|k} = H_k z_k + \left(A^{r_k} - H_k \sum_{i=1}^{r_k!} p_k^{(i)} \Theta_k^{(i)}\right) \hat{x}_{k-m_k|k-} \quad (37)$$

where k^- is the time instant just before k with a nonzero r_{k^-}, $\Theta_k^{(i)}$ is permuted from

$$\Theta_k = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{r_k-1} \end{bmatrix}$$

with the same permutation order as $z_k^{(i)}$,

$$H_k = \left(A^{r_k} P_{k-m_k|k^-} \left(\sum_{i=1}^{r_k} p_k^{(i)} \Theta_k^{(i)} \right)^T + \Omega_1^T \right) M_k^{-1}, \tag{38}$$

$$\begin{aligned} M_k = & \left(\sum_{i=1}^{r_k!} p_k^{(i)} \Theta_k^{(i)} \right) P_{k-m_k|k^-} \left(\sum_{i=1}^{r_k!} p_k^{(i)} \Theta_k^{(i)} \right)^T \\ & + \sum_{i=1}^{r_k!} p_k^{(i)} \left(1 - p_k^{(i)} \right) \Theta_k^{(i)} S_{k-m_k} \left(\Theta_k^{(i)} \right)^T + \Omega_2 + \Omega_3 \end{aligned} \tag{39}$$

$$S_{k-m_k} = \mathcal{E}\left(x_{k-m_k} x_{k-m_k}^T \right), \tag{40}$$

$$\Omega_1 = \mathcal{E}\left(\sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right) \left(\sum_{i=0}^{r_k-1} A^i v_{k-i-m_{k+1}} \right)^T \tag{41}$$

$$\Omega_2 = \mathcal{E}\left(\sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right) \left(\sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right)^T \tag{42}$$

$$\Omega_3 = \mathcal{E}\left(\sum_{i=1}^{r_k!} \gamma_k^{(i)} w^{(i)} \right) \left(\sum_{i=1}^{r_k!} \gamma_k^{(i)} w^{(i)} \right)^T \tag{43}$$

The corresponding estimation error covariance is given by

$$P_{k+1-m_{k+1}|k} = A^{r_k} P_{k-m_k|k^-} (A^{r_k})^T - H_k M_k H_k^T + \sum_{i=0}^{r_k-1} A^i Q_{k-i-m_{k+1}} (A^i)^T \tag{44}$$

Step 2: If $m_{k+1} > 0$, the estimate for x_{k+1} is given by

$$\hat{x}_{k+1|k} = A^{m_{k+1}} \hat{x}_{k+1-m_{k+1}|k} \tag{45}$$

with the corresponding estimation error covariance given by

$$P_{k+1|k} \quad = \quad A^{m_{k+1}} P_{k+1-m_{k+1}|k} (A^{m_{k+1}})^T + \sum_{i=0}^{m_{k+1}-1} A^i Q_{k-i} (A^i)^T \tag{46}$$

Proof

Given any time k with $r_k > 0$, $\hat{x}_{k+1-m_{k+1}|k}$ is a linear function with the following linear form:

$$\hat{x}_{k+1-m_{k+1}|k} = \Gamma_k \hat{x}_{k-m_k|k^-} + H_k z_k \tag{47}$$

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for some Γ_k and H_k. The estimation error is given by

$$\begin{aligned}
e_{k+1-m_{k+1}|k} &= x_{k+1-m_{k+1}} - \hat{x}_{k+1-m_{k+1}|k} \\
&= A^{r_k} x_{k-m_k} + \sum_{i=0}^{r_k-1} A^i v_{k-i-m_{k+1}} - \Gamma_k \hat{x}_{k-m_k|k-} - H_k z_k \\
&= \left(A^{r_k} - \sum_{i=1}^{r_k!} \gamma_k^{(i)} H_k \Theta_k^{(i)} \right) x_{k-m_k} - \Gamma_k \hat{x}_{k-m_k|k-} \\
&\quad + \sum_{i=0}^{r_k-1} A^i v_{k-i-m_{k+1}} - H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} - H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} w^{(i)}
\end{aligned} \tag{48}$$

where the noises $v^{(i)}$ and $w^{(i)}$ are respectively permuted from

$$v = \begin{bmatrix} 0 \\ C v_{k-m_k} \\ \cdots \\ \sum_{i=0}^{r_k-2} C A^i v_{k-m_k+r_k-2-i} \end{bmatrix}, w = \begin{bmatrix} w_{k-m_k} \\ w_{k-m_k+1} \\ \cdots \\ w_{k-m_k+r_k-1} \end{bmatrix}$$

To ensure unbiased state estimation, we obtain

$$\Gamma_k = A^{r_k} - \sum_{i=1}^{r_k!} p_k^{(i)} H_k \Theta_k^{(i)} \tag{49}$$

By following the definition in (7), the estimation error covariance is written as

$$\begin{aligned}
P_{k+1-m_{k+1}|k} &= \mathcal{E}\left[e_{k+1-m_{k+1}|k} e_{k+1-m_{k+1}|k}^T \right] \\
&= \left(A^{r_k} - \sum_{i=1}^{r_k!} p_k^{(i)} H_k \Theta^{(i)} \right) P_{k-m_k|k-} \left(A^{r_k} - \sum_{i=1}^{r_k!} p_k^{(i)} H_k \Theta^{(i)} \right)^T \\
&\quad + \mathcal{E}\left[\sum_{i=1}^{r_k!} \left(p_k^{(i)} - \gamma_k^{(i)} \right) H_k \Theta^{(i)} \right] S_{k-m_k} \left[\sum_{i=1}^{r_k!} \left(p_k^{(i)} - \gamma_k^{(i)} \right) H_k \Theta^{(i)} \right]^T \\
&\quad - \mathcal{E}\left(H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right) \left(\sum_{i=0}^{r_k-1} A^i v_{k-i-m_{k+1}} \right)^T + \sum_{i=0}^{r_k-1} A^i Q_{k-i-m_{k+1}} (A^i)^T \\
&\quad - \mathcal{E}\left(\sum_{i=0}^{r_k-1} A^i v_{k-i-m_{k+1}} \right) \left(H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right)^T \\
&\quad + \mathcal{E}\left(H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right) \left(H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} v^{(i)} \right)^T \\
&\quad + \mathcal{E}\left(H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} w^{(i)} \right) \left(H_k \sum_{i=1}^{r_k!} \gamma_k^{(i)} w^{(i)} \right)^T
\end{aligned}$$

By defining $H_k^* = -\left(A^{r_k} P_{k-m_k|k} \left(\sum_{i=1}^{r_k!} p_k^{(i)} \Theta^{(i)}\right)^T + \Omega_1^T\right) M_k^{-1}$ with M_k given in (39), the previous equation can be rewritten as

$$\begin{aligned} P_{k+1-m_{k+1}|k} &= (H_k + H_k^*) M_k (H_k + H_k^*)^T - H_k M_k H_k^{*T} - H_k^* M_k H_k^T - H_k^* M_k H_k^{*T} \\ &\quad + A^{r_k} P_{k-m_k|k} (A^{r_k})^T + \sum_{i=0}^{r_k-1} A^i Q_{k-i-m_{k+1}} (A^i)^T - \sum_{i=1}^{r_k!} p_k^{(i)} H_k \Theta^{(i)} P_{k-m_k|k} (A^{r_k})^T \\ &\quad - A^{r_k} P_{k-m_k|k} \left(\sum_{i=1}^{r_k!} p_k^{(i)} H_k \Theta^{(i)}\right)^T - H_k \Omega_1 - (H_k \Omega_1)^T + H_k \Omega_2 H_k^T + H_k \Omega_3 H_k^T \end{aligned}$$

Therefore, the estimator gain H_k is obtained to minimizing the estimate error covariance

$$H_k = -H_k^* = \left(A^{r_k} P_{k-m_k|k} \left(\sum_{i=1}^{r_k!} p_k^{(i)} \Theta^{(i)}\right)^T + \Omega_1^T\right) M_k^{-1} \tag{50}$$

□

7. SIMULATION EXAMPLE

In this section, we present a numerical example to illustrate the previous theoretical results with $N = 1$.

Consider a system described in (1) with the following specifications:

$$A = \begin{bmatrix} 1.1 & -0.1 \\ 0.5 & 0.9 \end{bmatrix}, C = [12]$$

and $R = 0.1$, $Q = 0.25 I_2$, $P_0 = 0.25 I_2$, where I_2 is the identity matrix.

We know that r_k is obtained according to the transition diagram in Figure 1, and suppose the transition probabilities are as follows:

$$\begin{aligned} p_{00} &= P(m(k+1) = 0 | m(k) = 0) = 0.75; \\ p_{01} &= P(m(k+1) = 1 | m(k) = 0) = 0.25; \\ p_{10} &= P(m(k+1) = 0 | m(k) = 1) = 0.65; \\ p_{11} &= P(m(k+1) = 1 | m(k) = 1) = 0.35 \end{aligned}$$

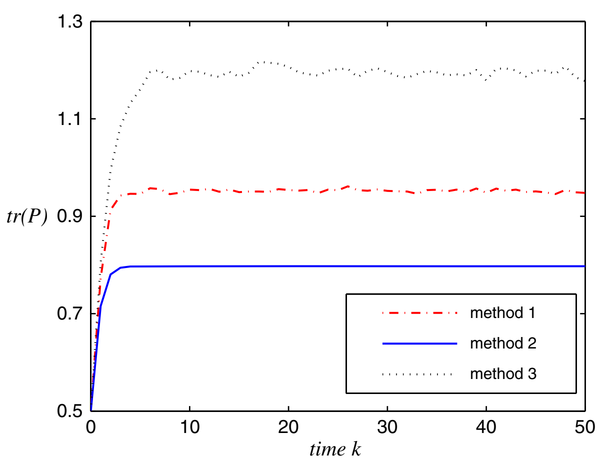

Figure 3. Comparison of the trace of error covariance.

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Figure 3 shows the comparison of the trace of the error covariance for three scenarios:

Method 1 : The proposed method in this paper;
Method 2 : The standard Kalman filtering, assuming that there is no time delay; and
Method 3 : When receiving two measurements, the estimator just uses the newest measurement.

It can be seen from the simulation results that the proposed estimator in the paper has a better performance over Method 3.

8. CONCLUSION

We have studied an optimal state estimation problem for unstable systems under the assumption that output measurements are subject to random time delays caused by network transmissions without time stamping. We have proposed a random time delay model that resembles many practical digital network systems. In particular, this model ensures that no packet loss is caused purely by inherent random transmission delays (contrary to the model (2)). Using the proposed model, we have given solutions to the optimal unbiased linear state estimators with uniformly bounded state estimation error covariance. Different maximum time delays have been considered. For unstable systems, a key observation we have made is that the uniform boundedness condition for the estimation error covariance implies that only limited information in the received measurements can be exploited when the time stamp of the measurements cannot be precisely inferred. In the case when multiple measurements are received and their set location can be determined except their order, only the averaged measurement can be used for state estimation. In the case when the set location is uncertain, the measurements cannot be used directly (although they may be used at a later time). That is, the statistics of random time delays cannot be used to tune the estimation gains. This is because, for unstable systems, the covariance of the system state is excessively large, and thus must be eliminated from the estimation error covariance by choosing appropriate estimation gains. For stable systems, the covariance of the system state may not be excessively large, thus the statistics of the random time delays may be used in optimizing the estimation gains.

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