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Introduction to quantized feedback control and estimation

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Abstract: This paper serves as a tutorial paper for a new area of research in control systems, namely, quantized feedback control and estimation. This area is motivated by the increasing need of incorporating communication networks in a control system. In such a framework, feedback information needs to be transmitted over a digital network, which results in a number of new challenges for estimation and control design. The focus of this paper is to introduce a number of recent results on the design of quantizers for the purposes of control design and state estimation. uantized feedback control, networked control, quantized estimation, robust control.

Key words: networked control; quantized feedback control; quantized estimation; quantization theory; separation principle

1 Introduction

Although often invisible to the general public, control and automation technologies are an integral part of modern industry. These technologies are essential in managing complex data and information, ensuring stable and safe operations, optimizing operational performances, guaranteeing economic viability and safeguarding environmental impacts. Unlike the traditional control technologies, modern control systems are typically implemented on a digital communication platform, forming the so-called networked control systems. With the increasing success and popularity of wireless communications, there is also an increasing need in deploying wireless network based control systems.

The introduction of networks has created many new challenges to the traditional control theory. Traditional control theory assumes that the feedback channel is analog and solely dedicated to control purposes. However, more and more industrial systems are controlled via digital communication links such as Fieldbus, local area networks, and wireless networks. These communication links are shared with other network functions. This means that feedback signals in the control loop are subject to a number of undesirable distortions. These include quantization errors, time delays, transmission errors and packet dropouts.

Quantization errors occur because a digital network is limited by the amount of data it can transmit per unit time. Such a limit can be severe especially in wireless networks. Time delays occur naturally in a network. The major problems here are random delays caused by congestions, packet collisions, re-transmissions and unpredictable routings. Transmission errors here refer to errors not recoverable through error-correction coding and decoding, thus unrecognizable by the control designer. Packet dropouts refer to the loss of transmission data, when transmission of a given packet can not take place within the required time limit. All these undesirable properties in a digital network require us to develop new theory and techniques for estimation and control design.

In this paper, we focus on the quantization issue and study the so-called quantized feedback control and estimation problems. The setting for these problems is that we assume that data transmission is only subject to quantization errors and ignore all other features such as time delays and packet dropouts. Likewise, many other researchers focus on other aspects of the networked control problems by, e.g., ignoring the quantization issue. This type of divide-and-conquer approach allows us to provide concrete solutions for each communication issue, and we can combine different solutions together for practical implementations.

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Control and estimation using quantized information can be traced back to early days of control research. In particular, research into the so-called quantized linear quadratic Gaussian (LQG) control problem started in early 1960's. This problem is the standard LQG control problem but subject to the constraint that the feedback information must be quantized by a fixed-rate quantizer. Related works include [19-23]. More broad attempts on quantized feedback control can be traced back further to the works of [17] and [18] on the effects of quantization errors to sampled-data feedback systems.

The overwhelming success of networked control systems, especially for industrial control and automation, has brought a resurgent interest in quantized feedback control. Examples of works include[1-4,6,9,8,7,10-11,13-14]. There are also many recent attempts on the quantized LQG problem; see[12,15,24].

Like in the classical control theory where state estimation plays in essential role, estimation based on quantized information is also critical to quantized feedback control. This has been well recognized in most of the references above. In addition, quantized estimation has a broad range of applications beyond feedback control. Examples include sensor network-based estimation and tracking[28-29] and consensus networks [26-27]. Examples where quantized estimation is a part of the solution to a more broad problem of network-based estimation subject to transmission delays, packet dropouts and other problems can be found in [25,28-29].

In the rest of the paper, we discuss how to jointly design quantizer and feedback controller or state estimator. We consider both static quantization and dynamic quantization.

2 Quantized feedback control via static quantization

Consider the following system:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}x(k) + \boldsymbol{B}u(k), \\ \boldsymbol{y}(k) &= \boldsymbol{C}x(k), \end{aligned} \tag{1}$$

where $\boldsymbol{x}(k) \in \mathbf{R}^n$ is the state, $\boldsymbol{u}(k) \in \mathbf{R}$ is the control input, $\boldsymbol{y}(k) \in \mathbf{R}$ is the measured output, $\boldsymbol{A} \in \mathbf{R}^{n\times n}$, $\boldsymbol{B} \in \mathbf{R}^{n\times 1}$ and $\boldsymbol{C} \in \mathbf{R}^{1\times n}$ are given. We will denote the transfer function from $\boldsymbol{u}(k)$ to $\boldsymbol{y}(k)$ by $\boldsymbol{G}(z)$. We assume that $\boldsymbol{A}$ is unstable and $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ is a minimal realization.

The quantized feedback control problem is is depicted in **Figure 1**, i.e., is to design a feedback quantizer

$$v(k) = Q(y(k)), \tag{2}$$

and a feedback controller of the form

$$\begin{aligned} \hat{x}(k+1) &= \boldsymbol{A}_c\hat{x}(k) + \boldsymbol{B}_c v(k),\ \hat{x}(0) = 0, \\ u(k) &= C_c\hat{x}(k) + D_c v(k), \end{aligned} \tag{3}$$

with $\hat{x}(k) \in \mathbf{R}^n$, such that the closed-loop system is stable and that the so-called quantization density [9] is coarsest. The quantization density of $Q(\cdot)$ is defined as follows:

$$\eta_Q = \lim\sup_{\varepsilon\to 0} \frac{\# g[\varepsilon]}{-\ln\varepsilon}, \tag{4}$$

where $\# g[\varepsilon]$ denotes the number of quantization levels in the interval $[\varepsilon, 1/\varepsilon]$.

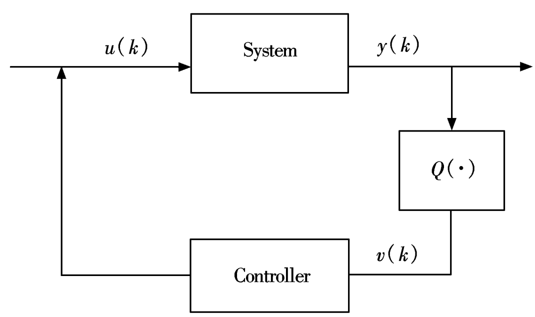

Fig.1 Quantized feedback control

In this section, restrict ourselves to a static quantizer. That is, the quantizer does not use past input and quantized values in order to quantize the given (current) input value. In this setting, it turns out that the so-called logarithmic quantizers are more appropriate than the commonly used (and commercially available) linear quantizers.

A logarithmic quantizer is described by

$$\mathscr{V} = \{\mu_i = \rho^i\mu_0 : i = 0, \pm 1, \pm 2, \cdots\} \cup \{0\}, \mu_0 > 0, \tag{5}$$

where $\rho \in (0,1)$ and

$$Q(y) = \begin{cases} \rho^i\mu_0, & \text{if } \frac{1}{1+\delta}\rho^i\mu_0 < y \leqslant \frac{1}{1-\delta}\rho^i\mu_0, \\ 0, & \text{if } y = 0, \\ -Q(-y), & \text{if } y < 0, \end{cases} \tag{6}$$

where

$$\delta = \frac{1-\rho}{1+\rho}. \tag{7}$$

A pictorial representation is given in **Figure 2.** The description above is for an infinite-level logarithmic quantizer. In practice, it is truncated when the input is too large (by a saturator) or too small (by a dead zone) in magnitude.

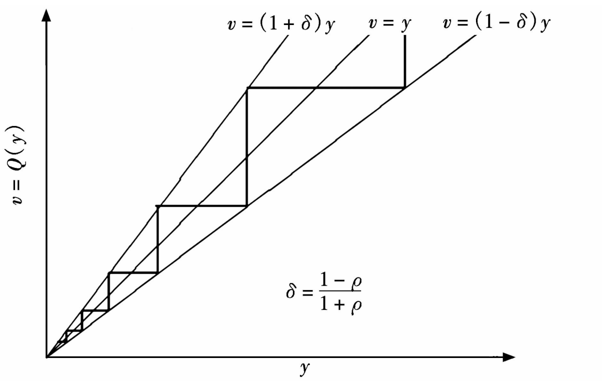

Fig.2　Logarithmic quantizer

The first case where logarithmic quantization is superior to linear quantization is in quantized feedback control where the objective is to drive the output or the state to the origin but the control signal or measurement signal need to be quantized [9,13]. This arises in stabilization, tracking and disturbance attenuation. The reason is that logarithmic quantization gives a multiplicative quantization error, which reduces as the input signal becomes small. As a tradeoff, the quantization error becomes large when the input signal is large, but this does not create problems.

The second case where logarithmic quantization is superior to linear quantization is in quantized state estimation where the state of a system needs to be estimated using quantized information [16]. If the measured signal is quantized directly, logarithmic quantization may not be appropriate because the measurement may be persistently large. However, one may quantize the estimation error instead. In doing so, logarithmic quantization is better because we want a small quantization error when the estimation error becomes small and we can tolerate a large quantization error when the estimation error is large.

Another case where logarithmic quantization is advantageous is when the signal to be quantized already has a multiplicative noise. Many sensors have the feature that measurement errors are specified using a relative error. For example, positions are often measured by range (distance) and most range sensors have accuracies specified by relative errors. Recall that logarithmic quantization also introduces a multiplicative error. When it is combined with a multiplicative noise, it is simply magnified without changing the noise structure.

It is interesting to note that most control and estimation settings deal with additive noises. We note here that this is indeed done mainly for mathematical convenience because multiplicative noises are somewhat more difficult to deal with; see [30].

It was shown in [13] that the optimal quantizer structure for the quadratic stabilization of (1) is given by logarithmic quantization. Moreover, under quadratic stabilization, quantized feedback control is equivalent to robust control with sector bounded uncertainty, and the coarsest quantization density (which is equivalent to the smallest ρ) can be found by standard H_∞ optimization as detailed below.

Theorem 1　Consider the system (1). For a given quantization density $\rho>0$, the system is quadratically stabilizable via a quantized controller (2) if and only if the following auxiliary system:

$$x(k+1)=\boldsymbol{A}x(k)+\boldsymbol{B}u(k)$$
$$v(k)=(1+\Delta)\boldsymbol{C}x(k),\ |\Delta|\leqslant\delta \tag{8}$$

is quadratically stabilizable via:

$$x_c(k+1)=\boldsymbol{A}_c x_c(k)+\boldsymbol{B}_c v(k)$$
$$u(k)=\boldsymbol{C}_c x_c(k)+\boldsymbol{D}_c v(k), \tag{9}$$

where δ, which is the sector bound produced by the quantization error, and ρ are related by (6).

The largest sector bound $\delta_{\sup}$ (which gives $\rho_{\inf}$) is given by

$$\delta_{\sup}=(\inf_{H(z)}\|\bar{G}_c(z)\|_\infty)^{-1}, \tag{10}$$

where $\bar{\boldsymbol{G}}_c(z)=(1-\boldsymbol{H}(z)\boldsymbol{G}(z))^{-1}\boldsymbol{H}(z)\boldsymbol{G}(z)$ and $\boldsymbol{H}(z)=\boldsymbol{D}_c+\boldsymbol{C}_c(z\boldsymbol{I}-\boldsymbol{A}_c)^{-1}\boldsymbol{B}_c$.

The result builds a fundamental bridge between quantized feedback control and robust control, paving way for a lot of further research on networked control.

3　Quantized state estimation

Consider the following linear system:

$$x(k+1)=\boldsymbol{A}x(k)+\boldsymbol{B}w(k),\ x(0)=x_0,$$
$$y(k)=\boldsymbol{C}x(k)+v(k), \tag{11}$$

where $w(k)\in\mathbf{R}^m$ is the process noise, $v(k)\in\mathbf{R}$ is the measurement noise. It is assumed that $x_0\in\mathbf{R}^n$ is a random variable with mean $\bar{x}_0$ and covariance Σ_0, and w and v are uncorrelated zero-mean white noises with covariances Σ_w and Σ_v, respectively, and they are uncorrelated with x_0.

We study the problem of state estimation using quantized measurement transmitted over a digital communication channel with a limited data rate. It is desirable to know how to quantize the measured signal so that good state estimation can be achieved using limited information.

The quantized estimator is shown in **Figure 3**. Instead of quantizing the measured signal directly, we choose to quantize the prediction error of the estimator. The estimator is chosen to be

$$\hat{x}(k+1) = A\hat{x}(k) + LQ(y(k) - \hat{y}(k)), \hat{x}(0) = \bar{x}_0,$$
$$\hat{y}(k) = C\hat{x}(k)), \qquad (12)$$

where $\hat{x}(k) \in \mathbf{R}^n$ is the estimate of $x(k)$, $\hat{y}(k) \in \mathbf{R}$ is the estimate of $y(k)$ based on $\hat{x}(k)$, $Q(\cdot)$ is the quantizer, and L is the estimator gain.

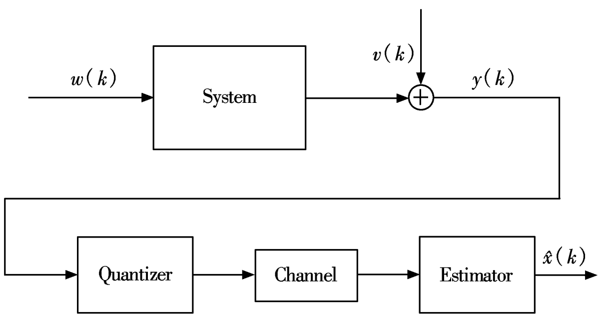

Fig.3 Quantized state estimation

Note in the above that state estimation is constructed only using the quantized prediction error. Therefore, under the ideal channel assumption, both sides of the channel can construct the same estimate using the quantized prediction error. In particular, the construction of $\hat{x}(k)$ on the transmission side does not require the estimated state to be transmitted back from the receiver side.

A logarithmic quantizer is used. Defining the estimation error

$$e(k) = x(k) - \hat{x}(k)$$

and its covariance matrix

$$E(k) = \varepsilon\{e(k)e^{\mathrm{T}}(k)\}$$

the aim is to design both the filter gain L and the quantizer so that the trace of the asymptotic $E(k)$, i.e., $E = \lim_{k\to\infty}$, is to be minimized. Details can be found in [16].

We now demonstrate quantized state estimation by an example. The system model is given by (11) with

$$\boldsymbol{A} = \begin{bmatrix} 2.4744 & -2.811 & 1.7038 & -.5444 & .0723 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 01 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{B}^{\mathrm{T}} = [1 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\boldsymbol{C} = [0.245 \quad 0.236 \quad 0.384 \quad 0.146 \quad 0.035], \qquad (13)$$

$\Sigma_w = 1$ and $\Sigma_v = 1/16$. The range of δ for the tests is chosen to be [0, 0.3]. For each δ, we try two estimator gains L, one taken as the Kalman gain designed by ignoring the quantization error and one being the robust gain computed by treating the quantization error as a multiplicative noise.

Figure 4 shows the simulated values of tr(E). Also shown in the figure are the estimates of tr(E) which we can ignore for this paper. We have two observations: (1) As the quantization becomes coarse (ρ becomes small or δ becomes large), the estimation error increases; (2) the robust gain outperforms the Kalman gain more significantly when the quantization becomes coarse.

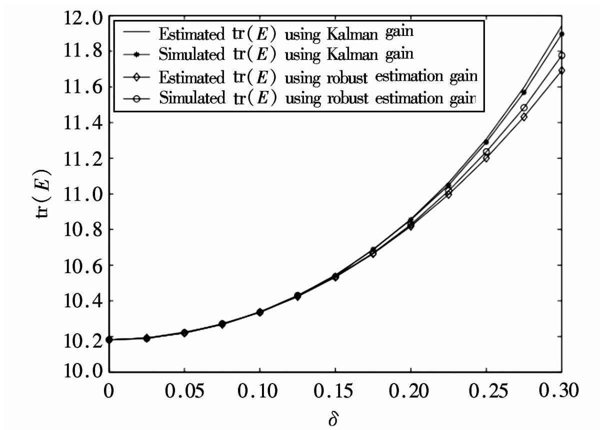

Fig.4 Intinite-level logarithmic quantization

When the quantizer is truncated to a finite-level one, additional estimation error arises. In this case, apart from the ρ, the parameter μ_0 in the quantizer needs to be designed as well. As a result, with about 4 ~ 5 bits of quantization, the quantized estimator has its estimation error variance only marginally larger than in the case without quantization. The details on the design of ρ and μ_0 can be found in [16]. **Figure 5** shows the result of estimation error vs. the number of quantization bits N_b.

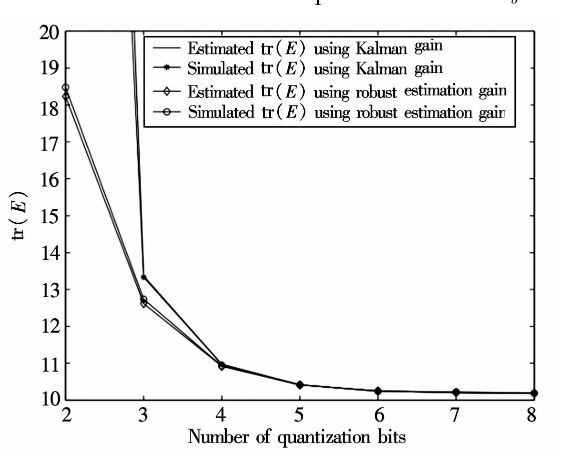

Fig.5 16-level quantization

A *dynamic quantizer* uses memory, i.e., it can use the past input-output values of the quantizer to determine how to quantize a current input value, and thus is more complex and potentially more powerful.

One type of dynamic quantizers uses *dynamic scaling* in conjunction with a static quantizer. That is, the input signal is pre-scaled so that its range is more suitable for quantization. The scaling parameter is dynamically adjusted (i.e., adjusted online). Noticeable work along this line includes [3,6,10-11]. In [3], it is pointed out that if a system is not excessively unstable, by employing a quantizer with various sensitivity a feedback strategy can be designed to bring the closed-loop state arbitrarily close to zero for an arbitrarily long time. The idea of quantizer with sensitivity is extended in [6] where it is shown that there exists a dynamic adjustment of the quantizer sensitivity and a quantized state feedback that asymptotically stabilizes the system. In the case of output feedback, a local (or semi-global) stabilization result is obtained.

It is shown in [8] that stabilization of a single-input-single-output linear time-invariant system (in some stochastic sense) can be achieved using only a finite number of quantization levels. In addition, the minimum number of quantization levels (also known as the minimum *feedback information rate*) is explicitly related to the unstable poles of the system, under the assumption of noise free communications. In this setting, the dynamic quantizer effectively consists of two parts: an *encoder* at the output end and a *decoder* at the input end. The problem of minimum feedback information rate is studied in more details in [10] by analyzing the structures of the encoder and decoder. We do caution that many results on quantized feedback with dynamic quantizers may be impractical due to three problems: (1) Most results are for stabilization only rather than for performance control; (2) The transient response is typically very poor due to the lack of good control design algorithms; (3) As pointed out in [31], the capacity results are in general not valid for practical communications channels which are not noise free.

In [14], a simple dynamic scaling method has been studied. This method employs a finite-level logarithmic quantizer $Q(\cdot)$ in conjunction with the following scaling:

$$v_k = g_k^{-1} Q(g_k y_k). \tag{14}$$

where the scaling gain g_k is adjusted by

$$g_{k+1} = \begin{cases} g_k \gamma_1, & \text{if } |Q(g_k y_k)| = \mu_0, \\ g_k / \gamma_2, & \text{if } |Q(g_k y_k)| = \rho^{N-1}\mu_0, \\ g_k, & \text{otherwise.} \end{cases} \tag{15}$$

with some initial $g_0 > 0$, where $\gamma_1, \gamma_2 \in (0,1)$ are design parameters. The basic idea is to scale down (resp. up) the next input if the current input is too large (resp. small) in magnitude.

Note that g_{k+1} is determined based on when $Q(g_k y_k)$ (quantized information), no additional information needs to be passed on from the transmit side to the receive side for updating g_k, provided both sides start with the same g_0 and there is no transmission error for the quantized information.

It is shown in [14] that it requires only a finite number of logarithmic quantization levels to quadratically stabilize a given linear system when the above dynamic scaling method is used. The detailed design of the dynamic quantizer and the controller are not discussed here.

Simulation results show that for most practical control systems, the number of quantization bits per time sample is very moderate [14]. To demonstrate this fact, we consider the system (1) with

$$\boldsymbol{A} = \begin{bmatrix} 2.7 & -2.41 & 0.507 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$\boldsymbol{C} = [1 \quad -0.5 \quad 0.04].$$

The system is unstable with two unstable open-loop poles at $1.2 \pm i0.5$ but without unstable zero and the relative degree is 1. **Figure 6** shows the state response of the closed-loop system with a 4-bit logarithmic quantizer.

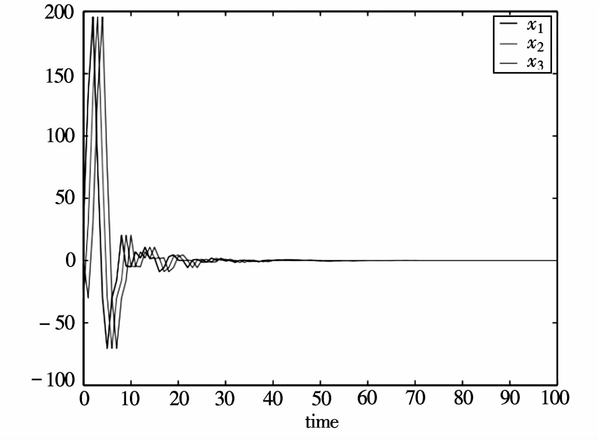

Fig.6　Closed-loop response with a 4-bit quantizer

5　Quantized linear quadratic gaussian control

In [24], we focus on the so-called *quantized LQG con-*

trol problem which is generalized from the standard LQG problem in discrete time but with the constraint that the feedback channel is a digital link with a fixed bit rate. The quantized LQG problem we study is the same as the standard LQG control problem but with the constraint that the feedback signal must be quantized and transmitted over a digital link with a fixed bit rate, as depicted in **Figure 7**.

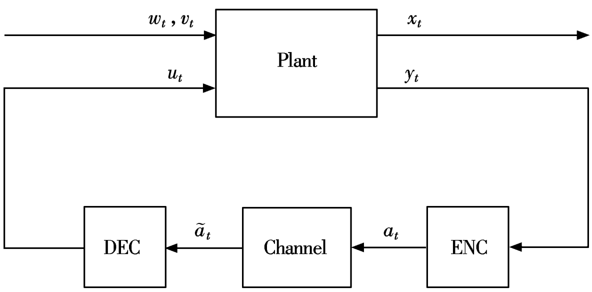

Fig.7 Quantized LQG control system

The system we consider is a discrete-time model given by

$$x_{t+1} = \boldsymbol{A}x_t + \boldsymbol{B}u_t + w_t,$$
$$y_t = \boldsymbol{C}x_t + v_t, \qquad (16)$$

where $x_t \in \mathbf{R}^n$ is the state, $u_t \in \mathbf{R}^m$ is the control input, $y_t \in \mathbf{R}^p$ is the measured output, $w_t \in \mathbf{R}^n$ and $v_t \in \mathbf{R}^p$ are independent Gaussian random distributions with zero mean and covariances $W_t > 0$ and $V_t > 0$, respectively, and the initial state x_0 is also assumed to be an independent zero-mean Gaussian distribution with covariance Σ_0.

In the sequel, we denote $z^t = \{z_0, z_1, \cdots, z_t\}$.

The communication channel we consider in this paper is assumed to be a memoryless and error-free channel with a fixed transmission rate of R bits per sample. The output signal y_t needs to be encoded first (as indicated by the ENC block in **Figure 2** before transmission, and the received signal is decoded which is then used to construct a control signal u_t (as indicated by the DEC block in **Figure 2**).

The encoder is required to be a causal mapping from the measured signal y_t, i.e.,

$$a_t = \alpha_t(y^t \mid a^{t-1}), \qquad (17)$$

where $\alpha(\cdot)$ takes values in a finite alphabet set $\mathscr{A}$ with size of 2^R. Without loss of generality, we take $\mathscr{A} = \{1, 2, \cdots, 2^R\}$.

Similarly, the decoder is required to be a causal mapping from the received quantized signal, i.e.,

$$u_t = \beta_t(\tilde{a}_t \mid \tilde{a}^{t-1}), \qquad (18)$$

where $\tilde{a}_t$ is the received version of a_t. Because the channel is error free, $\tilde{a}_t = a_t$, thus (22) can be rewritten as

$$u_t = \beta_t(a_t \mid a^{t-1}). \qquad (19)$$

We are interested in the following linear quadratic cost:

$$J = \varepsilon\left[x'_T Q_T x_T + \sum_{t=0}^{T-1} x'_t Q_t x_t + 2u'_t H_t x_t + u'_t S_t u_t\right], \qquad (20)$$

where $\varepsilon[\cdot]$ is the expectation operator and $Q_t = Q'_t$, $S_t = S'_t$ and H_t are weighting matrices with

$$S_t > 0, Q_t - H_t S_t^{-1} H'_t \geqslant 0 \qquad (21)$$

for all $t = 0, 1, \cdots, T-1$ and $Q_T = Q_T \geqslant 0$.

The problem of *quantized LQG control* is to jointly design the quantizer and controller (or encoder and decoder) to minimize the cost J, under the bit rate constraint.

In [24], we first look back at the history of the research on this problem and discuss many attempts to generalize the separation principle [19-23], some dated back to early 1960's. We point out that many of these generalizations contain technical errors and/or misinterpretations. This leads us to a number of results on quantized LQG control, as briefed below:

(1) A *weak separation principle* holds which states that optimal quantized LQG control can be achieved by separately designing state estimation, state feedback control and quantization. However, the separation is *weak* in two ways: (i) The quantization criterion depends on the control cost function; (ii) More seriously, optimal quantization can not be done by separately minimizing the quantization errors at different time instants. These weaknesses imply that optimal design for quantized LQG control is very complex numerically and is in huge contrast with the classical separation principle where state estimation is independent of the state feedback control and state estimation at each time instant can be done recursively without considering the future evolution of the system dynamics.

(2) The consequence of the weak separation principle is that the quantized LQG problem becomes a *quantized state estimation* problem. In this problem, the output signal of a system needs to be quantized by a fixed rate quantizer and the quantized information is used to construct an estimate of a linear function of the state of the system, the desired control signal in our case, in a way to minimize a given distortion function. We point out that

this can be viewed as a *generalized vector quantization* problem. We then use a linear predictive coding (LPC) type of approach to show that, under high resolution quantization and some mild rank condition, optimal quantization is done by using a memoryless quantizer. Using memoryless quantizers means that quantization can be done by considering each input sample separately. This result, together with the weak separation principle above, shows that a *full* separation principle holds for quantized LQG control under high resolution quantization and the mild rank condition. This rank condition essentially requires the dimension of output not to exceed the dimension of the input, which holds in particular for single-input-single-output systems.

Details on quantized LQG control can be found in [24].

6 Conclusion

In this paper, we have briefly discussed a number of quantized feedback control and estimation problems. This review is limited because many other results are not discussed. These include robust control using quantized feedback [32], control design with both input and output quantization [33] and feedback control with minimal feedback information in general [8,34].

Quantized feedback control is a relatively new area of research with many open and challenging questions. Although quantization is a well-studied subject in signal processing and digital communications, we caution that it is usually not appropriate to directly apply techniques in these areas to control problems. The main reason for this is that control systems involve feedback, which has two major implications: (1) The quantized signal re-enters the system; (2) The input signal to the quantizer is not known *a priori* to be bounded. Both of these implications make the analysis and design of quantizers much more difficult.

Future research work in this area should be directed at inter-disciplinary studies by incorporating knowledge in control, information theory, communication networks, sensor networks and quantization theory so that not only quantization problems but other network induced control and estimation problems can be solved.

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