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Quantized output feedback control with multiplicative measurement noises

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SUMMARY

In this paper, we consider the problem of quantized quadratic performance control for a class of stochastic systems, which are subject to multiplicative noises in the measurement. A dynamic output feedback controller is designed to guarantee a given level of performance. By using the sector bound approach to characterize the quantization error, the existence of a solution for the quantized quadratic performance control problem is found by solving the so-called guaranteed cost control problem of the associated system with a sector bounded uncertainty. We show that the latter problem can be solved using LMIs. Copyright © 2014 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Research on the problem of quantized feedback control can be traced back to 1956 [1], where Kalman investigated the effects of quantization in a sampled-data control system and pointed out that the quantized feedback system would exhibit limit cycles and chaotic behavior if a finite-alphabet quantizer is used. In the early works on quantized feedback control problem [2–5], the quantization error is always considered as undesirable, either as noise or state uncertainty, and most of the works try to eliminate its influence.

The widespread use of network-based control where the information between the system measurements and control input is exchanged through a network medium with a limited capacity has further strengthened the importance of the quantized feedback control problem. Different from the early views toward quantization, quantization is now considered to be useful instead of undesirable. As for the fundamental problem in networked control systems, how much is the least data rate that has to be sent to stabilize the system, [6] shows that the coarsest static quantizer for single-input deterministic systems to be stabilized via quantized state feedback is logarithmic, where the quantization density can be characterized by the unstable poles of the system matrix. The minimum quantization density with respect to state feedback subject to the Bernoulli packets dropouts is considered in [7], which is related to both the unstable modes and the statistical properties of Bernoulli noises. For the more general case with the input channel subject to an independent and identically distributed packet dropout process in [8], the minimum data rate for the mean-square stabilization is explicitly given in terms of the unstable eigenvalues of the open loop matrix and

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the packet dropout probability. For stochastic systems with multiplicative noise in both system and input matrices, not only the system matrices but also the statistical properties of the noises are influential to the coarsest quantization density for quantized state feedback. Sample results on feedback control with dynamic quantizers can be found in [9, 10]. Reference [11] focuses on the tradeoff between the stochastic noise and the coarsest quantization density, which permits mean-square stabilization of the system, and the exact solution to the coarsest quantization density is given in terms of a special Riccati equation, and an approximate numerical solution is given in terms of a linear matrix inequality. In works on quantized feedback control, most of them are confined to the problem of quantized stabilization, and control performance is usually not addressed. The quantized feedback control problem with a quadratic performance cost for deterministic systems is studied in [12], where a sector bound approach is used to characterize the quantization error, both quantized state feedback and dynamic output feedback control are considered. As for quantized stabilization and performance with a finite-level quantizer, [13] shows that asymptotic stabilization for the system can be achieved with a moderate number of quantization levels by introducing a dynamic scaling method for logarithmic quantizer, and the quantized feedback stabilization problem for systems with bounded noises is also studied. For deterministic discrete-time systems, [14] gives less conservative conditions on the quantization density for stabilization. By studying the properties of the logarithmic quantizer, [15] uses a method based on Tsypkin-type Lyapunov functions to give the absolute stability analysis for quantized feedback control, and less conservative conditions than those in the quadratic framework are derived. Tian *et al.*[16] have proposed a new model of the network control system in the unified framework, observer-based controller is developed for the asymptotical stabilization, which can be shown in terms of nonlinear matrices inequalities, which can be solved through a convex optimization problems. Niu *et al.* [17] considered the quantized output feedback control problem for networked systems with data packets dropout. By using an estimation method to cope with the effect of random packets loss, and the sector bound approach to treat with the quantization error, exponentially mean-square stability is achieved. For system with packets dropout and finite-level quantizer, sufficient condition for small l^∞ signal l^∞ stability is studied, which required the maximum number of conservative packet dropouts to be bounded [18]. In [19], the stability of system with quantized feedback subject to infinite time delay and packets dropout is studied. Most of the aforementioned work only considered the stability, performance index is not included.

In this paper, we consider a quantized quadratic performance problem for stochastic systems using output feedback control with a multiplicative noise in the measurement. This is motivated by the fact that there has been a lot of work on quantized control problems for additive noises, whereas multiplicative noises receive little attention. Multiplicative noises are indeed very common and thus deserve serious considerations. For example, in many distance-based measurements, the measurement error grows linearly with the distance; Packet loss in data transmission can be modeled as a multiplicative noise; Human response to a management command (e.g., evacuation order in a natural disaster or speed limit on the road) can also be modeled by a multiplicative noise. Ideally, we would like to be able to treat both additive and multiplicative noises. But this turns out to be a very hard problem, as noted by [20–25]. For simplicity, we assume that the system is subject only to multiplicative noises.

The major difficulty with output feedback control with multiplicative noise is that the well-known certainty equivalence principle fails to work, thus state feedback designs cannot be used to aid the design of output feedback controllers. In this paper, we turn to a very different approach. By using the sector bound approach to characterize the quantization error, we first show that the existence of the solutions to the quadratic quantized performance control problem is guaranteed by the existence of the solution to the so-called guaranteed cost control (GCC) problem. Using the Schur complement technique and the elimination lemma to deal with the stochastic noises, we show, through lengthy technical analysis, that the solutions to the latter problem can be expressed in terms of linear matrix inequalities with a simple scalar parameter.

Although the assumption of single-output is indeed some restrictive, it should be noted that the approaches used in this paper can be generalized to multi-output measurement systems. The main

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difference is that the conditions for the quantized quadratic performance problem may become sufficient only.

The paper is organized as follows. In Section 2, the system to be studied is introduced, some fundamentals about quantization is discussed, and the problem formulation is given. In Section 3, the relationship between the quadratic quantized performance control problem and the GCC problem is established, and the existence of the solution to the problem of GCC problem is given in terms of linear matrices inequalities. Section 4 gives an illustrative example to show the design of the controller and the effectiveness of the control strategy. Section 5 draws some conclusions.

2. PROBLEM FORMATION

Consider the following linear discrete-time system with a multiplicative noise in the measurement:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t), \\ y(t) &= (1+\gamma(t))Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the system state vector with initial state $x(0)$ assumed to be white noise with $Ex(0)x^T(0) = \Sigma_0$ for some positive-definite matrix Σ_0; $u(t) \in \mathcal{R}$ is the control input, $y(t) \in \mathcal{R}$ is the measurement, $\gamma(t)$ is a white scalar noise with $E\gamma^2(t) = \sigma^2 > 0$ for some $\sigma > 0$, and it is uncorrelated with the initial state $x(0)$. The measurement is sent through a band-limited channel that has to be quantized by a logarithmic quantizer in the following form:

$$Q(y) = \begin{cases} u_i, & if\ \frac{1}{1+\delta}u_i < y \leqslant \frac{1}{1-\delta}u_i,\ y > 0 \\ 0 & if\ y = 0 \\ -Q(-y) & if\ y < 0 \end{cases} \tag{2}$$

where $\delta = \frac{1-\rho}{1+\rho}$ with $0 < \rho < 1$ being the quantization density. The associated quantization levels are as follows:

$$U = \{\pm u_i : u_i = \rho^i u_0, i = 1, 2, \cdots\} \bigcup \{\pm u_0\} \bigcup \{0\},\ u_0 > 0. \tag{3}$$

As illustrated in [12], using the sector bound approach, we have

$$|y - Q(y)| = |\Delta(t)y| \leqslant \delta|y|, \tag{4}$$

where

$$\Delta(t) = \frac{y(t) - Q(y(t))}{y(t)}. \tag{5}$$

Consider the following quadratic performance cost function:

$$J(x(0)) = \sum_{t=0}^{\infty} \left(x^T(t)Sx(t) + u^T(t)Ru(t)\right),\ S \geqslant 0,\ R > 0. \tag{6}$$

We are interested in designing a quantized output feedback controller

$$\begin{aligned} x_c(t+1) &= A_c x_c(t) + B_c Q(y(t)),\ x_c(0) = 0, \\ u(t) &= C_c x_c(t) + D_c Q(y(t)), \end{aligned} \tag{7}$$

such that the closed-loop system is quadratically stable and that the performance cost function is minimized in the sense of

$$\min E[J(x(0), \xi(t))], \tag{8}$$

where the expectation E is performed over $x(0)$ and $\xi(t)$.

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3. SOLUTIONS

The exact solution for (8) is very difficult to obtain. Instead, we will solve a relaxed problem, which we will call a *quantized quadratic performance control problem*.

From the system (1) and the controller (7), we can write the closed-loop system as

$$\xi(t+1) = [\bar{A} + \bar{B}(\Delta(t)) + \gamma(t)\hat{B}(\Delta(t))]\xi(t), \tag{9}$$

with the system state as

$$\xi(t) = \begin{bmatrix} x_c(t) \\ x(t) \end{bmatrix}, \tag{10}$$

and parameters are defined as

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A_c & B_c C \\ BC_c & A + BD_c C \end{bmatrix}, \bar{B}(\Delta(t)) = \begin{bmatrix} 0 & \Delta(t)B_c C \\ 0 & \Delta(t)BD_c C \end{bmatrix}, \\ \hat{B}(\Delta(t)) &= \begin{bmatrix} 0 & B_c C(1+\Delta(t)) \\ 0 & BD_c C(1+\Delta(t)) \end{bmatrix}. \end{aligned} \tag{11}$$

Using the system state $\xi(t)$ of the closed-loop system (9), the performance cost (6) can be rewritten as

$$J(\xi(0)) = \sum_{t=0}^{\infty} \left(\xi^T(t)\bar{S}\xi(t) + u^T(t)Ru(t)\right) \tag{12}$$

with

$$\xi(0)) = \begin{bmatrix} 0 \\ x(0) \end{bmatrix}, \bar{S} = \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}. \tag{13}$$

For the closed-loop system (9) to be quadratically mean-square stable, there exists an associated Lyapunov function $V(\xi) = \xi^T \bar{P}\xi$ with $\bar{P} = \bar{P}^T$ such that

$$E[\nabla V(\xi(t))] = E[V(\xi(t+1))] - E[V(\xi(t))] < 0, \tag{14}$$

for all $t \geqslant 0$. The performance cost (8) is given by

$$\begin{aligned} E[J(\xi(0))] &= E\left[\xi^T(0)\bar{P}\xi(0)\right] + E\left[\sum_{t=0}^{\infty}\left(\nabla V(\xi(t)) + \xi^T(t)\bar{S}\xi(t) + u^T(t)Ru(t)\right)\right] \\ &= E\left[\xi^T(0)\bar{P}\xi(0)\right] + E\left[\sum_{t=0}^{\infty}\xi^T(t)\bar{\Omega}(\Delta(t))\xi(t)\right], \end{aligned} \tag{15}$$

where

$$\begin{aligned} \bar{\Omega}(\Delta(t)) = {} & [\bar{A} + \bar{B}(\Delta(t)) + \gamma(t)\hat{B}(\Delta(t))]^T \bar{P}[\bar{A} + \bar{B}(\Delta(t)) + \gamma(t)\hat{B}(\Delta(t))] - \bar{P} + \bar{S} \\ & + [C_c \ D_c C(1+\Delta(t))(1+\gamma(t))]^T R[C_c \ D_c C(1+\Delta(t))(1+\gamma(t))]. \end{aligned} \tag{16}$$

It follows that

$$\begin{aligned} E[\bar{\Omega}(\Delta(t))] = {} & [\bar{A} + \bar{B}(\Delta(t))]^T \bar{P}[\bar{A} + \bar{B}(\Delta(t))] - \bar{P} + \bar{S} \\ & + \sigma^2[0 \ D_c C(1+\Delta(t))]^T R[0 \ D_c C(1+\Delta(t))] \\ & + [C_c \ D_c C(1+\Delta(t))]^T R[C_c \ D_c C(1+\Delta(t))] \\ & + \sigma^2 \hat{B}^T(\Delta(t))\bar{P}\hat{B}(\Delta(t)). \end{aligned} \tag{17}$$

The performance control problem to be studied in this paper can be formally formulated as follows: *Given a performance bound $\tau > 0$ and quantization density $\rho > 0$, find $\bar{P}$ and (7), if there exist, such that*

$$E\left[\xi^T(0)\bar{P}\xi(0)\right] < \tau \tag{18}$$

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subject to

$$E\left[\xi^T\bar{\Omega}(\Delta(t))\xi\right] \leqslant 0,\ \forall\ \xi \in R^{2n},\ t \geqslant 0. \tag{19}$$

We call this problem a quantized quadratic performance control (QQPC) problem.

In order to solve the QQPC problem, we define the following auxiliary system:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t),\ \ x(0) = x_0 \\ y(t) &= (1+\gamma(t))Cx(t) \\ v(t) &= (1+\Delta(t))y(t),\ \ |\ \Delta(t)\ | \leqslant \delta, \end{aligned} \tag{20}$$

where $v(t)$ is the output available for feedback. The solution to QQPC is related to the so-called guaranteed cost control (GCC) problem for the auxiliary system (20), that is, we want to find $\bar{P}$ and (7), if there exists, such that (18) holds subject to

$$E[\bar{\Omega}(\Delta)] < 0,\ \forall\ |\Delta| \leqslant \delta. \tag{21}$$

Remark 1

It is clear that if there exists a solution to the GCC problem, the same solution works for the QQPC problem. We show in the succeeding text that the converse is also true, with a trival gap.

Theorem 1

Consider the system (1) with performance cost in (6), controller (7), performance bound $\tau > 0$, and quantization density as ρ. Suppose the GCC problem has a solution, then there exists a solution to the QQPC problem. Conversely, if the QQPC problem has a solution, then for any arbitrarily small $\eta > 0$, the GCC problem for (21) has a solution with δ replaced with $\delta - \eta$.

Proof

It is easy to know that (21) implies (19). Next, we prove that if the QQPC problem has a solution, then, for any given arbitrarily small ε, the GCC problem for $\varepsilon > 0$ for (21) has a solution for $|\Delta| \leqslant \delta - \eta$. To see this, we assume that (19) holds but (21) fails. Then there exist some ξ_0 and Δ_0 with $E\left[\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0\right] \neq 0$ and $|\Delta_0| \leqslant \delta$ such that

$$E\left[\xi_0^T\bar{\Omega}(\Delta_0)\xi_0\right] \geqslant 0. \tag{22}$$

If Δ_0 is a boundary point, that is $\Delta_0 = \delta$, then the GCC problem has a solution for $|\Delta| \leqslant \delta - \eta$ for all $\eta > 0$. In the sequel, we assume that Δ_0 is an interior point.

We claim that $E[[\ 0\ \ C(1+\gamma(t))\ ]\xi_0] \neq 0$. Suppose that $E[[\ 0\ \ C(1+\gamma(t))\ ]\xi_0] = 0$, then from (16) and (22), we can obtain that

$$E\left[\xi_0^T\bar{\Omega}(\Delta\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0)\xi_0\right] = E\left[\xi_0^T\Omega_0\xi\right] = E\left[\xi_0^T\bar{\Omega}(\Delta_0)\xi_0\right] \geqslant 0. \tag{23}$$

which contradicts with (19), so $E\left[\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0\right] \neq 0$. For the strict convexity of $E[\bar{\Omega}(\Delta)]$, there exists Δ^1 with $|\Delta^1| \leqslant \delta - \eta_1$, for some $\eta_1 > 0$ such that

$$E\left[\xi_0^T\bar{\Omega}(\Delta^1)\xi_0\right] \geqslant 0. \tag{24}$$

For it is continuous in ξ_0, we can perturb ξ_0 slightly such that (24) holds and with every element of $E\left[\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0\right] \neq 0$. Now, for $\Delta\left(\alpha\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0\right)$ covers $[-\delta, \delta]$ densely as α varies from $-\infty$ to ∞. Hence, that $\alpha \neq 0$ such that

$$E\left[\xi_0^T\bar{\Omega}(\Delta(\alpha\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0))\xi_0\right] > 0. \tag{25}$$

Define $\xi_1 = \alpha\xi_0$, we obtain that

$$E\left[\xi_1^T\bar{\Omega}(\Delta(\alpha\left[\ 0\ \ C(1+\gamma(t))\ \right]\xi_0))\xi_1\right] > 0 \tag{26}$$

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which contradicts (19), which means Δ^0 cannot be an interior point. Hence, (19) implies (21) has a solution for $|\Delta| \leqslant \delta - \eta$. □

Next, we give necessary and sufficient conditions for the existence of the solution to the GCC problem. To this end, we first introduce some notations:

$$K = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}, A_0 = \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix}, B_0 = \begin{bmatrix} I & 0 \\ 0 & B \end{bmatrix}, I_{10} = \begin{bmatrix} I & 0 \end{bmatrix}, I_{20} = \begin{bmatrix} 0 & I \end{bmatrix}, \\ L = \begin{bmatrix} 0 & C \end{bmatrix}, I_1 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}. \tag{27}$$

Using (17) and applying the well-known S-procedure [26] on (21), we obtain the following result:

Lemma 1

For system (1) and controller (7), (21) holds if and only if there exists $\varepsilon > 0$, such that the following matrix inequality holds:

$$\begin{bmatrix} -\bar{P}^{-1} & \bar{A} & 0 & 0 & 0 \\ \bar{A}^T & -\bar{P} + \bar{S} + \varepsilon L^T L & \bar{C}^T & \sigma \hat{D}^T & \sigma \bar{B}^T \\ 0 & \bar{C} & -R^{-1} & 0 & 0 \\ 0 & \sigma \hat{D} & 0 & -R^{-1} & 0 \\ 0 & \sigma \bar{B} & 0 & 0 & -\bar{P}^{-1} \end{bmatrix} + \varepsilon^{-1} H H^T < 0, \tag{28}$$

where

$$H = \begin{bmatrix} \bar{H} \\ 0 \\ \bar{D} \\ \sigma \bar{D} \\ \sigma \bar{H} \end{bmatrix}, \hat{D} = \begin{bmatrix} 0 & D_c C \end{bmatrix}, L = \begin{bmatrix} 0 & C \end{bmatrix}, \bar{C} = \begin{bmatrix} C_c & D_c C \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 0 & B_c C \\ 0 & B D_c C \end{bmatrix}, \bar{D} = \delta D_c, \bar{H} = \begin{bmatrix} \delta B_c \\ \delta B D_c \end{bmatrix}.$$

To simplify (28), we note that (28) is equivalent to the following inequality:

$$\begin{bmatrix} -\bar{P}^{-1} & \bar{A} & 0 & 0 & 0 & \bar{H} \\ \bar{A}^T & -\bar{P} + \bar{S} + \varepsilon L^T L & \bar{C}^T & \sigma \hat{D}^T & \sigma \bar{B}^T & 0 \\ 0 & \bar{C} & -R^{-1} & 0 & 0 & \bar{D} \\ 0 & \sigma \hat{D} & 0 & -R^{-1} & 0 & \sigma \bar{D} \\ 0 & \sigma \bar{B} & 0 & 0 & -\bar{P}^{-1} & \sigma \bar{H} \\ \bar{H}^T & 0 & \hat{D}^T & \sigma \hat{D}^T & \sigma \bar{H}^T & -\varepsilon I \end{bmatrix} < 0. \tag{29}$$

The conversion earlier is performed using Schur complement [26].

Define $K_1 = \begin{bmatrix} A_c \\ C_c \end{bmatrix}$, $K_2 = \begin{bmatrix} B_c \\ D_c \end{bmatrix}$, then the inequality (29) can be written as follows:

$$\Omega_0 + F_1^T K_1 W_1 + W_1^T K_1^T F_1 + F_2^T K_2 W_2 + W_2^T K_2^T F_2 < 0, \tag{30}$$

where

$$\Omega_0 = \begin{bmatrix} -\bar{P}^{-1} & A_0 & 0 & 0 & 0 & 0 \\ A_0^T & \Psi & 0 & 0 & 0 & 0 \\ 0 & 0 & -R^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -R^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{P}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}, \tag{31}$$

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$$\Psi = -\bar{P} + \bar{S} + \varepsilon L^T L,$$

$$F_1^T = \begin{bmatrix} B_0 \\ 0 \\ I_{20} \\ 0 \\ 0 \\ 0 \end{bmatrix}, W_1^T = \begin{bmatrix} 0 \\ I_{10}^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_2^T = \begin{bmatrix} B_0 \\ 0 \\ I_{20} \\ \sigma I_{20} \\ \sigma B_0 \\ 0 \end{bmatrix}, W_2^T = \begin{bmatrix} 0 \\ L^T \\ 0 \\ 0 \\ 0 \\ \delta I \end{bmatrix}. \tag{32}$$

Lemma 2

The inequality (30) holds if and only if

$$F_{10}^T \left(\Omega_0 + F_2^T K_2 W_2 + W_2^T K_2^T F_2\right) F_{10} < 0 \tag{33}$$

and

$$W_{10}^T \left(\Omega_0 + F_2^T K_2 W_2 + W_2^T K_2^T F_2\right) W_{10} < 0 \tag{34}$$

where F_{10} and W_{10} denote the kernels of F_1 and W_1, respectively, given by

$$F_{10} = Ker(F_1) = \begin{bmatrix} I_{20}^T & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ -B^T & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, \tag{35}$$

$$W_{10} = Ker(W_1) = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{20}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}. \tag{36}$$

Proof

This follows by using the elimination Lemma [26]. The kernels of F_1 and W_1 can be verified directly. □

Denote $\Omega_{01} = F_{10}^T \Omega_0 F_{10}$, $\Omega_{02} = W_{10}^T \Omega_0 W_{10}$, $F_{11}^T = F_{10}^T F_2^T$, $W_{11} = W_2 F_{10}$, $F_{12}^T = W_{10}^T F_2^T$, and $W_{12} = W_2 W_{10}$, we obtain

$$\Omega_{01} = \begin{bmatrix} -I_{20}\bar{P}^{-1} I_{20}^T - BR^{-1}B^T & I_{20}A_0 & 0 & 0 & 0 \\ A_0^T I_{20}^T & \Psi & 0 & 0 & 0 \\ 0 & 0 & -R^{-1} & 0 & 0 \\ 0 & 0 & 0 & -\bar{P}^{-1} & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}, \tag{37}$$

$$\Omega_{02} = \begin{bmatrix} -\bar{P}^{-1} & A_0 I_{20}^T & 0 & 0 & 0 & 0 \\ I_{20}A_0^T & I_{20}\Psi I_{20}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & -R^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -R^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{P}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}, \tag{38}$$

$$F_{11}^T = \begin{bmatrix} 0 & 0 & \sigma I_{20}^T & \sigma B_0^T & 0 \end{bmatrix}^T, \tag{39}$$

$$W_{11} = \begin{bmatrix} 0 & L & 0 & 0 & \delta I \end{bmatrix}, \tag{40}$$

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$$F_{12}^T = \left[\, B_0^T \;\; 0 \;\; I_{20}^T \;\; \sigma I_{20}^T \;\; \sigma B_0^T \;\; 0 \,\right]^T, \tag{41}$$

$$W_{12} = \left[\, 0 \;\; L \begin{bmatrix} 0 \\ I \end{bmatrix} \;\; 0 \;\; 0 \;\; 0 \;\; \delta I \,\right]. \tag{42}$$

Then the inequalities of (33)–(34) become

$$\begin{aligned} \Omega_{01} + F_{11}^T K_2 W_{11} + W_{11}^T K_2 F_{11} < 0 \\ \Omega_{02} + F_{12}^T K_2 W_{12} + W_{12}^T K_2 F_{12} < 0 \end{aligned} \tag{43}$$

Define $F_{21} = ker(F_{11})$, $W_{21} = ker(W_{11})$, $W_{21} = ker(W_{11})$, $F_{22} = ker(F_{12})$, and $W_{22} = ker(W_{12})$. By computation, we obtain that

$$F_{21} = \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ -B^T & 0 & 0 & 0 \\ I_{20}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \tag{44}$$

$$W_{21} = \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ \delta I_{20}^T & 0 & I_{10}^T & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ -C & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{45}$$

$$F_{22} = \begin{bmatrix} -\sigma I_{10}^T & I_{20}^T & -\sigma I_{20}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & -B^T & 0 & -\sigma I & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ I_{10}^T & 0 & I_{20}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}, \tag{46}$$

$$W_{22} = \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ -\delta I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ C & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{47}$$

Denoting

$$\bar{P} = \begin{bmatrix} X & X_1 \\ X_1^T & X_2 \end{bmatrix}, \; \bar{P}^{-1} = \begin{bmatrix} Y & Y_1 \\ Y_1^T & Y_2 \end{bmatrix}, \tag{48}$$

the following theorem holds:

Theorem 2

The GCC problem (18) and (21) for the auxiliary system (20) has a solution if and only if there exist $\bar{P}$ and $\bar{P}^{-1}$ as in (48) such that $tr(X_2\Sigma_0) < \tau$ subject to

$$\begin{bmatrix} -Y_2 - BR^{-1}B^T & 0 & A \\ 0 & -X & -X_1 \\ A^T & -X_1^T & -X_2 + S + \varepsilon C^T C \end{bmatrix} < 0, \tag{49}$$

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$$\begin{bmatrix} -(1+\sigma^2)Y & \sigma Y_1 & -(1+\sigma^2)Y_1 & 0 & 0 \\ \sigma Y_1^T & -Y_2 - BR^{-1}B^T & \sigma Y_2 & -\sigma BR^{-1} & A \\ -(1+\sigma^2)Y_1^T & \sigma Y_2^T & -(1+\sigma^2)Y_2 & 0 & -\sigma A \\ 0 & -\sigma R^{-1}B^T & 0 & -(1+\sigma^2)R^{-1} & 0 \\ 0 & A^T & -\sigma A^T & 0 & -X_2 + S + \varepsilon C^T C \end{bmatrix} < 0, \quad (50)$$

$$\begin{bmatrix} \delta^2(-X_2 + S + \varepsilon C^T C) - \varepsilon C^T C & 0 & -\delta A^T \\ 0 & -Y & -Y_1 \\ -\delta A & -Y_1^T & -Y_2 \end{bmatrix} < 0. \quad (51)$$

Proof

It is straightforward to check that

$$E\left[\xi^T(0)\bar{P}\xi(0)\right] = E\left[x^T(0)X_2x(0)\right] = E\left[x^T(0)X_2x(0)\right] = E\left[tr(X_2x(0)x^T(0))\right] = tr(X_2\Sigma_0). \quad (52)$$

Applying the elimination lemma to the inequalities (43), we find that (43) holds if and only if

$$\begin{aligned} F_{21}^T\Omega_{01}F_{21} &< 0 \\ W_{21}^T\Omega_{01}W_{21} &< 0, \end{aligned} \quad (53)$$

and

$$\begin{aligned} F_{22}^T\Omega_{02}F_{22} &< 0 \\ W_{22}^T\Omega_{02}W_{22} &< 0. \end{aligned} \quad (54)$$

Plugging (37), (44), and (45) into the inequalities of (53), we obtain (49) and the following inequality:

$$\begin{bmatrix} \delta^2(-X_2 + S + \varepsilon C^T C) - \varepsilon C^T C & \delta A^T & -\delta X_1^T \\ \delta A & -Y_2 - BR^{-1}B^T & 0 \\ -\delta X_1 & 0 & -X \end{bmatrix} < 0, \quad (55)$$

Multiplying the first row and first column of (55) by δ^{-1}, then permuting its rows and columns, (55) is converted into

$$\begin{bmatrix} -Y_2 - BR^{-1}B^T & 0 & A \\ 0 & -X & -X_1 \\ A^T & -X_1^T & -X_2 + S + \varepsilon C^T C - \delta^{-2}\varepsilon C^T C \end{bmatrix} < 0. \quad (56)$$

It is clear that this inequality is implied by (49) and thus it is not necessarily required. Plugging (38), (46), and (47) into the inequalities of (54), we obtain (50) and (51). □

Next, we want to revise the inequalities (49)–(51) further by changing variables so that they become linear inequalities. Define $S = M^T M$ with a full column rank matrix M, and let $\tilde{Y}_2 = \varepsilon Y_2$, $\tilde{X}_2 = \varepsilon^{-1}X_2$, then we have the following theorem:

Theorem 3

The inequalities of (49)–(51) are equivalent to

$$\begin{bmatrix} -\tilde{Y}_2 - \varepsilon BR^{-1}B^T & A\tilde{Y}_2 & 0 & 0 \\ \tilde{Y}_2 A^T & -\tilde{Y}_2 & \tilde{Y}_2 M^T & \tilde{Y}_2 C^T \\ 0 & M\tilde{Y}_2 & -\varepsilon I & 0 \\ 0 & C\tilde{Y}_2 & 0 & -I \end{bmatrix} < 0, \quad (57)$$

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$$\begin{bmatrix} -\frac{1}{1+\sigma^2}(\tilde{Y}_2+\varepsilon BR^{-1}B^T) & \frac{1}{1+\sigma^2}A & 0 \\ \frac{1}{1+\sigma^2}A^T & -\tilde{X}_2+C^TC+\frac{\sigma^2}{1+\sigma^2}A^T\tilde{X}_2A & M^T \\ 0 & M & -\varepsilon I \end{bmatrix} < 0, \tag{58}$$

$$\begin{bmatrix} \delta^2(-\tilde{X}_2+C^TC+A^T\tilde{X}_2A)-C^TC & \delta M^T \\ \delta M & -\varepsilon I \end{bmatrix} < 0. \tag{59}$$

Proof

Using Schur complement, (49) can be rewritten as

$$\begin{bmatrix} -Y_2-BR^{-1}B^T & A \\ A^T & -X_2+X_1^TX^{-1}X_1+M^TM+\varepsilon C^TC \end{bmatrix} < 0. \tag{60}$$

Using the fact that $Y_2^{-1} = X_2 - X_1^T X^{-1} X_1$ and multiplying the second row and column by Y_2, (60) is equivalent to

$$\begin{bmatrix} -Y_2-BR^{-1}B^T & AY_2 \\ Y_2A^T & -Y_2+Y_2M^TMY_2+\varepsilon Y_2C^TCY_2 \end{bmatrix} < 0. \tag{61}$$

Substituting Y_2 with $\varepsilon^{-1}\tilde{Y}_2$, the aforementioned equation becomes

$$\begin{bmatrix} -\varepsilon^{-1}\tilde{Y}_2-BR^{-1}B^T & \varepsilon^{-1}A\tilde{Y}_2 \\ \varepsilon^{-1}\tilde{Y}_2A^T & -\varepsilon^{-1}\tilde{Y}_2+\varepsilon^{-2}\tilde{Y}_2M^TM\tilde{Y}_2+\varepsilon^{-1}\tilde{Y}_2C^TC\tilde{Y}_2 \end{bmatrix} < 0. \tag{62}$$

Multiplying it by ε and applying Schur complement change the aforementioned equation to (57).

For (50), multiplying $\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & -\frac{\sigma}{1+\sigma^2}B & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$ and $\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & -\frac{\sigma}{1+\sigma^2}B^T & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$ to the left and right side of it, it is changed to

$$\begin{bmatrix} -(1+\sigma^2)Y & \sigma Y_1 & -(1+\sigma^2)Y_1 & 0 & 0 \\ \sigma Y_1^T & -Y_2-\frac{1}{1+\sigma^2}BR^{-1}B^T & \sigma Y_2 & 0 & A \\ -(1+\sigma^2)Y_1^T & \sigma Y_2^T & -(1+\sigma^2)Y_2 & 0 & -\sigma A \\ 0 & 0 & 0 & -(1+\sigma^2)R^{-1} & 0 \\ 0 & A^T & -\sigma A^T & 0 & -X_2+S+\varepsilon C^TC \end{bmatrix} < 0. \tag{63}$$

Deleting the fourth row and fourth column, which correspond to a negative-definite diagonal term, the aforementioned equation becomes

$$\begin{bmatrix} -(1+\sigma^2)Y & \sigma Y_1 & -(1+\sigma^2)Y_1 & 0 \\ \sigma Y_1^T & -Y_2-\frac{1}{1+\sigma^2}BR^{-1}B^T & \sigma Y_2 & A \\ -(1+\sigma^2)Y_1^T & \sigma Y_2^T & -(1+\sigma^2)Y_2 & -\sigma A \\ 0 & A^T & -\sigma A^T & -X_2+S+\varepsilon C^TC \end{bmatrix} < 0. \tag{64}$$

Swapping the second and third rows as well as the second and third columns, the aforementioned equation is equivalent to

$$\begin{bmatrix} -(1+\sigma^2)Y & -(1+\sigma^2)Y_1 & \sigma Y_1 & 0 \\ -(1+\sigma^2)Y_1^T & -(1+\sigma^2)Y_2 & \sigma Y_2^T & -\sigma A \\ \sigma Y_1^T & \sigma Y_2 & -Y_2-\frac{1}{1+\sigma^2}BR^{-1}B^T & A \\ 0 & -\sigma A^T & A^T & -X_2+S+\varepsilon C^TC \end{bmatrix} < 0, \tag{65}$$

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Using Schur complement, it follows that

$$\begin{bmatrix} -Y_2 - \frac{1}{1+\sigma^2} BR^{-1}B^T & A \\ A^T & -X_2 + S + \varepsilon C^T C \end{bmatrix} + \frac{\sigma^2}{1+\sigma^2} \begin{bmatrix} Y_1^T & Y_2 \\ 0 & -A^T \end{bmatrix} \begin{bmatrix} Y & Y_1 \\ Y_1^T & Y_2 \end{bmatrix}^{-1} \begin{bmatrix} Y_1 & 0 \\ Y_2^T & -A \end{bmatrix} < 0. \tag{66}$$

Using (48), the aforementioned equation is equivalent to

$$\begin{bmatrix} -\frac{1}{1+\sigma^2}[Y_2 + BR^{-1}B^T] & \frac{1}{1+\sigma^2} A \\ \frac{1}{1+\sigma^2} A^T & -X_2 + S + \varepsilon C^T C + \frac{\sigma^2}{1+\sigma^2} A^T X_2 A \end{bmatrix} < 0. \tag{67}$$

Substituting $\tilde{X}_2$ and $\tilde{Y}_2$, we obtain

$$\begin{bmatrix} -\frac{1}{1+\sigma^2}\left(\varepsilon^{-1}\tilde{Y}_2 + BR^{-1}B^T\right) & \frac{1}{1+\sigma^2} A \\ \frac{1}{1+\sigma^2} A^T & -\varepsilon\tilde{X}_2 + M^T M + \varepsilon C^T C + \frac{\sigma^2}{1+\sigma^2}\varepsilon A^T \tilde{X}_2 A \end{bmatrix} < 0. \tag{68}$$

Multiplying both sides by $\begin{bmatrix} \sqrt{\varepsilon} I & 0 \\ 0 & \frac{1}{\sqrt{\varepsilon}} I \end{bmatrix}$ and applying Schur complement, we obtain (58).

For (51), using Schur complement, it is equivalent to

$$\delta^2\left(-X_2 + S + \varepsilon C^T C\right) - \varepsilon C^T C - \begin{bmatrix} 0 & -\delta A^T \end{bmatrix} \begin{bmatrix} -Y & -Y_1 \\ -Y_1^T & -Y_2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -\delta A \end{bmatrix} < 0, \tag{69}$$

which is equivalent to

$$\delta^2\left(-\varepsilon\tilde{X}_2 + M^T M + \varepsilon C^T C\right) - \varepsilon C^T C + \delta^2 \varepsilon A^T \tilde{X}_2 A < 0. \tag{70}$$

Dividing the aforementioned equation by ε and applying Schur complement, we obtain (59). □

Next, we point out that the constraint that $\bar{P}\bar{P}^{-1} = I$ can be characterized using $\tilde{X}_2$ and $\tilde{Y}_2$.

Lemma 3

Given $\varepsilon > 0$, the constraint $\bar{P}\bar{P}^{-1} = I$ implies that

$$\begin{bmatrix} \tilde{X}_2 & I \\ I & \tilde{Y}_2 \end{bmatrix} > 0. \tag{71}$$

Conversely, given $\tilde{X}_2$ and $\tilde{Y}_2$ satisfying (71) and $\varepsilon > 0$, $\bar{P}$ can be constructed.

Proof

Because that $X_1^T Y_1 + X_2 Y_2 = I$, $X_1^T Y + X_2 Y_1^T = 0$ (following from (48)), solving X_1^T from the second equation and plugging it into the first equation gives

$$X_2 = \left(Y_2 - Y_1^T Y^{-1} Y_1\right)^{-1} > Y_2^{-1} \tag{72}$$

from which we can obtain the linear matrix inequality

$$\begin{bmatrix} X_2 & I \\ I & Y_2 \end{bmatrix} > 0, \tag{73}$$

which is equivalent to

$$\begin{bmatrix} \varepsilon\tilde{X}_2 & I \\ I & \varepsilon^{-1}\tilde{Y}_2 \end{bmatrix} > 0. \tag{74}$$

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Multiplying both sides by $\begin{bmatrix} \frac{1}{\sqrt{\varepsilon}} I & 0 \\ 0 & \sqrt{\varepsilon} I \end{bmatrix}$, we can obtain (71). Conversely, given $\tilde{X}_2$ and $\tilde{Y}_2$ satisfying (71) and $\varepsilon > 0$, X_2 and Y_2 are obtained to satisfy (71). X_1 and Y_1 can be computed from $X_1^T Y_1 + X_2 Y_2 = I$, $X_1^T Y + X_2 Y_1^T = 0$. □

Considering Theorems 2–3 and Lemma 3, we obtain the following main result, which together with Theorem 1 solves the QQPC problem.

Theorem 4

The GCC problem (18) and (21) for the auxiliary system (20) has a solution if and only if there exist positive matrices $\tilde{X}_2$ and $\tilde{Y}_2$ and $\varepsilon > 0$ such that $\varepsilon tr(\tilde{X}_2 \Sigma_0) < \tau$, subject to the inequalities (57)–(59) and (71).

Remark 2

Note that the inequalities (57)–(59) and (71) are jointly linear in $\tilde{X}_2$ and $\tilde{Y}_2$ and ε. Thus, the set satisfying the constraints (57)–(59) and (71) is a convex set. The only nonlinear inequality is $\varepsilon tr(\tilde{X}_2 \Sigma_0) < \tau$. To get around this difficulty, we note that ε is a scalar parameter, thus the solution to the GCC problem can be obtained by sweeping the ε parameter. Once $\tilde{X}_2$ $\tilde{Y}_2$ and ε are found, $\bar{P}$ and $\bar{P}^{-1}$ can be constructed. Afterwards, we obtain Ω_0 in (31). Then the inequality (30) becomes a linear inequality in terms of K_1 and K_2, which can be easily solved.

4. SIMULATION

In this part, we give an example to show the design of the controller and the effectiveness of the approach proposed in this paper. The system under consideration is in the form of

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} -1.5 \\ 2.5 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \; R = 1, \; S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \; M = 1, \; \tau = 60. \tag{75}$$

It can be verified that the system is stabilizable, and the system is observable. For the case of $\sigma = 0$, the coarsest quantization density for the system to be stabilized by quantized output feedback is $\rho = 0.8182$, which is given in [12], so for the system with multiplicative noise, the coarsest quantization density should be greater than this. Consider the case with $\sigma = 0.2$. Select $\varepsilon = 0.8$ and $\delta = 0.053$, which gives $\rho = 0.9$. By solving the linear matrices inequalities (57)–(59) and (71) as required by theorem 4, and use Remark 2, we obtain that $K = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \begin{bmatrix} -0.7260 & 0.3306 & 0.9582 \\ -0.2447 & 0.1087 & 0.3247 \\ 0.3580 & -0.1616 & -0.4721 \end{bmatrix}$. The response of the state of the system is illustrated in the Figures 1 and 2. It is confirmed that the required H_∞ performance index τ is satisfied.

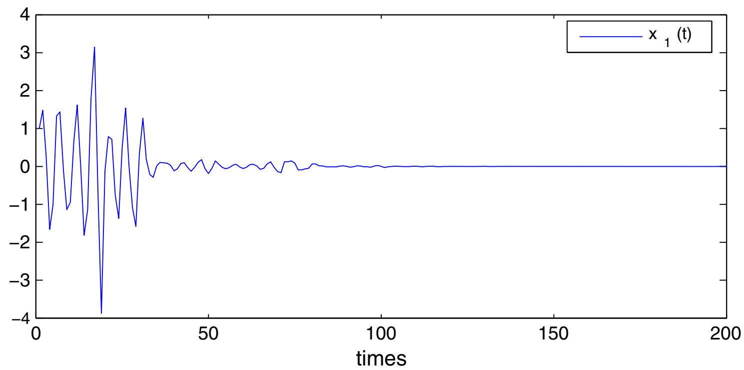

Figure 1. The response of $x_1(t)$.

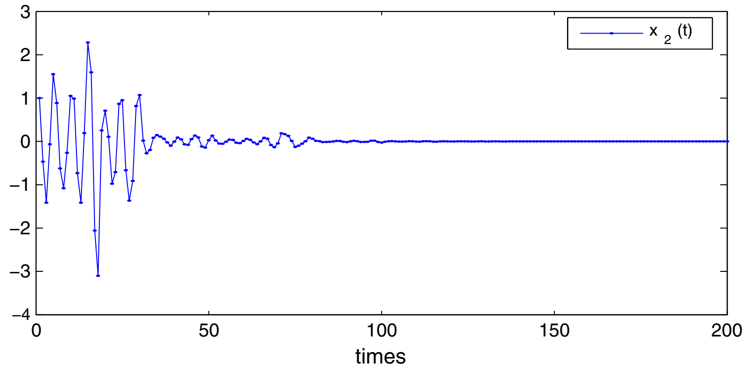

Figure 2. The response of $x_2(t)$.

5. CONCLUSION

This paper has studied a quantized output feedback control for linear systems with multiplicative measurement noises. To make the problem computationally tractable, we have defined the quadratic quantized performance control problem. Using the sector bound approach to characterizing the quantized error, we have shown that the solution to this problem can be solved through GCC problem. Using S-procedure, Schur complement, and the elimination lemma, we obtain numerically efficient necessary and sufficient conditions for the problem using linear matrices inequalities.

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