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Journal of the Franklin Institute 355 (2018) 6617–6637
www.elsevier.com/locate/jfranklin

Observer-based control for singular nonhomogeneous Markov jump systems with packet losses ☆

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Received 2 January 2018; received in revised form 7 June 2018; accepted 27 June 2018
Available online 25 July 2018

Abstract

This paper is concerned with the observer-based H_∞ control for a class of singular Markov jump systems over a finite-time interval, where the transition probability (TP) is time-varying and is limited to a convex hull. Due to the limited capacity of network medium, packet losses are presented in the underlying systems. Firstly, using a stochastic Lyapunov functional, a sufficient condition on singular stochastic H_∞ finite-time boundedness for the corresponding closed-loop error systems is provided. Subsequently, a linear matrix inequality (LMI) condition on the existence of the H_∞ observer-based controller is developed from a new perspective. Finally, three numerical examples are provided to illustrate the effectiveness of the proposed controller design method, wherein it is shown that the proposed method yields less conservative results than those in the literature.

☆ This work is supported by National Natural Science Foundation of China (61473173), Major International (Regional) Joint Research Project of the National Natural Science Foundation of China (NSFC)(61320106011).

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https://doi.org/10.1016/j.jfranklin.2018.06.032
0016-0032/

1. Introduction

The past several decades have witnessed a great deal of interest in Markov jump systems (MJSs). This increased interest is due to their strong ability to describe systems subject to abrupt variation in their structures or parameters [1]. As a result, MJSs can be utilized to characterize and model many types of systems in applications, such as communication systems, networked control systems, economics systems, and others. It is a fact that TPs play an important role in the performance of such systems. Based on the assumption that the TPs are time invariant, the stability analysis and synthesis for MJSs have been studied in [2–8]. For example, the problem of adaptive sliding-mode stabilization for MJSs with actuator faults was discussed in [7]; the energy-to-peak state estimation for Markov jump recurrent neural networks with time-varying delays was studied in [8]. However, in practical applications involving economic systems, flight control systems and networked control systems, the TPs are not time invariant. As it is well known that packet dropout and stochastic delays in the networked control systems can be expressed by a Markov process or Markov chain. In practice, delay or packet dropouts are changing in different periods, which results in the time-varying transition probabilities, so the investigation of the control problem on MJSs with nonhomogeneous Markov process or Markov chain becomes important. Recently, the issue of state estimation for Markov jump neural networks with piecewise homogeneous Markov chain was concerned in [9,10]. For nonhomogeneous MJSs (NMJSs), the stability analysis and controller design have been investigated in [11–15]. Especially, when the time-varying TPs are assumed to be in a polytopic sense, the design of controller for NMJSs was investigated in [12–15]; the filtering problem for Markov jump neural networks was investigated in [16].

Singular systems, also referred to as descriptor systems, differential-algebraic systems, generalized state-space systems or semi-state systems, have attracted a large number of researchers' attention. The reason is that they have widespread applications in biological systems, networked control systems, economic systems, power systems, and so on [17,18]. Recently, the observer-based controller for descriptor system with Brownian motions was investigated in [19]. Singular MJSs, as a special class of MJSs, have been widely studied due to their perfect application in the real system [20–32]. Many interesting results for singular MJSs are produced, for example stabilization [20,21,23–26], sliding mode control [22,27], finite-time control [28–32]. In particular, the observer-based finite-time control problem for discrete-time singular MJSs has been studied in [28]. However, in order to use the existing LMIs method, there is mandatory restriction on the Lyapunov variables in [28], which will lead to conservative results. By invoking equality constraints $P_{ia}B_i = B_i\theta_i$, the reliable sliding mode finite-time control for discrete-time singular MJSs with sensor fault and randomly occuring nonlinearities has been discussed in [31]. In this case, checking the conditions may involve numerical difficulties. Thus, developing a method to give a less conservative condition on the existence of an observer-based controller for singular MJSs in terms of strict LMIs motivates our current study.

Networked control systems have many advantages such as lower cost, higher reliability and easier maintenance. The network-induced problems has been attracted lots of researchers in the past decades, such as network-induced time delays [33,34], event-triggered control [35,36]. It is worth mentioning that in networked control systems, the data may be damaged in the network due to limited bandwidth, sensor failure and noisy measurements. This can degrade the system performance or even cause system-level faults. Consequently, many useful results on designing networked control systems against the packet losses have been developed

[37–43]. The H_∞ control problem for nonlinear systems with missing measurements between the sensor, controller and actuator was studied in [38]. Authors in [41] considered the H_∞ filtering problem for discrete-time singular systems with lossy measurements. For singular MJSs with missing measurements, the design of filter was given in [42,43]. But up to now, the issue of observer-based controller design for singular NMJSs in the presence of packet losses has not been addressed.

Inspired by the aforementioned works, we study the observer-based finite-time control problem for a type of discrete-time singular NMJSs subject to packet losses in this paper. Bernoulli processes are introduced to describe the intermittent measurements caused by the packet losses in the forward and feedback channels. First, based on stochastic Lyapunov functional, considering the influence of packet losses, a sufficient condition on singular stochastic H_∞ finite-time boundedness for the corresponding closed-loop error systems is given. Then, we design the observer-based controllers in terms of strict LMIs from a new perspective. The innovations of this paper are outlined as follows:

(1) A new discrete-time singular MJSs model is proposed, which takes singular systems, nonhomogeneous Markov chain and packet losses into account. In contrast to [30,31,32], a new method is introduced to better eliminate the coupling between Lyapunov variables and system matrices.
(2) Different from [5,13,28,31], a new observer-based controller design method is presented in our paper, which leads to a less conservative result.
(3) To show the practicability of the proposed method, the DC motor controlled inverted pendulum is applied.

Notations: Throughout this paper, $X \geq 0$ $(X > 0)$ means that the symmetric matrix X is semi-positive definite (positive definite). I and 0 represent, respectively, the identity matrix and zero matrix with appropriate dimensions. The superscript 'T' denotes the transpose of a matrix, diag$\{\cdots\}$ represents a block-diagonal matrix. $\|x\|$ refers to the Euclidean norm of the vector x. $\mathbf{E}[\cdot]$ stands for the mathematical expectation. In addition, in symmetric block matrices, * represents as an ellipsis for the terms that are introduced by symmetry, and $sym(X)$ represents $X + X^T$. $\star$ represents matrix components that are not relevant in the discussion.

2. Preliminaries

Consider the following discrete-time singular NMJSs:

$$\begin{cases} Ex(k+1) = A(\theta_k)x(k) + B_1(\theta_k)u(k) + B_2(\theta_k)\omega(k), \\ y(k) = \alpha_k C(\theta_k)x(k), \\ z(k) = H(\theta_k)x(k) + D(\theta_k)\omega(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^l$, and $z(k) \in \mathbb{R}^p$ are the system state, the measurement output, and the controlled output of the system, respectively. The matrix $E \in \mathbb{R}^{n\times n}$ is singular with rank$(E) = r_e \leq n$. $\omega(k) \in \mathbb{R}^q$ is the exogenous disturbance input that is of the following form:

$$\mathbf{E}\left\{ \sum_{k=0}^{N} \omega^T(k)\omega(k) \right\} \leq d^2, d \geq 0. \tag{2}$$

In this paper, the stochastic variables α_k represents the possibility of occurring networked induced packet losses, which is a Bernoulli distributed white sequence with the following probability distribution laws:

$$\text{Prob}\{\alpha_{\text{k}}=1\}=\mathbf{E}\{\alpha_k\}=\alpha,$$
$$\text{Prob}\{\alpha_k=0\}=1-\mathbf{E}\{\alpha_k\}=1-\alpha, \tag{3}$$

where $\alpha \in [0,\ 1]$ is a known constant.

$\{\theta_k,\ k\geq 0\}$ is a discrete-time Markov stochastic process taking values in a finite state space $\mathcal{S}=\{1,2,\ldots,S\}$, the evolution of $\{\theta_k,\ k\geq 0\}$ is governed by the following TPs:

$$\pi_{ij}(k)=\Pr\{\theta_{k+1}=j|\theta_k=i\}, \tag{4}$$

with the restrictions $\pi_{ij}(k)\geq 0$ and $\sum_{j=1}^{S}\pi_{ij}(k)=1$. $\pi_{ij}(k)$ are the entries of the TP matrix $\Pi(k)$. $\Pi(k)$ is a time-varying matrix that resides in a polytope:

$$\Pi(k)\in \text{co}\{\Pi^s : s=1,2,\ldots,\mathcal{M}\}, \tag{5}$$

where $\Pi^s : s=1,2,\ldots,\mathcal{M}$ are constant TP matrices that are the vertices of the polytope and co stands for convex hull, namely,

$$\Pi(k)=\sum_{s=1}^{\mathcal{M}}\alpha_s(k)\Pi^s, \tag{6}$$

where $\alpha_s(k)\in[0,1], s=1,2,\ldots,\mathcal{M}$, and $\sum_{s=1}^{\mathcal{M}}\alpha_s(k)=1$.

In this paper, we design an observer-based controller for system (1) of the following form:

$$\begin{cases}E\overline{x}(k+1)=A(\theta_k)\overline{x}(k)+B_1(\theta_k)u(k)+F(\theta_k)(y(k)-\alpha_kC(\theta_k)\overline{x}(k)),\\ u(k)=\beta_kK(\theta_k)\overline{x}(k).\end{cases} \tag{7}$$

where $\overline{x}(k)\in\mathbb{R}^m$ is the estimated state. $F(\theta_k)$, $K(\theta_k)$ are the observer and controller gains to be designed later, respectively. The stochastic variables β_k, mutually independent of α_k, is also a Bernoulli distributed white sequence with the following probability distribution laws:

$$\text{Prob}\{\beta_{\text{k}}=1\}=\mathbf{E}\{\beta_k\}=\beta,$$
$$\text{Prob}\{\beta_k=0\}=1-\mathbf{E}\{\beta_k\}=1-\beta, \tag{8}$$

where $\beta\in[0,\ 1]$ is a known constant.

Remark 1. In our paper, as depicted in Fig. 1, it is assumed that the packet losses occur in controller-to-actuator and sensor-to-controller communication links. In this case, two stochastic variables $\alpha(k)$, $\beta(k)$, which follow the Bernoulli distribution, are respectively introduced to model the packet losses.

For notational simplicity, in the sequel, for every $\theta_k=i$, we denote $A(\theta_k)$ by A_i, $B_1(\theta_k)$ by B_{1i}, $B_2(\theta_k)$ by B_{2i}, $C(\theta_k)$ by C_i, $D(\theta_k)$ by D_i, $H(\theta_k)$ by H_i, $K(\theta_k)$ by K_i, and $F(\theta_k)$ by F_i. Define $e(k)=x(k)-\overline{x}(k)$, and $\zeta(k)=\begin{bmatrix}x^T(k) & e^T(k)\end{bmatrix}^T$, then the corresponding closed-loop error systems formed by system (1) and controller (7) can be written as follows:

$$\begin{cases}\overline{E}\zeta(k+1)=\overline{A}_i\zeta(k)+\overline{B}_i\omega(k),\\ z(k)=\overline{C}_i\zeta(k)+\overline{D}_i\omega(k),\end{cases} \tag{9}$$

where

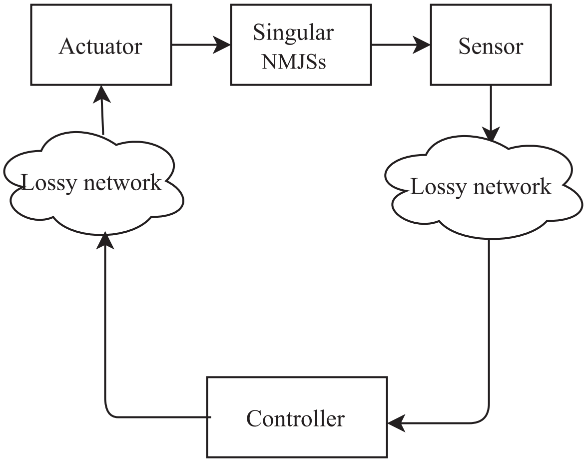

Fig. 1. Block diagram of networked singular NMJSs.

$$\overline{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \overline{B}_i = \begin{bmatrix} B_{2i} \\ B_{2i} \end{bmatrix}, \overline{C}_i = \begin{bmatrix} H_i & 0 \end{bmatrix},$$
$$\overline{A}_i = \begin{bmatrix} A_i + \beta_k B_{1i} K_i & -\beta_k B_{1i} K_i \\ 0 & A_i - \alpha_k F_i C_i \end{bmatrix}, \overline{D}_i = D_i. \tag{10}$$

Before establishing the main results, we first recall the following lemmas and definitions:

Definition 1 [21,23]**.** System (9) with $\omega(k) = 0$ is said to be

(i) regular if $\det(s\overline{E} - \overline{A}_i) \not\equiv 0$ for $\forall\ i \in \mathcal{S}$,
(ii) causal if degree $\{\det(s\overline{E} - \overline{A}_i)\}=$ rank$(\overline{E})$ for $\forall\ i \in \mathcal{S}$.

Definition 2 [28]**.** (singular stochastic finite-time boundedness (SSFTB)) System (9) is said to be SSFTB with respect to (c_1, c_2, G_i, N, d), where $0 < c_1 < c_2$, $G_i > 0$ and $N \in \mathbb{Z}$, if system (9) is regular and causal, and satisfies

$$\begin{aligned} &\mathbf{E}\{\zeta^T(0)\overline{E}^T G_i \overline{E}\zeta(0)\} \le c_1^2 \\ \Rightarrow\quad &\mathbf{E}\{\zeta^T(k)\overline{E}^T G_i \overline{E}\zeta(k)\} < c_2^2, \quad \forall k \in 1, 2, \ldots, N. \end{aligned} \tag{11}$$

Definition 3 [28]**.** (singular stochastic H_∞ finite-time boundedness (SSH_∞FTB)) System (9) is said to be SSH_∞FTB with respect to $(c_1, c_2, G_i, N, d, \gamma)$, where γ is a prescribed positive scalar, if system (9) is SSFTB with respect to (c_1, c_2, G_i, N, d) and under zero initial condition, the controlled output $z(k)$ satisfies

$$\mathbf{E}\left\{\sum_{k=0}^{N} z^T(k)z(k)\right\} < \gamma^2 \sum_{k=0}^{N} \omega^T(k)\omega(k). \tag{12}$$

Lemma 1 [3,6]**.** *The following conditions are equivalent.*

(1) *There exists a symmetric matrix $P > 0$ such that*

$$A^T P A - P < 0.$$

(2) *There exist a symmetric matrix* P *and* $\mathcal{G}$ *such that*

$$\begin{bmatrix} P & (\mathcal{G}A)^T \\ * & sym(\mathcal{G}) - P \end{bmatrix} > 0.$$

Lemma 2 [44]**.** *Given any real matrices* X, Y *and* Z *with appropriate dimensions and such that* $Y > 0$ *and symmetric. Then, we have*

$$X^T Z + Z^T X \le X^T Y X + Z^T Y^{-1} Z.$$

The purpose of the paper is to design the observer-based controllers in the form of Eq. (7) for system (1) such that system (9) is $SSH_\infty FTB$ *with respect to* $(c_1, c_2, G_i, N, d, \gamma)$.

3. Main results

In this section, taking into account the influence of the packet losses, we aim to study the SSH_∞FTB problem for system (9). The result is given in the following theorem:

Theorem 1. *For given scalars* $\mu \ge 1$, $c_1 > 0$, $N > 0$, $d > 0$, *and matrices* $G_i > 0$, *system (9) is* $SSH_\infty FTB$ *with respect to* $(c_1, c_2, G_i, N, d, \gamma)$, *where* $\gamma = \sqrt{\rho\mu^N}$, *if there exist constants* $c_2 > 0$, $\lambda_2 > 0$, $\rho > 0$, *a set of positive definite symmetric matrices* $\overline{P}_i^s$ *and* $\overline{P}_{il}^s$, *matrices* U_i, V_i, J_i, $\forall i, j \in \mathcal{S}$, *such that*

$$\begin{bmatrix} \Phi_{1i} & \Phi_{2i} & U_i\overline{B}_i & \overline{C}_i^T \\ * & -V_i - V_i^T + \overline{P}_{il}^s & V_i\overline{B}_i & 0 \\ * & * & -\rho I & \overline{D}_i^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{13}$$

$$G_i < \overline{P}_i^s < \lambda_2 G_i, \tag{14}$$

$$\lambda_2 c_1^2 + \rho d^2 < \mu^{-N} c_2^2, \tag{15}$$

where

$$\begin{cases} \Phi_{1i} = sym\{U_i(\overline{A}_i - \overline{E})\} + \overline{E}^T \overline{P}_{il}^s \overline{E} - \mu \overline{E}^T \overline{P}_i^s \overline{E}, \\ \Phi_{2i} = -U_i + (\overline{A}_i - \overline{E})^T V_i^T + \overline{E}^T \overline{P}_{il}^s + J_i S_i^T, \\ \overline{P}_{il}^s = \sum\limits_{j=1}^{S} \sum\limits_{s=1}^{\mathcal{M}} \sum\limits_{l=1}^{\mathcal{M}} \alpha_s(k)\sigma_l(k)\pi_{ij}^s P_j^l, \\ \overline{P}_i^s = \sum\limits_{s=1}^{\mathcal{M}} \alpha_s(k) P_i^s. \end{cases}$$

$\overline{A}_i$ *is defined as in Eq. (10) with* α_k, β_k *replaced by* α, β. $S_i \in \mathbb{R}^{(2n)\times(2n-2r)}$ *is the arbitrary matrix satisfying* $\overline{E}^T S_i = 0$ *and rank* $(S_i) = 2n - 2r$.

Proof. Firstly, we prove that system (9) with $\omega(k) = 0$ is regular and causal.

From Eq. (13), we have

$$\begin{bmatrix} \Phi_{1i} & \Phi_{2i} \\ * & -V_i - V_i^T + \overline{P}_{il}^s \end{bmatrix} < 0. \tag{16}$$

Setting $\mathscr{G}_i = \begin{bmatrix} I & (\overline{A}_i - \overline{E})^T \end{bmatrix}$, which is of full row rank. Then we pre- and post-multiply Eq. (16) by $\mathscr{G}_i$ and $\mathscr{G}_i^T$, it follows that:

$$\overline{A}_i^T \overline{P}_{il}^s \overline{A}_i + sym\{\overline{A}_i^T S_i J_i^T\} - \mu \overline{E}^T \overline{P}_i^s \overline{E} < 0. \tag{17}$$

Now we choose two nonsingular matrices $\mathscr{K}_i$ and $\mathscr{N}_i$ such that

$$\begin{aligned}
\mathscr{K}_i \overline{E} \mathscr{N}_i &= \begin{bmatrix} I_{2r} & 0 \\ 0 & 0 \end{bmatrix}, \mathscr{K}_i \overline{A}_i \mathscr{N}_i = \begin{bmatrix} \overline{A}_{1i} & \overline{A}_{2i} \\ \overline{A}_{3i} & \overline{A}_{4i} \end{bmatrix}, \\
\mathscr{K}_i^{-T} \overline{P}_{il}^s \mathscr{K}_i^{-1} &= \begin{bmatrix} \overline{P}_{1il}^s & \overline{P}_{2il}^s \\ * & \overline{P}_{3il}^s \end{bmatrix}, \\
\mathscr{K}_i^{-T} \overline{P}_i^s \mathscr{K}_i^{-1} &= \begin{bmatrix} \overline{P}_{1i}^s & \overline{P}_{2i}^s \\ * & \overline{P}_{3i}^s \end{bmatrix}, \\
\mathscr{K}_i^{-T} S_i &= \begin{bmatrix} S_{1i} \\ S_{2i} \end{bmatrix}, J_i^T \mathscr{N}_i = \begin{bmatrix} J_{1i} & J_{2i} \end{bmatrix}.
\end{aligned} \tag{18}$$

From $\overline{E}^T S_i = 0$, it follows that $S_{1i} = 0$. Then pre- and post-multiply Eq. (17) by $\mathscr{N}_i^T$ and $\mathscr{N}_i$, it follows from Eq. (18) that:

$$\begin{bmatrix} \bigstar & \bigstar \\ \bigstar & \Delta_i \end{bmatrix} < 0, \tag{19}$$

where

$$\Delta_i = \overline{A}_{2i}^T \overline{P}_{1il}^s \overline{A}_{2i} + sym\{\overline{A}_{4i}^T (\overline{P}_{2il}^s)^T \overline{A}_{2i} + \overline{A}_{4i}^T S_{2i} J_{2i}\} + \overline{A}_{4i}^T \overline{P}_{3il}^s \overline{A}_{4i}.$$

From $\overline{P}_{1il}^s > 0$ and Eq. (19), it is obtained that

$$sym\{\overline{A}_{4i}^T (\overline{P}_{2il}^s)^T \overline{A}_{2i} + \overline{A}_{4i}^T S_{2i} J_{2i}\} + \overline{A}_{4i}^T \overline{P}_{3il}^s \overline{A}_{4i} < 0. \tag{20}$$

Then, Eq. (20) implies that $\overline{A}_{4i}$ is nonsingular. From Definition 1 and reference [18], we have system (9) with $\omega(k) = 0$ is regular and causal.

Next, we prove that system (9) is SSFTB, i.e. Eq. (11) holds. To this end, we assume that the left hand side of Eq. (11) holds for the given $c_1 > 0$. We construct the Lyapunov functional as:

$$V(\zeta(k), \theta_k, \alpha_k) = \zeta^T(k) \overline{E}^T \overline{P}_i^s \overline{E} \zeta(k) = \sum_{s=1}^{\mathcal{M}} \alpha_s(k) \zeta^T(k) \overline{E}^T P_i^s \overline{E} \zeta(k), \tag{21}$$

where $P_i^s > 0, \forall i \in \mathcal{S}$. Then we have

$$\mathbf{E}[V(\zeta(k+1), \theta_{k+1}, \alpha_{k+1}) | \theta_k, \alpha_k] = \sum_{j=1}^{S} \sum_{s=1}^{\mathcal{M}} \sum_{s=1}^{\mathcal{M}} \alpha_s(k) \alpha_s(k+1) \pi_{ij}^s \zeta^T(k+1) \overline{E}^T P_j^s \overline{E} \zeta(k+1). \tag{22}$$

Denote

$$\sum_{s=1}^{\mathcal{M}} \alpha_s(k+1)P_j^s = \sum_{l=1}^{\mathcal{M}} \sigma_l(k)P_j^l, \tag{23}$$

where $0 \leq \sigma_l(k) \leq 1$, $\sum_{l=1}^{\mathcal{M}} \sigma_l(k) = 1$, we rewrite Eq. (22) as

$$\mathbf{E}[V(\zeta(k+1), \theta_{k+1}, \alpha_{k+1})|\theta_k, \alpha_k] = \zeta^T(k+1)\overline{E}^T\overline{P}_{il}^s\overline{E}\zeta(k+1), \tag{24}$$

where $\overline{P}_{il}^s = \sum_{j=1}^{S}\sum_{s=1}^{\mathcal{M}}\sum_{l=1}^{\mathcal{M}} \alpha_s(k)\sigma_l(k)\pi_{ij}^s P_j^l$.

Let

$$\tau(k) = \overline{E}\zeta(k+1) - \overline{E}\zeta(k). \tag{25}$$

From Eq. (25), we rewrite Eq. (24) as the following equivalent form:

$$\begin{aligned}&\mathbf{E}[V(\zeta(k+1), \theta_{k+1}, \alpha_{k+1})|\theta_k, \alpha_k]\\ &\quad = (\overline{E}\zeta(k) + \tau(k))^T\overline{P}_{il}^s(\overline{E}\zeta(k) + \tau(k)).\end{aligned} \tag{26}$$

From Eq. (9) and Eq. (25), it follows that:

$$(\overline{A}_i - \overline{E})\zeta(k) - \tau(k) + \overline{B}_i\omega(k) = 0, \tag{27}$$

then for any matrices U_i and V_i, we get

$$2\varpi(k)\mathcal{M}_i^T\{(\overline{A}_i - \overline{E})\zeta(k) - \tau(k) + \overline{B}_i\omega(k)\} = 0, \tag{28}$$

where

$$\begin{aligned}&\varpi(k) = \begin{bmatrix}\zeta^T(k) & \tau^T(k) & \omega^T(k)\end{bmatrix},\\ &\mathcal{M}_i = \begin{bmatrix}U_i^T & V_i^T & 0\end{bmatrix}.\end{aligned}$$

Noting that $2\tau^T(k)S_iJ_i^T\zeta(k) = 0$, along with Eqs. (25) and (27), it is obtained that

$$\mathbf{E}[V(\zeta(k+1), \theta_{k+1}, \alpha_{k+1})|\theta_k, \alpha_k] = \varpi(k)\Theta_i\varpi^T(k), \tag{29}$$

where

$$\Theta_i = \begin{bmatrix}\Theta_{1i} & \Phi_{2i} & U_i\overline{B}_i\\ * & -V_i - V_i^T + \overline{P}_{il}^s & V_i\overline{B}_i\\ * & * & 0\end{bmatrix},$$

$$\Theta_{1i} = sym\{U_i(\overline{A}_i - \overline{E})\} + \overline{E}^T\overline{P}_{il}^s\overline{E}.$$

From Eq. (13), we have

$$\mathbf{E}[V(\zeta(k+1), \theta_{k+1}, \alpha_{k+1})|\theta_k, \alpha_k] \leq \mu V(\zeta(k), \theta_k, \alpha_k) + \rho\omega^T(k)\omega(k). \tag{30}$$

Further, we iterate this process of Eq. (29), and it follows from Eq. (2) that

$$\begin{aligned}\mathbf{E}[V(\zeta(k), \theta_k, \alpha_k)] &\leq \mu\mathbf{E}[V(\zeta(k-1), \theta_{k-1}, \alpha_{k-1})] + \rho\mathbf{E}[\omega^T(k-1)\omega(k-1)]\\ &\quad \cdots\cdots\\ &\leq \mu^k\mathbf{E}[V(\zeta(0), \theta_0, \alpha_0)] + \rho\mathbf{E}\left[\sum_{n=0}^{k-1}\mu^{k-1-n}\omega^T(n)\omega(n)\right]\\ &\leq \mu^k\mathbf{E}[V(\zeta(0), \theta_0, \alpha_0)] + \rho\mu^k d^2.\end{aligned} \tag{31}$$

Define $\mathcal{P}_i^s = G_i^{-\frac{1}{2}} \overline{P}_i^s G_i^{-\frac{1}{2}}$, considering $\mathbf{E}\{\zeta^T(0)\overline{E}^T G_i \overline{E}\zeta(0)\} \leq c_1^2$ and Eq. (14), it is obtained that

$$\begin{aligned}\mathbf{E}[V(\zeta(0), \theta_0, \alpha_0)] &= \mathbf{E}[\zeta^T(0)\overline{E}^T \overline{P}_i^s \overline{E}\zeta(0)] \\ &= \mathbf{E}[\zeta^T(0)\overline{E}^T G_i^{\frac{1}{2}} \mathcal{P}_i^s G_i^{\frac{1}{2}} \overline{E}\zeta(0)] \\ &\leq \max_{i\in\mathcal{S}} \lambda_{\max}(\mathcal{P}_i^s)\mathbf{E}\{\zeta^T(0)\overline{E}^T G_i \overline{E}\zeta(0)\} \\ &\leq \lambda_2 c_1^2. \end{aligned} \tag{32}$$

On the other hand, from Eq. (14), it follows that:

$$\begin{aligned}\mathbf{E}[V(\zeta(k), \theta_k, \alpha_k)] &= \mathbf{E}[\zeta^T(k)\overline{E}^T \overline{P}_i^s \overline{E}\zeta(k)] \\ &= \mathbf{E}[\zeta^T(k)\overline{E}^T G_i^{\frac{1}{2}} \mathcal{P}_i^s G_i^{\frac{1}{2}} \overline{E}\zeta(k)] \\ &\geq \mathbf{E}\{\zeta^T(k)\overline{E}^T G_i \overline{E}\zeta(k)\}. \end{aligned} \tag{33}$$

Considering the proof process between Eqs. (30) and (32), we have

$$\mathbf{E}\{\zeta^T(k)\overline{E}^T G_i \overline{E}\zeta(k)\} < \mu^k(\lambda_2 c_1^2 + \rho d^2) < \mu^N(\lambda_2 c_1^2 + \rho d^2).$$

Then one obtains from Eq. (15) that $\mathbf{E}\{\zeta^T(k)\overline{E}^T G_i \overline{E}\zeta(k)\} < c_2^2, \forall k \in \{1, 2, \ldots, N\}$. Based on this, it is obtained that system (9) is SSFTB with respect to (c_1, c_2, G_i, N, d).

Finally, we discuss the H_∞ performance of system (9), that is, under zero initial condition, Eq. (12) holds. From Eq. (13), it is obtained that

$$\mathbf{E}[V(\zeta(k+1), \theta_{k+1}, \alpha_{k+1})] \leq \mu V(\zeta(k), \theta_k, \alpha_k) + \rho\omega^T(k)\omega(k) - z^T(k)z(k). \tag{34}$$

In the following, we give the iteration process of Eq. (34)

$$\begin{aligned}&\mathbf{E}[V(\zeta(k), \theta_k, \alpha_k)] \\ &\quad \leq \mu\mathbf{E}[V(\zeta(k-1), \theta_{k-1}, \alpha_{k-1})] + \rho\mathbf{E}[\omega^T(k-1)\omega(k-1)] - \mathbf{E}[z^T(k-1)z(k-1)] \\ &\quad \cdots\cdots \\ &\quad \leq \mu^k\mathbf{E}[V(\zeta(0), \theta_0, \alpha_0)] + \rho\mathbf{E}\left[\sum_{n=0}^{k-1}\mu^{k-1-n}\omega^T(n)\omega(n)\right] - \mathbf{E}\left[\sum_{n=0}^{k-1}\mu^{k-1-n}z^T(n)z(n)\right]. \end{aligned} \tag{35}$$

Then under zero initial condition, together with $V(\zeta(k), \theta_k, \alpha_k) \geq 0$, it follows from Eq. (35) that

$$\rho\mathbf{E}\left[\sum_{n=0}^{k-1}\mu^{k-1-n}\omega^T(n)\omega(n)\right] \geq \mathbf{E}\left[\sum_{n=0}^{k-1}\mu^{k-1-n}z^T(n)z(n)\right], \tag{36}$$

Since $\mu \geq 1$, it implies from Eq. (36) that

$$\begin{aligned}\mathbf{E}\left[\sum_{n=0}^{k-1} z^T(n)z(n)\right] &\leq \mathbf{E}\left[\sum_{n=0}^{k-1} \mu^{k-1-n} z^T(n)z(n)\right] \\ &\leq \rho\mathbf{E}\left[\sum_{n=0}^{k-1} \mu^{k-1-n}\omega^T(n)\omega(n)\right] \\ &\leq \rho\mu^{k-1}\mathbf{E}\left[\sum_{n=0}^{k-1} \omega^T(n)\omega(n)\right],\end{aligned} \tag{37}$$

which further implies that

$$\mathbf{E}\left[\sum_{n=0}^{N} z^T(n)z(n)\right] \leq \gamma^2\mathbf{E}\left[\sum_{n=0}^{N} \omega^T(n)\omega(n)\right],$$

with $\gamma = \sqrt{\rho\mu^N}$. Thus, by Definition 3, system (9) is SSH_∞FTB with respect to (c_1, c_2, G_i, N, d, γ). The proof is completed. □

Remark 2. A sufficient condition on SSH_∞FTB for a class of singular NMJSs with packet losses is presented in Theorem 1. In contrast to [30–32], where the traditional inequality $P_1P^{-1}P_1 \geq 2P_1 - P$ is introduced to eliminate the coupling between Lyapunov variables and system matrices, the slack variables U_i and V_i in Eq. (28) are used in the paper. It is noted that this alternative controller synthesis method will give less conservative results by means of the incremental flexibility from Lyapunov variables and additional slack variables.

We have analyzed the SSH_∞FTB problem of system (9). On the basis of the obtained results, we will design the parameters for the observer-based controller of Eq. (7) in terms of strict LMIs.

Theorem 2. *For given scalars $\mu \geq 1$, α, β, a_1, a_2, a_3, a_4, $c_1 > 0$, $N > 0$, $d > 0$, and matrices $G_i > 0$, system (9) is SSH_∞FTB with respect to (c_1, c_2, G_i, N, d, γ), where $\gamma = \sqrt{\rho\mu^N}$, if there exist constants $c_2 > 0$, $\lambda_2 > 0$, $\rho > 0$, matrices P^s_{1i}, P^s_{2i}, P^s_{3i}, U_{1i}, U_{2i}, V_{1i}, V_{2i}, J_{1i}, J_{2i}, J_{3i}, J_{4i}, Y_i, $Q_i > 0$, L_i, T_i, N_i, $\forall i, j \in \mathcal{S}$, such that Eq. (15) and*

$$\begin{bmatrix} \Pi_i + Q_i & \beta\mathcal{W}_i^T L_i^T & 0 \\ * & sym\{-N_i\} & \mathcal{T}_i^T \\ * & * & -Q_i \end{bmatrix} < 0, \tag{38}$$

$$G_i < \begin{bmatrix} P^s_{1i} & P^s_{2i} \\ * & P^s_{3i} \end{bmatrix} < \lambda_2 G_i, \tag{39}$$

where

$$
\begin{cases}
\Pi_i = \begin{bmatrix} \Pi_{1i} & \Pi_{2i} & \Pi_{3i} & \Pi_{4i} & \Pi_{5i} & H_i^T \\ * & \Pi_{6i} & \Pi_{7i} & \Pi_{8i} & \Pi_{9i} & 0 \\ * & * & \Pi_{10i} & \Pi_{11i} & \Pi_{12i} & 0 \\ * & * & * & \Pi_{13i} & \Pi_{14i} & 0 \\ * & * & * & * & -\rho I & D_i^T \\ * & * & * & * & * & -I \end{bmatrix}, \\
\Pi_{1i} = sym\{U_{1i}(A_i - E) + \beta B_{1i} L_i\} + E^T \mathcal{P}_{1i} E - \mu E^T P_{1i}^s E, \\
\Pi_{2i} = -\beta B_{1i} L_i - a_1 \alpha T_i C_i + a_1 Y_i (A_i - E) + \beta L_i^T B_{1i}^T + (A_i - E)^T U_{2i}^T + E^T \mathcal{P}_{2i} E - \mu E^T P_{2i}^s E, \\
\Pi_{3i} = -U_{1i} + \beta L_i^T B_{1i}^T + (A_i - E)^T V_{1i}^T + E^T \mathcal{P}_{1i} + J_{1i} R_i^T, \\
\Pi_{4i} = -a_1 Y_i + \beta L_i^T B_{1i}^T + (A_i - E)^T V_{2i}^T + E^T \mathcal{P}_{2i} + J_{2i} R_i^T, \\
\Pi_{5i} = U_{1i} B_{2i} + a_1 Y_i B_{2i}, \\
\Pi_{6i} = sym\{-\beta B_{1i} L_i - a_2 \alpha T_i C_i + a_2 Y_i (A_i - E)\} + E^T \mathcal{P}_{3i} E - \mu E^T P_{3i}^s E, \\
\Pi_{7i} = -U_{2i} - \beta L_i^T B_{1i}^T - a_3 \alpha C_i^T T_i^T + a_3 (A_i - E)^T Y_i^T + E^T \mathcal{P}_{2i}^T + J_{3i} R_i^T, \\
\Pi_{8i} = -a_2 Y_i - \beta L_i^T B_{1i}^T - a_4 \alpha C_i^T T_i^T + a_4 (A_i - E)^T Y_i^T + E^T \mathcal{P}_{3i} + J_{4i} R_i^T, \\
\Pi_{9i} = U_{2i} B_{2i} + a_2 Y_i B_{2i}, \\
\Pi_{10i} = -V_{1i} - V_{1i}^T + \mathcal{P}_{1i}, \\
\Pi_{11i} = -a_3 Y_i - V_{2i}^T + \mathcal{P}_{2i}, \\
\Pi_{12i} = V_{1i} B_{2i} + a_3 Y_i B_{2i}, \\
\Pi_{13i} = -a_4 Y_i - a_4 Y_i^T + \mathcal{P}_{3i}, \\
\Pi_{14i} = V_{2i} B_{2i} + a_4 Y_i B_{2i}, \\
\mathcal{T}_i = \begin{bmatrix} B_{1i}^T U_{1i}^T - N_i^T B_{1i}^T & B_{1i}^T U_{2i}^T - N_i^T B_{1i}^T & B_{1i}^T V_{1i}^T - N_i^T B_{1i}^T & B_{1i}^T V_{2i}^T - N_i^T B_{1i}^T & 0 & 0 \end{bmatrix}^T, \\
\mathcal{W}_i = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathcal{P}_{\iota i} = \sum_{j=1}^{S} \pi_{ij}^s P_{\iota j}^l, \ \iota = 1, 2, 3.
\end{cases}
$$

$R_i \in \mathbb{R}^{n\times(n-r)}$ is the arbitrary matrix satisfying $E^T R_i = 0$ and rank $(R_i) = n - r$. Moreover, the state feedback controller gains are $K_i = N_i^{-1} L_i$ and the observer gains are $F_i = Y_i^{-1} T_i$.

Proof. Firstly, from Eq. (37), we have $-Y_i - Y_i^T + \mathcal{P}_{3i} < 0$ and $-N_i - N_i^T < 0$. Since $\mathcal{P}_{3i} > 0$, it implies that Y_i and N_i are nonsingular. From $K_i = N_i^{-1} L_i$ and $F_i = Y_i^{-1} T_i$, it is obtained that

$$
L_i = N_i K_i, \quad T_i = Y_i F_i. \tag{40}
$$

Setting

$$
\begin{aligned}
U_i &= \begin{bmatrix} U_{1i} & a_1 Y_i \\ U_{2i} & a_2 Y_i \end{bmatrix}, \quad V_i = \begin{bmatrix} V_{1i} & a_3 Y_i \\ V_{2i} & a_4 Y_i \end{bmatrix}, \\
J_i &= \begin{bmatrix} J_{1i} & J_{2i} \\ J_{3i} & J_{4i} \end{bmatrix}, \quad S_i = \begin{bmatrix} R_i & 0 \\ 0 & R_i \end{bmatrix}, \\
P_i^s &= \begin{bmatrix} P_{1i}^s & P_{2i}^s \\ * & P_{3i}^s \end{bmatrix}, \overline{P}_i^s = \begin{bmatrix} \overline{P}_{1i}^s & \overline{P}_{2i}^s \\ * & \overline{P}_{3i}^s \end{bmatrix}, \overline{P}_{il}^s = \begin{bmatrix} \overline{P}_{1il}^s & \overline{P}_{2il}^s \\ * & \overline{P}_{3il}^s \end{bmatrix}.
\end{aligned} \tag{41}
$$

Then, we substitute Eq. (10) with α_k, β_k replaced by α, β, Eqs. (39) and (40) into Eq. (13), we have

$$\begin{bmatrix} \Delta_{1i} & \Delta_{2i} & \Delta_{3i} & \Delta_{4i} & \Pi_{5i} & H_i^T \\ * & \Delta_{6i} & \Delta_{7i} & \Delta_{8i} & \Pi_{9i} & 0 \\ * & * & \Delta_{10i} & \Delta_{11i} & \Pi_{12i} & 0 \\ * & * & * & \Delta_{13i} & \Pi_{14i} & 0 \\ * & * & * & * & -\rho I & D_i^T \\ * & * & * & * & * & -I \end{bmatrix} < 0,$$

where

$$\begin{cases} \Delta_{1i} = sym\{U_{1i}(A_i + \beta B_{1i}K_i - E)\} + E^T\overline{P}^s_{1il}E - \mu E^T\overline{P}^s_{1i}E, \\ \Delta_{2i} = -\beta U_{1i}B_{1i}K_i - a_1\alpha T_iC_i + a_1Y_i(A_i - E) + (A_i + \beta B_{1i}K_i - E)^TU_{2i}^T + E^T\overline{P}^s_{2il}E - \mu E^T\overline{P}^s_{2i}E, \\ \Delta_{3i} = -U_{1i} + (A_i + \beta B_{1i}K_i - E)^TV_{1i}^T + E^T\overline{P}^s_{1il} + J_{1i}R_i^T, \\ \Delta_{4i} = -a_1Y_i + (A_i + \beta B_{1i}K_i - E)^TV_{2i}^T + E^T\overline{P}^s_{2il} + J_{2i}R_i^T, \\ \Delta_{6i} = sym\{-\beta U_{2i}B_{1i}K_i - a_2\alpha T_iC_i + a_2Y_i(A_i - E)\} + E^T\overline{P}^s_{3il}E - \mu E^T\overline{P}^s_{3i}E, \\ \Delta_{7i} = -U_{2i} - \beta K_i^TB_{1i}^TV_{1i}^T - a_3\alpha C_i^TT_i^T + a_3(A_i - E)^TY_i^T + E^T(\overline{P}^s_{2il})^T + J_{3i}R_i^T, \\ \Delta_{8i} = -a_2Y_i - \beta K_i^TB_{1i}^TV_{2i}^T - a_4\alpha C_i^TT_i^T + a_4(A_i - E)^TY_i^T + E^T\overline{P}^s_{3il} + J_{4i}R_i^T, \\ \Delta_{10i} = -V_{1i} - V_{1i}^T + \overline{P}^s_{1il}, \\ \Delta_{11i} = -a_3Y_i - V_{2i}^T + \overline{P}^s_{2il}, \\ \Delta_{13i} = -a_4Y_i - a_4Y_i^T + \overline{P}^s_{3il}. \end{cases}$$

By considering the nature of the convex combination, the above inequality holds if the following inequality is satisfied:

$$\begin{bmatrix} \Xi_{1i} & \Xi_{2i} & \Xi_{3i} & \Xi_{4i} & \Pi_{5i} & H_i^T \\ * & \Xi_{6i} & \Xi_{7i} & \Xi_{8i} & \Pi_{9i} & 0 \\ * & * & \Pi_{10i} & \Pi_{11i} & \Pi_{12i} & 0 \\ * & * & * & \Pi_{13i} & \Pi_{14i} & 0 \\ * & * & * & * & -\rho I & D_i^T \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{42}$$

where

$$\begin{cases} \Xi_{1i} = sym\{U_{1i}(A_i + \beta B_{1i}K_i - E)\} + E^T\mathcal{P}_{1i}E - \mu E^TP^s_{1i}E, \\ \Xi_{2i} = -\beta U_{1i}B_{1i}K_i - a_1\alpha T_iC_i + a_1Y_i(A_i - E) + (A_i + \beta B_{1i}K_i - E)^TU_{2i}^T + E^T\mathcal{P}_{2i}E - \mu E^TP^s_{2i}E, \\ \Xi_{3i} = -U_{1i} + (A_i + \beta B_{1i}K_i - E)^TV_{1i}^T + E^T\mathcal{P}_{1i} + J_{1i}R_i^T, \\ \Xi_{4i} = -a_1Y_i + (A_i + \beta B_{1i}K_i - E)^TV_{2i}^T + E^T\mathcal{P}_{2i} + J_{2i}R_i^T, \\ \Xi_{6i} = sym\{-\beta U_{2i}B_{1i}K_i - a_2\alpha T_iC_i + a_2Y_i(A_i - E)\} + E^T\mathcal{P}_{3i}E - \mu E^TP^s_{3i}E, \\ \Xi_{7i} = -U_{2i} - \beta K_i^TB_{1i}^TV_{1i}^T - a_3\alpha C_i^TT_i^T + a_3(A_i - E)^TY_i^T + E^T\mathcal{P}_{2i}^T + J_{3i}R_i^T, \\ \Xi_{8i} = -a_2Y_i - \beta K_i^TB_{1i}^TV_{2i}^T - a_4\alpha C_i^TT_i^T + a_4(A_i - E)^TY_i^T + E^T\mathcal{P}_{3i} + J_{4i}R_i^T. \end{cases}$$

Now, we decouple the terms in Eq. (41), Eq. (41) is rewritten as the following equivalent form:

$$\begin{bmatrix} \Lambda_{1i} & \Lambda_{2i} & \Lambda_{3i} & \Lambda_{4i} & \Pi_{5i} & H_i^T \\ * & \Lambda_{6i} & \Lambda_{7i} & \Lambda_{8i} & \Pi_{9i} & 0 \\ * & * & \Pi_{10i} & \Pi_{11i} & \Pi_{12i} & 0 \\ * & * & * & \Pi_{13i} & \Pi_{14i} & 0 \\ * & * & * & * & -\rho I & D_i^T \\ * & * & * & * & * & -I \end{bmatrix} + sym\{\beta \mathcal{Q}_i K_i \mathcal{W}_i\} < 0, \tag{43}$$

where

$$\begin{cases} \Lambda_{1i} = sym\{U_{1i}(A_i - E)\} + E^T \mathcal{P}_{1i} E - \mu E^T P_{1i}^s E, \\ \Lambda_{2i} = -a_1 \alpha T_i C_i + a_1 Y_i (A_i - E) + (A_i - E)^T U_{2i}^T + E^T \mathcal{P}_{2i} E - \mu E^T P_{2i}^s E, \\ \Lambda_{3i} = -U_{1i} + (A_i - E)^T V_{1i}^T + E^T \mathcal{P}_{1i} + J_{1i} R_i^T, \\ \Lambda_{4i} = -a_1 Y_i + (A_i - E)^T V_{2i}^T + E^T \mathcal{P}_{2i} + J_{2i} R_i^T, \\ \Lambda_{6i} = sym\{-a_2 \alpha T_i C_i + a_2 Y_i (A_i - E)\} + E^T \mathcal{P}_{3i} E - \mu E^T P_{3i}^s E, \\ \Lambda_{7i} = -U_{2i} - a_3 \alpha C_i^T T_i^T + a_3 (A_i - E)^T Y_i^T + E^T \mathcal{P}_{2i}^T + J_{3i} R_i^T, \\ \Lambda_{8i} = -a_2 Y_i - a_4 \alpha C_i^T T_i^T + a_4 (A_i - E)^T Y_i^T + E^T \mathcal{P}_{3i} + J_{4i} R_i^T, \\ \mathcal{Q}_i = \begin{bmatrix} B_{1i}^T U_{1i}^T & B_{1i}^T U_{2i}^T & B_{1i}^T V_{1i}^T & B_{1i}^T V_{2i}^T & 0 & 0 \end{bmatrix}^T. \end{cases}$$

Further, Eq. (42) has the following equivalent form:

$$\Pi_i + sym\{\beta \mathcal{T}_i K_i \mathcal{W}_i\} < 0. \tag{44}$$

On the other hand, from Lemma 2, we have

$$sym\{\beta \mathcal{T}_i K_i \mathcal{W}_i\} \le Q_i + (\beta \mathcal{T}_i K_i \mathcal{W}_i)^T Q_i^{-1} (\beta \mathcal{T}_i K_i \mathcal{W}_i). \tag{45}$$

From Eq. (37), using Schur complement, we have

$$\begin{bmatrix} \Pi_i + Q_i & \beta \mathcal{W}_i^T L_i^T \\ * & sym\{-N_i\} + \mathcal{T}_i^T Q_i^{-1} \mathcal{T}_i \end{bmatrix} < 0. \tag{46}$$

Applying Lemma 1 to Eq. (45) with $\mathcal{G} = N_i$, then Eq. (45) is equivalent to

$$\begin{bmatrix} \Pi_i + Q_i & \beta \mathcal{W}_i^T K_i^T \\ * & -(\mathcal{T}_i^T Q_i^{-1} \mathcal{T}_i)^{-1} \end{bmatrix} < 0, \tag{47}$$

which further implies that

$$\Pi_i + Q_i + (\beta \mathcal{T}_i K_i \mathcal{W}_i)^T Q_i^{-1} (\beta \mathcal{T}_i K_i \mathcal{W}_i) < 0. \tag{48}$$

From Eq. (44), it is obtained that if Eq. (47) holds, then Eq. (43) holds, which further implies that Eq. (13) holds. On the other hand, it follows from Eq. (38) that Eq. (14) holds. Therefore, according to Theorem 1, if Eqs. (15), (37), (38) hold, one has system (9) is SSH_∞FTB with respect to $(c_1, c_2, G_i, N, d, \gamma)$. The proof is completed. □

Remark 3. In Theorem 2, the criterion on finite-time observer-based control is established for a class of discrete-time singular NMJSs with packet losses. In the case of $E = I$, the problem studied in this paper reduces to the finite-time observer-based controller design for NMJSs. Compared with the results in [13], where the Lyapunov variables have a diagonal form, U_i and V_i are introduced in our conditions to eliminate the restriction form. This suggests that our method is less conservative.

Remark 4. It should be pointed out that when $\Pi(k) = \Pi$ for some constant matrix Π, the problem studied in this paper reduces to finite-time observer-based controller design for discrete-time singular MJSs with packet losses. And the result is correspondingly elaborated as follows:

Corollary 1. *For given scalars $\mu \geq 1$, α, β, a_1, a_2, a_3, a_4, $c_1 > 0$, $N > 0$, $d > 0$, and matrices $G_i > 0$, system (9) is SSH$_\infty$FTB with respect to $(c_1, c_2, G_i, N, d, \gamma)$, where $\gamma = \sqrt{\rho\mu^N}$, if there exist constants $c_2 > 0$, $\lambda_2 > 0$, $\rho > 0$, matrices P_{1i}, P_{2i}, P_{3i}, U_{1i}, U_{2i}, V_{1i}, V_{2i}, J_{1i}, J_{2i}, J_{3i}, J_{4i}, Y_i, $Q_i > 0$, L_i, T_i, N_i, $\forall i, j \in \mathcal{S}$, such that Eq. (15) and*

$$\begin{bmatrix} \Gamma_i + Q_i & \beta \mathcal{W}_i^T L_i^T & 0 \\ * & sym\{-N_i\} & \mathcal{T}_i^T \\ * & * & -Q_i \end{bmatrix} < 0, \tag{49}$$

$$G_i < \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} < \lambda_2 G_i, \tag{50}$$

where

$$\begin{cases} \Gamma_i = \begin{bmatrix} \Gamma_{1i} & \Gamma_{2i} & \Gamma_{3i} & \Gamma_{4i} & \Pi_{5i} & H_i^T \\ * & \Gamma_{6i} & \Gamma_{7i} & \Gamma_{8i} & \Pi_{9i} & 0 \\ * & * & \Pi_{10i} & \Pi_{11i} & \Pi_{12i} & 0 \\ * & * & * & \Pi_{13i} & \Pi_{14i} & 0 \\ * & * & * & * & -\rho I & D_i^T \\ * & * & * & * & * & -I \end{bmatrix}, \\ \Gamma_{1i} = sym\{U_{1i}(A_i - E) + \beta B_{1i} L_i\} + E^T \mathbb{P}_{1i} E - \mu E^T P_{1i} E, \\ \Gamma_{2i} = -\beta B_{1i} L_i - a_1 \alpha T_i C_i + a_1 Y_i (A_i - E) + \beta L_i^T B_{1i}^T + (A_i - E)^T U_{2i}^T + E^T \mathbb{P}_{2i} E - \mu E^T P_{2i} E, \\ \Gamma_{3i} = -U_{1i} + \beta L_i^T B_{1i}^T + (A_i - E)^T V_{1i}^T + E^T \mathbb{P}_{1i} + J_{1i} R_i^T, \\ \Gamma_{4i} = -a_1 Y_i + \beta L_i^T B_{1i}^T + (A_i - E)^T V_{2i}^T + E^T \mathbb{P}_{2i} + J_{2i} R_i^T, \\ \Gamma_{6i} = sym\{-\beta B_{1i} L_i - a_2 \alpha T_i C_i + a_2 Y_i (A_i - E)\} + E^T \mathbb{P}_{3i} E - \mu E^T P_{3i} E, \\ \Gamma_{7i} = -U_{2i} - \beta L_i^T B_{1i}^T - a_3 \alpha C_i^T T_i^T + a_3 (A_i - E)^T Y_i^T + E^T \mathbb{P}_{2i}^T + J_{3i} R_i^T, \\ \Gamma_{8i} = -a_2 Y_i - \beta L_i^T B_{1i}^T - a_4 \alpha C_i^T T_i^T + a_4 (A_i - E)^T Y_i^T + E^T \mathbb{P}_{3i} + J_{4i} R_i^T, \\ \mathbb{P}_{\iota i} = \sum_{j=1}^{S} \pi_{ij} P_{\iota j}, \ \ \iota = 1, 2, 3. \end{cases}$$

Π_{5i}, Π_{9i}, Π_{10i}, Π_{11i}, Π_{12i}, Π_{13i}, Π_{14i}, $\mathcal{T}_i$, $\mathcal{W}_i$, R_i are the same as Theorem 2. Moreover, the state feedback controller gains are $K_i = N_i^{-1} L_i$ and the observer gains are $F_i = Y_i^{-1} T_i$.

Remark 5. When there are no packet losses affecting the underlying system, i.e. $\alpha = 1$, $\beta = 1$, Corollary 1 reduces to the finite-time observer-based control criterion for discrete-time singular MJSs. Note that similar problems were investigated in [5,28,31]. However, the equality constraints $C_{yi} X_i = W_i C_{yi}$ in [5], $P_{ia} B_i = B_i \theta_i$ in [31] are involved, which may make it difficult to check the condition numerically. The conditions given in our paper are in the form of strict LMIs without invoking equality constraint, which are reliable and tractable in numerical computation. Compared with [28], where special structure was imposed on the Lyapunov variables, more flexible conditions are given in our paper via introducing slack variables U_i and V_i.

4. Examples

In this section, three numerical examples are given to show the effectiveness of the proposed design methods. First, we present two examples to show the benefit of our methods over the existing ones. Then, an inverted pendulum is provided to illustrate the application of the proposed methodologies.

Example 1. Consider the following MJSs with two operation modes, which were given in [5].

- Mode 1

$$A_1 = \begin{bmatrix} 1.5 & 0 \\ 1.8 & 0.6 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.5 & 1 \\ 0.8 & 1 \end{bmatrix},$$

- Mode 2

$$A_2 = \begin{bmatrix} 1.2 & 1 \\ 0.8 & 1 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$B_{22} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix},$$

The TP matrix is given by

$$\Pi = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}.$$

In addition, similar to [5], we choose $G_1 = G_2 = I_4,\ c_1 = 1,\ N = 5,\ d = 1$. In order to compare with [5], we set $\alpha = 1,\ \beta = 1,\ H_1 = 0,\ H_2 = 0, D_1 = 0,\ D_2 = 0,\ E = I_2,\ R_1 = R_2 = 0$. By using the method in [5], we can find feasible solution when $1.92 \le \mu \le 43.75$. However, according to Corollary 1, we can find a feasible solution when $1.05 \le \mu \le 47.2$, which is larger (better) than the one given in [5]. Particularly, when $\mu = 1.05$, the state feedback controller gains and observer gains are given below:

$$K_1 = \begin{bmatrix} -0.0916 & -0.2708 \\ -0.7964 & -0.2795 \end{bmatrix}, K_2 = \begin{bmatrix} -0.6899 & -0.7332 \\ -0.5569 & -0.5614 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} -0.0061 & 0.6597 \\ 1.1320 & 0.0241 \end{bmatrix}, F_2 = \begin{bmatrix} 0.4267 & 1.0753 \\ 0.9293 & 0.5592 \end{bmatrix}.$$

Example 2. Consider the following singular MJSs with the parameters, which were given in [28].

- Mode 1

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.3 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 0 & 0.2 \end{bmatrix}, D_1 = 0.1,$$

- Mode 2

$$A_2 = \begin{bmatrix} 0.8 & 0.1 \\ 0.7 & 0.5 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 & 0 \\ 0.8 & 0.6 \end{bmatrix},$$
$$B_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, D_1 = 0.1.$$

The singular matrix and TP matrix are given by

$$E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \Pi = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}.$$

Setting $R_1 = R_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$, $G_1 = G_2 = I_4$, $c_1 = 1$, $d = 2$, $\mu = 1$, $N = 6$. It should be noted that there is no feasible solution by using the method given in [28] (Remark 5). However, if we choose $\alpha = 1$, $\beta = 1$, $a_1 = 0$, $a_2 = 1$, $a_3 = 0$, $a_4 = 1$, by using Corollary 1, it is feasible. Then, we have the following state feedback controller gains and observer gains

$$K_1 = \begin{bmatrix} -0.5267 & 0.2214 \\ 0.2214 & -0.6991 \end{bmatrix}, F_1 = \begin{bmatrix} 1.5995 \\ 1.8397 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.4852 & 0.0302 \\ 0.0302 & -0.6534 \end{bmatrix}, F_2 = \begin{bmatrix} 2.0920 \\ 3.9520 \end{bmatrix}.$$

Thus, from the above discussion, it is obtained that the methods given in our paper are less conservative.

Example 3. In this example, we consider a DC motor device driving an inverted pendulum, which is shown in Fig. 2. As noted in [24,27], the following equations of motion are used to represent the inverted pendulum:

$$E\dot{x}(t) \quad = A(\theta_t)x(t) + B_1(\theta_t)u(t).$$

By setting a certain sampling time, such as $T_s = T/10$, we can discretize the obtained continuous-time singular MJSs. The system parameters are given by

- Mode 1

$$A_1 = \begin{bmatrix} -3 & 2.4 \\ 1.4 & 0.8 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} 0.2 & 1.1 \\ 0.2 & 0.6 \end{bmatrix}, C_1 = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} -0.5 & 0.3 \end{bmatrix}, D_1 = \begin{bmatrix} 1.2 & 0.1 \end{bmatrix},$$

- Mode 2

$$A_2 = \begin{bmatrix} -2 & 0.6 \\ 1.6 & 1.2 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.4 \\ 0 \end{bmatrix},$$
$$B_{22} = \begin{bmatrix} 0.1 & 0.5 \\ 0 & 0.5 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} -0.4 & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}.$$

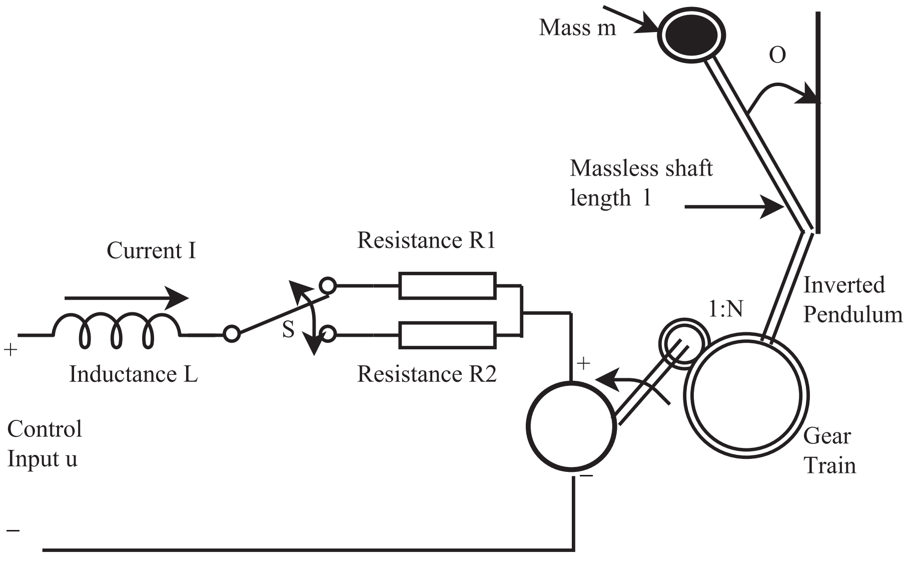

Fig. 2. DC motor controlled inverted pendulum.

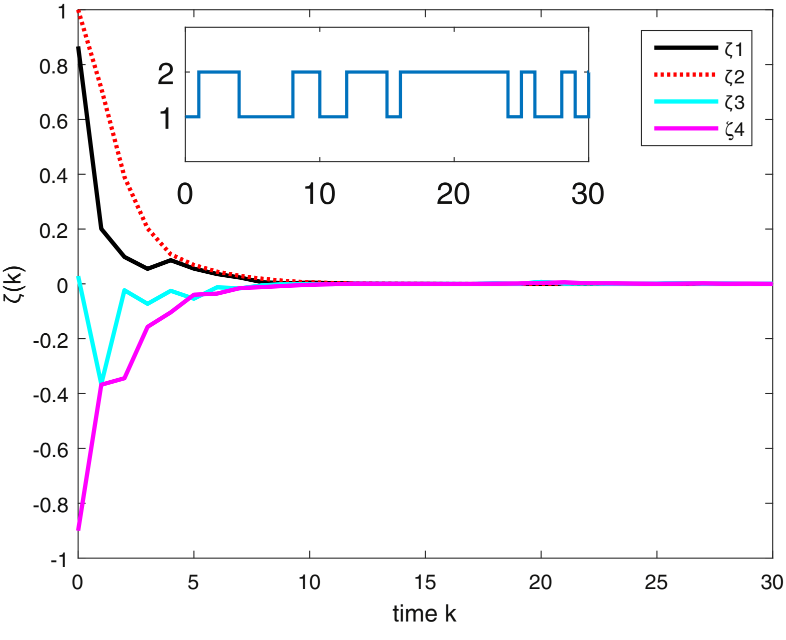

Fig. 3. State responses of the closed-loop error system (9).

The motor is subject to abrupt failures, and the equipment is altered to take these failures into account according to a prescribed Markov chain. The TP matrix is known to be cumbersome in some circumstances but is assumed in [24,27] to be precisely known. In our paper, we assume that the TP matrix is not precisely known but belongs to the polytope defined by

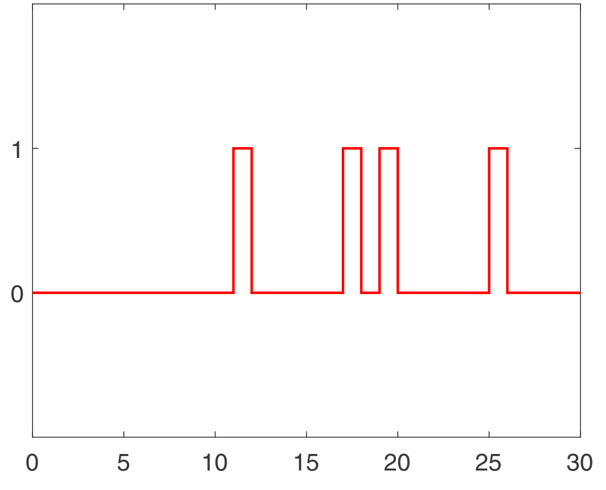

Fig. 4. Data packet losses $\alpha(k)$.

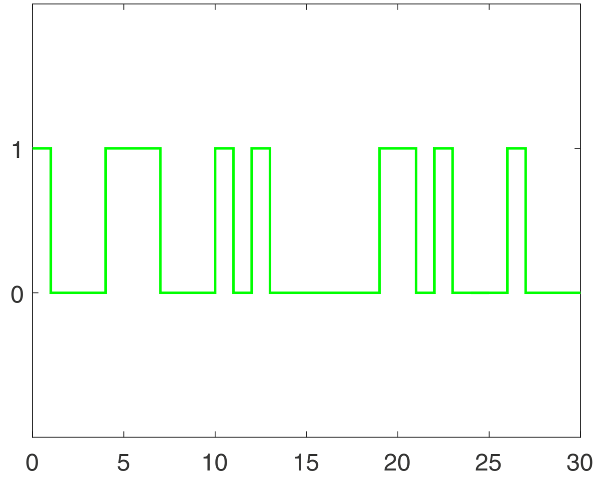

Fig. 5. Data packet losses $\beta(k)$.

the following two vertices:

$$\Pi^1 = \begin{bmatrix} 0.2 & 0.8 \\ 0.65 & 0.35 \end{bmatrix}, \Pi^2 = \begin{bmatrix} 0.6 & 0.4 \\ 0.53 & 0.47 \end{bmatrix}.$$

The singular matrix is given by $E = [\begin{smallmatrix} 0 & 0 \\ 0 & 3 \end{smallmatrix}]$, therefore, we take R_1 and R_2 as $R_1 = R_2 = [\begin{smallmatrix} 2 \\ 0 \end{smallmatrix}]$. Additionally, we suppose $G_1 = G_2 = I_4$, $c_1 = 1$, $d = 2$, $N = 5$, $\mu = 1.01$, $\alpha = \beta = 0.2$, $a_1 = 0.3$, $a_2 = 1$, $a_3 = -0.1$, $a_4 = 1$. According to Theorem 2, solving the LMIs (15), (37), (38), the optimal performance index, γ, is calculated as $\gamma = 5.9351$, the optimal c_2 is $c_2 = 13.1014$, the associated state feedback controller gains and observer gains are given below

$$K_1 = \begin{bmatrix} -1.1037 & 0.2540 \end{bmatrix}, F_1 = \begin{bmatrix} 13.5615 \\ 12.4724 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.3910 & -0.4803 \end{bmatrix}, F_2 = \begin{bmatrix} -31.8526 \\ 34.4809 \end{bmatrix}.$$

For simulation, we choose the disturbance input as $\omega(k) = [\begin{smallmatrix} \frac{\sqrt{2}}{2}\exp(-k)\sin k \\ \frac{\sqrt{2}}{2}\exp(-k)\cos k \end{smallmatrix}]$. Fig. 3 shows the response of states of the closed-loop error systems. From Fig. 3, it can be seen that system (9) is SSH_∞FTB with respect to (1, 13.1014,I_4, 5,2,5.9351).

5. Conclusions

The problem of finite-time observer-based H_∞ control for singular MJSs subject to packet losses has been investigated in this paper. Packet losses, which follow the Bernoulli distribution, occur both in the forward and feedback channels. Based on a stochastic Lyapunov functional and considering the impact of packet losses, a sufficient condition on SSH_∞FTB for the closed-loop error systems is given. Then, the controller gains and the observer gains are designed in terms of strict LMIs from a new perspective. Numerical examples demonstrate the effectiveness of the proposed method. Applying the theoretical results developed in this paper to some practical applications such as bio-economic systems, oil catalytic cracking process, and so on, will be part of our future research. Inspired by [45,46], the adaptive observer-based controller design for singular switched nonlinear systems will be another interesting future research topic.

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