Introduction to the Parametric Approach to Robust Stability

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ABSTRACT: This paper discusses recent results relating to the parametric approach to robust stability. The general problem of robust stability is defined, and a brief review of Kharitonov-type, edge, and zero exclusion results is given. Several open research directions are presented, followed by some concluding remarks on the future of the approach.

Introduction

The fundamental justification for using feedback control is "to enforce good performance of control systems in the presence of uncertainty. Also, feedback is used to enable a process to work in the neighborhood of open-loop unstable operating conditions, that is, to stabilize unstable plants" [1]. Although most systems to be controlled are open-loop stable, the introduction of integrators in the feedback loop (to suppress steady-state errors) or more general compensators (to improve dynamic performance) makes the stabilization, in the presence of uncertain parameters and/or delays, very difficult. Contrary to the decade of the 1970s, when most control efforts were concentrated on known mathematical models, now "control engineers must live with uncertainty and understand that the impact of the level of modeling uncertainty on the design of controllers is crucial" [1]. Thus, questions of stability and stabilization of uncertain systems are among the most important issues in control engineering today. The aim of this paper is to review and discuss recent results on some of these issues, i.e., what is now commonly called robust stability. The reader interested in tracing the history of robust stability in the parameter space is referred to a recent survey [2] and a number of books, e.g., [3]–[7].

When dealing with a nominal system, there are a number of well-known methods of verifying stability. These include analytical tests, such as the Routh-Hurwitz criterion or Lyapunov method as well as graphical tests, such as the Nyquist criterion. Ensuring the practical stability of a control system, however, requires taking into consideration not only the nominal system but also all its reasonable (expected) perturbations. In the classical Bode-Nyquist design techniques, the goal of practical stability is achieved by introducing uncertainty in gain and phase, leading to the notion of a stability (gain and phase) margin. In many applications, however, the uncertain parameters are not the gain and/or phase but some other well-defined physical quantities, such as time constants, friction coefficients, loads, interconnection gains, chemical reaction rates, time delays, etc.

There are basically two approaches available to problems of robust stability. The first approach uses additive $[G(s) + \Delta(s)]$ or multiplicative $[G(s)[1 + \Delta(s)]]$ unstructured perturbations of the nominal system transfer function (c.f., [8]–[10] and references therein) with H-infinity (H^∞)-norm bounds on the perturbation $\Delta(s)$. The second approach takes into account the structure of the perturbations by assuming that some parameters (usually physical parameters) are known to lay within some bounds or tolerances (c.f., [4], [6], [11] for examples of uncertainty models in the parameter space). We will focus on the parametric approach since

(1) Parameter bounds usually can be obtained from physical considerations, while it is difficult to find bounds on $\Delta(s)$.
(2) Models of perturbations in the frequency domain that proved successful (e.g., gain or phase perturbations) cannot represent parameter perturbation as a special case.
(3) In most practical cases, after deriving an appropriate uncertainty model in the parameter space, the remaining uncertainty in the frequency domain is negligible; it perturbs the Nyquist plot in a small neighborhood of the origin.
(4) The classical gain and phase margin model becomes a special case of a more general uncertainty model in the space of transfer-function coefficients, after introducing an uncertain complex gain coefficient.

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Problem Definition and Preliminary Remarks

Consider the continuous- or discrete-time closed-loop system shown in Fig. 1. For simplicity, the controller is incorporated into the GH block in the forward loop. The characteristic equation of the system is obtained from the determinant.

$$\det(1 + GH) = 0 \qquad (1)$$

This usually can be represented in the equivalent form shown in Eq. (2), where p is a function (called the "characteristic function") in the vector of variables $\mathbf{r}$ and the uncertain coefficient vector $\mathbf{a}$ belongs to some known region of tolerances.

$$p(\mathbf{r}, \mathbf{a}) = 0 \qquad (2)$$

The components of $\mathbf{r}$ are functions of one complex variable associated with either the Laplace transform or the z-transform. For example, they may be of the form s^k, $\exp(-sT)$, etc. The general robust stability problem can now be formulated as follows: *Given a family of characteristic functions P associated with an uncertain system, and a set D in the complex plane, provide computationally tractable techniques for determining the D stability of P, i.e., for checking whether the zeros of the functions in P remain within the set D.*

Typical choices for set D are the open left half-plane (for continuous-time systems), the open unit disk (for discrete-time systems), or specified subsets thereof. The general problem is very difficult, and additional assumptions are required before concrete results can be given. To better understand some of the difficulties, consider the particular case where the characteristic function of a continuous-time system is given by a polynomial in s with uncertain coefficients:

$$p(s, \mathbf{a}) = \mathbf{a}_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0, \quad \text{with } a_n > 0 \qquad (3)$$

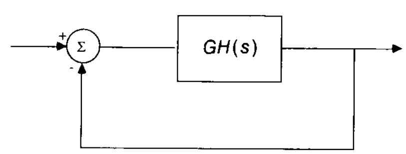

Fig. 1. Closed-loop system.

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We assume that the uncertain coefficients vary independently in the intervals $a_i \in [\alpha_i, \beta_i]$, $i = 0, 1, \ldots, n$. One way of checking the stability (in this case, D is the open left half-plane) would be to discretize the admissible parameter space and to check the stability of each $p(s, \mathbf{a})$ at all discrete values of a_i. This is tractable only when n is small and the discretization is coarse; for example, for $n = 3$, 10 values of a_i, $i = 0, 1, 2, 3$, would require checking 10,000 polynomials. This approach is clearly not a good method to use, since it would be necessary to check thousands of polynomials, even for relatively small n. Furthermore, the discretization might be too coarse, and this approach would not guarantee the correctness of the result. On the other hand, it is obvious (in view of the Hurwitz conditions) that: for $n = 2$, it is sufficient to check whether $\alpha_0 > 0$ and $\alpha_1 > 0$; for $n = 3$, it is sufficient to check the positivity of the lower bounds of the coefficients and the inequality $\alpha_2\alpha_1 - \beta_3\beta_0 > 0$. However, for $n > 3$, the problem becomes cumbersome. Fortunately, recent results [12] obviate the necessity of pursuing this approach, at least for the polynomial in Eq. (3) and for independent coefficient variations.

Recent Results on the Robust Stability Problem

In this section, recent results for special cases of the general robust stability problem are reviewed.

Interval Polynomials and Kharitonov-Type Results

For the polynomial $p(s, \mathbf{a})$ given in Eq. (3), a complete solution has been obtained by Kharitonov [12] under two assumptions.

The first assumption (K1) is that the family P is a so-called "interval polynomial," meaning that the coefficient variations are independent and the coefficient a_i can take positive values from α_i (minimum) to β_i (maximum), i.e.,

$$a_i \in [\alpha_i, \beta_i], \quad 0 < \alpha_i \le \beta_i, \quad i = 0, \ldots, n \tag{4}$$

The second assumption (K2) is that region D is the open left half-plane.

Kharitonov demonstrated that the family P is D stable if, and only if, exactly four polynomials corresponding to specially chosen extreme values of the coefficients are stable. The four polynomials are as follows:

$$\begin{aligned}
p_{\alpha\alpha}(s) &= \alpha_0 + \alpha_1 s + \beta_2 s^2 + \beta_3 s^3 + \alpha_4 s^4 + \alpha_5 s^5 + \ldots \\
p_{\beta\beta}(s) &= \beta_0 + \beta_1 s + \alpha_2 s^2 + \alpha_3 s^3 + \beta_4 s^4 + \beta_5 s^5 + \ldots \\
p_{\alpha\beta}(s) &= \alpha_0 + \beta_1 s + \beta_2 s^2 + \alpha_3 s^3 + \alpha_4 s^4 + \beta_5 s^5 + \ldots \\
p_{\beta\alpha}(s) &= \beta_0 + \alpha_1 s + \alpha_2 s^2 + \beta_3 s^3 + \beta_4 s^4 + \alpha_5 s^5 + \ldots
\end{aligned} \tag{5}$$

If the coefficients in Eq. (3) are complex [i.e., (K1) is modified such that each coefficient varies independently in a rectangle in the complex plane], Kharitonov [13] has shown that only eight polynomials must be checked. Kharitonov's results make the solution to the particular robust stability problem considered earlier straightforward; it is necessary and sufficient to check no more than four (eight in the case of complex coefficients) polynomials, rather than potentially millions. (Recently, a number of "simple" proofs [14]–[18] of Kharitonov's results have appeared in the literature. Although simplicity, like beauty, is in the eye of the beholder, one cannot help but ask: If the proofs are so simple, why are there so many?)

The types of systems that can be treated using Kharitonov's results are limited by assumptions (K1) and (K2). For example, discrete-time systems cannot be treated since the stability region is the open unit disk, violating assumption (K2). A considerable research effort has gone into attempts to relax these assumptions ([19]–[34] and references therein), i.e., to consider other regions D and/or dependent coefficient variations and/or systems with $p(s, \mathbf{a})$ different than the one in Eq. (3). We note that, if the system possesses only one unknown coefficient satisfying Eq. (4), then Eq. (5) reduces to checking two polynomials differing only in that one involves the minimum and the other the maximum value of the unknown coefficient. We refer to this as checking extreme values, and say that a given class of systems will not generally admit a Kharitonov-like result if checking extreme values does not guarantee stability. Based on this: *Neither discrete-time [23], [29] nor delay systems [28] admit Kharitonov-like results.*

For some special D regions and/or restricted types of polynomials, however, checking the extreme points may be sufficient [31], [32], [35].

Assumption (K1) often is not satisfied in typical situations. To see this, consider the transfer function (with uncertain gain parameter $k \in [0.5, 3]$)

$$GH(s) = \frac{k(s^2 + s + 5)}{s^3 + 1.4s^2 + 1.4s + 1} \tag{6}$$

The characteristic equation is given by Eq. (3) with $n = 3$, $a_3 = 1$, $a_1 = a_2 = 1.4 + k$, and $a_0 = 1 + 5k$. Clearly, the coefficient variations are linearly dependent and do not satisfy assumption (K1). On the other hand, the stability is preserved if, and only if, $a_1 a_2 > a_0$ (note that the positivity of a_i, $i = 0, \ldots, 3$, is assured). Checking this inequality for $k = 0.5$, 1, and 3, it is found that the system is stable, unstable, and again stable, respectively. Hence, stability for the extreme values of the uncertain parameter does not guarantee stability for the entire region of uncertainty, and a Kharitonov-like result cannot, in general, be obtained for systems where the coefficient variations are linearly dependent. This seems to indicate that progress in the theory of robust stability to include linearly dependent coefficients is difficult to achieve. Fortunately, however, some recent results bring positive reactions.

Polytopes of Polynomials and Edge Results

Now consider coefficient variations that span a polytope in the space of coefficients. A polytope in a vector space is a set of all convex combinations of a finite number of fixed vectors. For instance, a triangle, rectangle, and cube are polytopes (convex combinations of their corners). A general expression for a polytope of polynomials is

$$P = \left\{ p(s, \lambda) : p(s, \lambda) = \lambda_1 p_1(s) + \cdots + \lambda_m p_m(s),\ \lambda_i \ge 0,\ \sum_{i=1}^{m} \lambda_i = 1 \right\} \tag{7}$$

Note that the characteristic polynomial in Eq. (6) belongs to the polytope

$$\begin{aligned}
&\{s^3 + (1.4 + k)s^2 + (1.4 + k)s + 1 + 5k : k \in [0, 3]\} \\
&= \{\lambda_1(s^3 + 1.4s^2 + 1.4s + 1) + \lambda_2(s^3 + 4.4s^2 + 4.4s + 16) : \\
&\quad \lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_1 + \lambda_2 = 1\}
\end{aligned}$$

As an introduction to edge results, consider the polynomial given in Eq. (3), with $n = 3$, $a_3 = 1$, $a_2 = a_1$, and assume that a_1 and a_0 are bounded by a triangular region (a polytope), as shown in Fig. 2. Using the Routh or Hurwitz test, it can be seen that, in order to check the stability of the whole triangular region, it is sufficient to check the stability of its edges (note that $a_0 = a_1^2$ defines the boundary of the stability region). This reduces the dimensionality of the problem, since it is not necessary to check the stability of interior points of the triangle.

The preceding example generalizes to the multidimensional case with the so-called

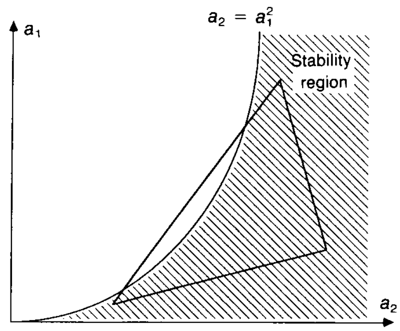

Fig. 2. Example showing that checking the edges is sufficient for checking the stability of the triangular region.

"edge theorem" of Bartlett et al. [21], which states that, given a polytope in the space of coefficients $a_0, \ldots, a_n$, the stability of the whole polytope is equivalent to the stability of its edges. This yields a great reduction in computational complexity; not as dramatic as Kharitonov's result, but still substantial since checking the stability of the edges can be executed by checking the root locations of certain easily derivable matrices. A necessary and sufficient condition for the stability of an edge

$$\{\lambda_1 p_1(s) + \lambda_2 p_2(s): \lambda_1 + \lambda_2 = 1, \lambda_i \geq 0\}$$

is that p_1 be stable and the matrix $H_1^{-1}H_2$ (or $H_2H_1^{-1}$) does not have eigenvalues in $(-\infty, 0]$, where H_1 and H_2 are Hurwitz matrices corresponding to p_1 and p_2, respectively [22]. (The counterpart of this condition for discrete-time systems can be found in [11] and [36].) In addition, an attractive graphical test has been found for checking edges [28]: It is sufficient to check stability of p_1 and then verify whether for all ω,

$$-\pi < \arg\left(\frac{p_2(j\omega)}{p_1(j\omega)}\right) < \pi \qquad (7)$$

For example, if $p_1(s) = (s + 1)^2$ and $p_2(s) = (s - 1)^2$, then the continuous function

$$\arg\left(\frac{p_2(j\omega)}{p_1(j\omega)}\right) = -4 \text{ arc } tg(\omega)$$

reaches $-\pi$ at $\omega = 1$, thus indicating instability of some convex combination of $p_1(s)$ and $p_2(s)$. In general, the practical computation can be executed over a finite interval, since for large ω the ratio $p_2(j\omega)/p_1(j\omega)$ approaches a constant value.

We can now ask whether the edge theorem holds for a wider class of systems. The answer is that: *The edge theorem holds for the case with a reduced system order (vanishing highest order coefficients) [30], for discrete-time systems [21], and can be extended to time-delay systems [28] as well as almost arbitrary D regions (some mild assumptions on D are required [27], [28]).*

Note that, for time-delay systems, we encounter polynomials in s, and $\exp(-h_i s)$, $i = 1, \ldots, k$, where $h_1, \ldots, h_k$ are positive numbers (time delays). In addition, the graphical tests for checking the edges extend to this case [28], where they are of even greater interest than in the case of polynomials since no general analytic tests exist for checking stability of time-delay systems.

Results Based on the Zero Exclusion Principle

The main idea behind the so-called "zero exclusion principle" is the fundamental property that the roots of polynomials are continuous as functions of coefficients. To explain this, consider the robust stability of a continuous-time system with coefficients of the characteristic polynomial depending continuously on a number of independent parameters located within specified intervals. Then the robust stability is guaranteed if (1) a nominal system is stable and, (2) for any admissible values of the parameters, the characteristic polynomial does not have zeros on the imaginary axis. Statement 2 means that no polynomial can assume a zero value at the boundary of the left half-plane (zero exclusion).

This principle can be developed further to obtain numerical tests for robust stability. A number of results [19], [20], [25], [26], [37], [38] are based on this approach. In [19], Barmish uses the zero exclusion principle to generalize the concept of four Kharitonov functions for a polytope of polynomials. This generalization would seem to be of real interest for reducing the computation time when treating cases involving many uncertain parameters. Barmish and Khargonekar [20] extend the results of [19] to include unmodeled dynamics. Dasgupta et al. [37] address the issue of robust stability of polytopes of functions that are not necessarily polynomials and develop a more general form of the zero exclusion principle. Fu [26] proposes an approach based on the zero exclusion principle for the robust stability of polytopes of polynomials, which provides a closed-form description for the "maximal" size of a polytope of robustly stable polynomials. Simple and efficient numerical algorithms are also given in [26] for calculating the maximal size.

Open Research Directions

In this section, three directions for future research are presented.

Direction 1

An important class of open problems is the multilinear case. This case assumes that the characteristic function is given by Eq. (3), where a_i is multilinearly dependent on a set of other parameters. To illustrate this case, consider the following transfer function, where the gain and poles are uncertain:

$$GH(s) = \frac{k}{(s + \lambda_1)(s + \lambda_2)} \qquad (8)$$

Then the characteristic polynomial has uncertain parameters k, λ_1, and λ_2, which are assumed to vary independently within certain intervals.

$$p(s, \mathbf{a}) = s^2 + a_1 s + a_0 = s^2 + (\lambda_1 + \lambda_2)s + k + \lambda_1\lambda_2 \qquad (9)$$

Unfortunately, no efficient results currently exist for treating the general case of multilinear parameter perturbations. Since the multilinear case is encountered often in applications, it represents a major barrier that needs to be overcome through future research on the parametric approach to robust stability.

Direction 2

When the state-space approach is used for modeling, the system uncertainties lead to perturbations of the elements of the matrices relating the state variables, the inputs, and the outputs. Typically, the problem of robust stability becomes that of the robust stability of a polytope of matrices (i.e., convex combinations of finitely many matrices) or an interval matrix (i.e., a matrix whose elements vary independently in given intervals). This problem, however, is much more complicated than the one of a polytope of polynomials because the coefficients of the characteristic function of the system are multivariable or, in special cases, multilinear functions. Consequently, the results on the polynomial counterpart no longer hold (see [39] for some false conjectures and insights into the difficulties). Nevertheless, there are some useful sufficient robust stability conditions available in the literature [34], [40]–[48].

Direction 3

An interesting open area in which results are just beginning to appear [26], [42], [45], [48]–[50] is that of finding "maximal perturbation bounds"—finding the maximal value of a scaling factor for the set of uncertain parameters such that robust stability is preserved. The interest here is that, in contrast to cases where the perturbation bounds are assumed known, we are attempting to find the largest set of perturbations for which the system remains robustly stable. This may be of particular interest for the robust controller synthesis problem, where design parameters need to be chosen. In [50], a closed-form description is given for the maximal perturbation bound of Hurwitz stable interval polynomials. For the polytope of polynomials defined in Eq. (9), a solution based on the zero exclusion principle is given in [26]. Although analytical results for more general cases probably will be difficult to obtain, computational methods offer a promising avenue of approach.

Remarks and Conclusions

The necessity of developing tools to deal with uncertainty in control systems is now widely recognized, and in the last five years the parametric approach to robust stability has been receiving increasing interest. If knowledge of the system justifies assuming independent or linearly dependent coefficient perturbations in the characteristic function, the theory underlying the parametric approach to robust stability is reasonably well developed. Furthermore, conditions under which Kharitonov-type results or edge results can be applied are now fairly clear. In addition, the zero exclusion principle offers a method of determining the maximal size of the polytope of polynomials for which stability can be achieved. Thus, for a wide class of systems, we believe the theory is sufficiently well developed that work can begin on developing efficient software to aid control engineers in incorporating the parametric approach into their analysis and design toolboxes.

The major stumbling block to the extension of the parametric approach to a wider class of systems is the lack of a theory for treating multilinear and/or nonlinear cases. Currently, the only results concern sufficient conditions for limited cases. Undoubtedly, this is the main area that begs for results, and we feel that numerical approaches represent the most promising direction.

References

[1] A. Levis et al., "Challenges to Control: A Collective View, Report of a Workshop Held at the University of Santa Clara on Sept. 18-19, 1986," *IEEE Trans. Automat. Contr.*, vol. AC-32, no. 4, pp. 275-285, 1987.

[2] D. D. Siljak, "Parameter Space Methods for Robust Control Design: A Guided Tour," School of Engineering, Santa Clara Univ., Tech. Rept. EECS-031588, Jan. 1988.

[3] S. P. Bhattacharyya, *Robust Stabilization Against Structured Perturbations*, Lecture Notes in Control and Information Sciences, vol. 99, New York, NY: Springer-Verlag, 1987.

[4] I. Horowitz, *Synthesis of Feedback Systems*, New York, NY: Academic Press, 1963.

[5] M. G. Safonov, *Stability and Robustness of Multivariable Feedback Systems*, Cambridge, MA: MIT Press, 1980.

[6] D. D. Siljak, *Nonlinear Systems: The Parameter Analysis and Design*, New York, NY: Wiley, 1969.

[7] G. J. Thaler and R. G. Brown, *Analysis and Design of Feedback Control Systems*, New York, NY: McGraw-Hill, 1960.

[8] P. Dorato, "A Historical Review of Robust Control," *IEEE Contr. Syst. Mag.*, vol. 7, pp. 44-47, 1987.

[9] J. C. Doyle and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis," *IEEE Trans. Automat. Contr.*, vol. AC-26, no. 1, pp. 4-16, 1981.

[10] Q. Huang and R. Liu, "A Necessary and Sufficient Condition for Stability of a Perturbed System," *IEEE Trans. Automat. Contr.*, vol. AC-32, no. 4, pp. 337-340, 1987.

[11] J. Ackermann and B. R. Barmish, "Robust Shur Stability of a Polytope of Polynomials," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 10, pp. 984-986, 1988.

[12] V. L. Kharitonov, "Asymptotic Stability of an Equilibrium Position of a Family of Systems of Linear Differential Equations," *Differentsyalnye Uravnenya*, vol. 14, no. 11, pp. 1483-1485, 1978.

[13] V. L. Kharitonov, "On a Generalization of a Stability Criterion," *Izv. Akad. Nauk. Kazakh. SSR Ser. Fiz. Mat.*, vol. 1, pp. 53-57, 1979 (in Russian).

[14] Y. Bistritz, "Stability Criterion for Continuous-Time System Polynomials with Uncertain Complex Coefficients," *IEEE Trans. Circuits Syst.*, vol. 35, no. 4, pp. 442-448, 1988.

[15] N. K. Bose, "A System-Theoretic Approach to Stability of Sets of Polynomials," *Linear Algebra and Its Role in Systems Theory*, Contemporary Mathematics, vol. 47, Providence, RI: American Math Soc., pp. 25-34, 1985.

[16] N. K. Bose and Y. Q. Shi, "A Simple General Proof of Kharitonov's Generalized Stability Criterion," *IEEE Trans. Circuits Syst.*, vol. CAS-34, no. 8, pp. 1233-1237, 1987.

[17] N. K. Bose and Y. Q. Shi, "Network Realizability Theory Approach to Stability of Complex Polynomials," *IEEE Trans. Circuits Syst.*, vol. CAS-34, no. 2, pp. 216-218, 1987.

[18] K. S. Yeung and S. S. Wang, "A Simple Proof of Kharitonov's Theorem," *IEEE Trans. Automat. Contr.*, vol. AC-32, no. 9, pp. 822-823, 1987.

[19] B. R. Barmish, "A Generalization of Kharitonov's Four Polynomial Concept for Robust Stability Problems with Linearly Dependent Coefficient Perturbations," *Proc. 1988 ACC*, Atlanta, GA, pp. 1869-1875, June 1988.

[20] B. R. Barmish and P. P. Khargonekar, "Robust Stability of Feedback Control Systems with Uncertain Parameters and Unmodelled Dynamics," *Proc. 1988 ACC*, Atlanta, GA, pp. 1857-1862, June 1988.

[21] A. C. Bartlett, C. V. Hollot, and H. Lin, "Root Locations of an Entire Polytope of Polynomials: It Suffices to Check the Edges," *Proc. 1987 ACC*, Minneapolis, MN, pp. 1611-1616, June 1987; also, *Math. Contr. Signals Syst.*, vol. 1, pp. 61-71, 1987.

[22] S. Bialas, "A Necessary and Sufficient Condition for the Stability of Convex Combinations of Stable Polynomials and Matrices," *Bull. Polish Acad. Sci., Tech. Sci.*, vol. 33, no. 9-10, pp. 473-480, 1985.

[23] S. Bialas and J. Garloff, "Convex Combinations of Stable Polynomials," *J. Franklin Inst.*, vol. 319, no. 3, pp. 373-377, 1985.

[24] J. Cieslik, "On Possibilities of the Extension of Kharitonov's Stability Test for Interval Polynomials to the Discrete Time Case," *IEEE Trans. Automat. Contr.*, vol. AC-32, no. 3, pp. 237-238, 1987.

[25] R. R. E. de Gaston and M. G. Safonov, "Exact Calculation of Multiloop Stability Margin," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 2, pp. 156-171, 1988.

[26] M. Fu, "Polytopes of Polynomials with Zeros in a Prescribed Region: New Criteria and Algorithms," to appear in *Robustness in Identification and Control*, M. Milanese, R. Tempo, and A. Pecile, eds.; also presented at the International Workshop on Robustness in Identification and Control, Turin, Italy, June 1988.

[27] M. Fu and B. R. Barmish, "Polytopes of Polynomials with Zeros in a Prescribed Set," *Proc. 1988 ACC*, Atlanta, GA, pp. 2461-2464, June 1988; also, to appear in *IEEE Trans. Automat. Contr.*

[28] M. Fu, A. W. Olbrot, and M. P. Polis, "Robust Stability for Time-Delay Systems: The Edge Theorem and Graphical Tests," to appear in *IEEE Trans. Automat. Contr.*

[29] C. V. Hollot and A. C. Bartlett, "Some Discrete-Time Counterparts to Kharitonov's Stability Criterion for Uncertain Systems," *IEEE Trans. Automat. Contr.*, vol. AC-31, no. 4, pp. 355-356, 1986.

[30] C. V. Hollot, D. P. Looze, and A. C. Bartlett, "Unmodeled Dynamics: Performance and Stability via Parameter Space Methods," *Proc. 26th IEEE CDC*, pp. 2076-2081, 1987.

[31] F. J. Kraus, M. Mansour, and B. D. O. Anderson, "Robust Schur Polynomial Sta-

bility and Kharitonov's Theorem," *Proc. 26th IEEE CDC*, Los Angeles, CA, pp. 2088–2095, Dec. 1987.

[32] I. R. Petersen, "A Class of Stability Regions for Which a Kharitonov-like Theorem Holds," *Proc. 26th IEEE CDC*, Los Angeles, CA, pp. 440–444, Dec. 1987.

[33] I. R. Petersen, "A New Extension to Kharitonov's Theorem," *Proc. 26th IEEE CDC*, Los Angeles, CA, pp. 2070–2075, Dec. 1987.

[34] K. M. Zhou and P. P. Khargonekar, "Stability Robustness Bounds for Linear State Space Models with Structured Uncertainty," *IEEE Trans. Automat. Contr.*, vol. AC-32, no. 7, pp. 621–623, 1987.

[35] N. K. Bose, E. I. Jury, and E. Zeheb, "On Robust Hurwitz and Schur Polynomials," *Proc. 25th IEEE CDC*, Athens, Greece, pp. 739–744, Dec. 1986.

[36] A. C. Bartlett and C. V. Hollot, "A Necessary and Sufficient Condition for Schur Invariance and Generalized Stability of Polytopes of Polynomials," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 6, pp. 577–578, 1988.

[37] S. Dasgupta, P. J. Parker, B. D. O. Anderson, F. J. Kraus, and M. Mansour, "Frequency Domain Conditions for the Robust Stability of Linear and Nonlinear Dynamic Systems," *Proc. 1988 ACC*, Atlanta, GA, pp. 1863–1868, June 1988.

[38] A. Vicino, "Some Results on Robust Stability of Discrete Time Systems," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 9, pp. 844–847, 1988.

[39] B. R. Barmish, M. Fu, and S. Saleh, "Stability of a Polytope of Matrices: Counterexamples," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 6, pp. 569–571, 1988.

[40] R. Byers, "A Bisection Method for Measuring the Distance of a Stable Matrix to the Unstable Matrices," to appear in *SIAM J. Scientific and Statistical Computing*.

[41] J. A. Heinen, "Sufficient Conditions for Stability of Interval Matrices," *Intl. J. Contr.*, vol. 39, no. 6, pp. 1323–1328, 1984.

[42] D. Hinrichsen and A. J. Pritchard, "Stability Radii of Linear Systems," *Syst. Contr. Lett.*, vol. 7, no. 1, pp. 1–10, 1986.

[43] W. H. Lee, "Robustness Analysis for State Space Models," Alphatech, Inc., Rept. TP-151, 1982.

[44] R. V. Patel and M. Toda, "Quantitative Measures of Robustness for a Multivariable System," *Proc. JACC*, San Francisco, CA, Paper TP8-A, 1980.

[45] L. Qui and E. J. Davison, "New Perturbation Bounds for the Robustness Stability of Linear State Space Models," *Proc. 25th IEEE CDC*, Athens, Greece, pp. 751–755, 1986.

[46] L. X. Xin, "Stability of Interval Matrices," *Intl. J. Contr.*, vol. 45, pp. 203–214, 1987.

[47] D. Y. Xu, "Simple Criteria for Stability of Interval Matrices," *Intl. J. Contr.*, vol. 41, no. 1, pp. 289–295, 1985.

[48] R. K. Yedavalli and Z. Liang, "Reduced Conservatism in Stability Robustness Bounds by State Transformation," *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 863–866, 1986.

[49] B. R. Barmish, "Invariance of the Strict Hurwitz Property for Polynomials with Perturbed Coefficients," *IEEE Trans. Automat. Contr.*, vol. AC-29, no. 10, pp. 935–936, 1984.

[50] M. Fu and B. R. Barmish, "Maximal Unidirectional Perturbation Bounds for Stability of Polynomials and Matrices," *Syst. Contr. Lett.*, vol. 11, pp. 173–179, 1988.

[51] R. M. Biernacki, H. Hwang, and S. P. Bhattacharyya, "Robust Stability with Structured Parameter Perturbations," *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 495–506, 1987.

[52] C. B. Soh, C. S. Berger, and K. P. Dabke, "On the Stability Properties of Polynomials with Perturbed Coefficients," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 1033–1036, 1985.

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